Quantum Theory as operational probabilistic theory: what we have learnt

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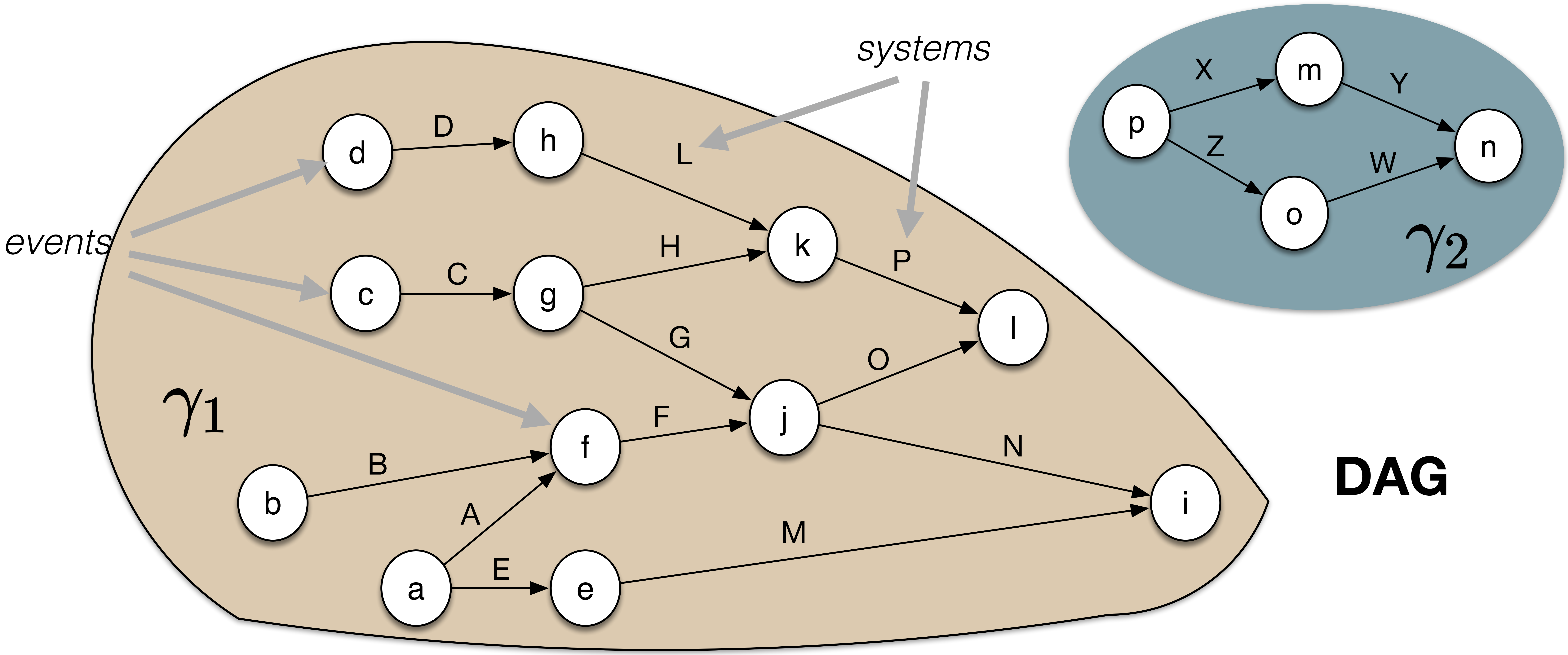
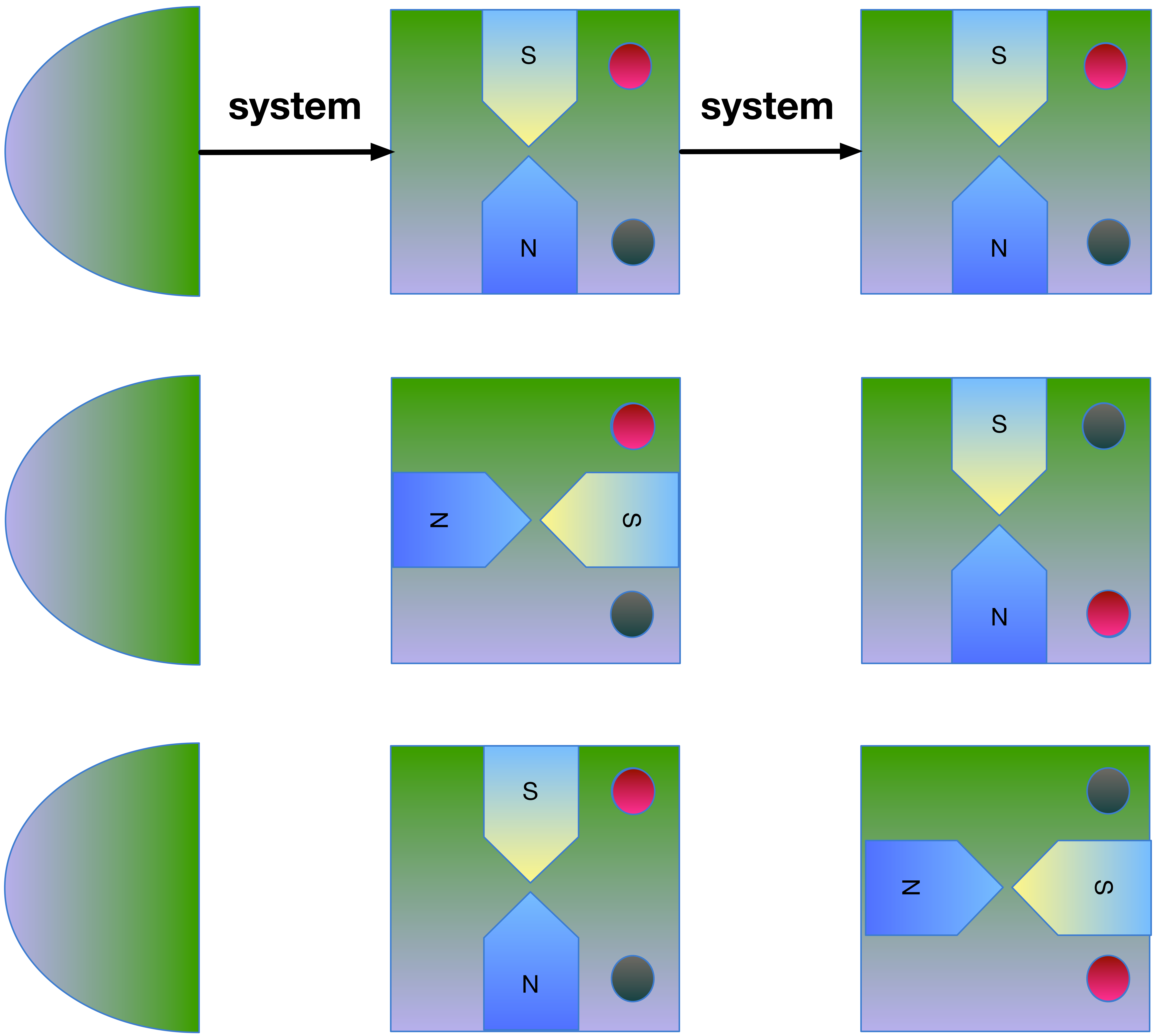
Operational probabilistic theory (OPT)



$$p(abc,\ldots,o|\gamma_1 \cup \gamma_2) = p(abc,\ldots,l|\gamma_1)p(n,\ldots,p|\gamma_2)$$

NOTICE: marginals depend on the marginalised part of the graph!



system
S
N
system
S
N
N
S
S
N
S
N
N
S

Goal of Science

1. To connect “objective things happening” (events)
2. To devise a theory of such “connections” (systems)
3. To make predictions for future occurrence (predict joint probabilities of events depending on their connections).

Which **events** happen is **objective**

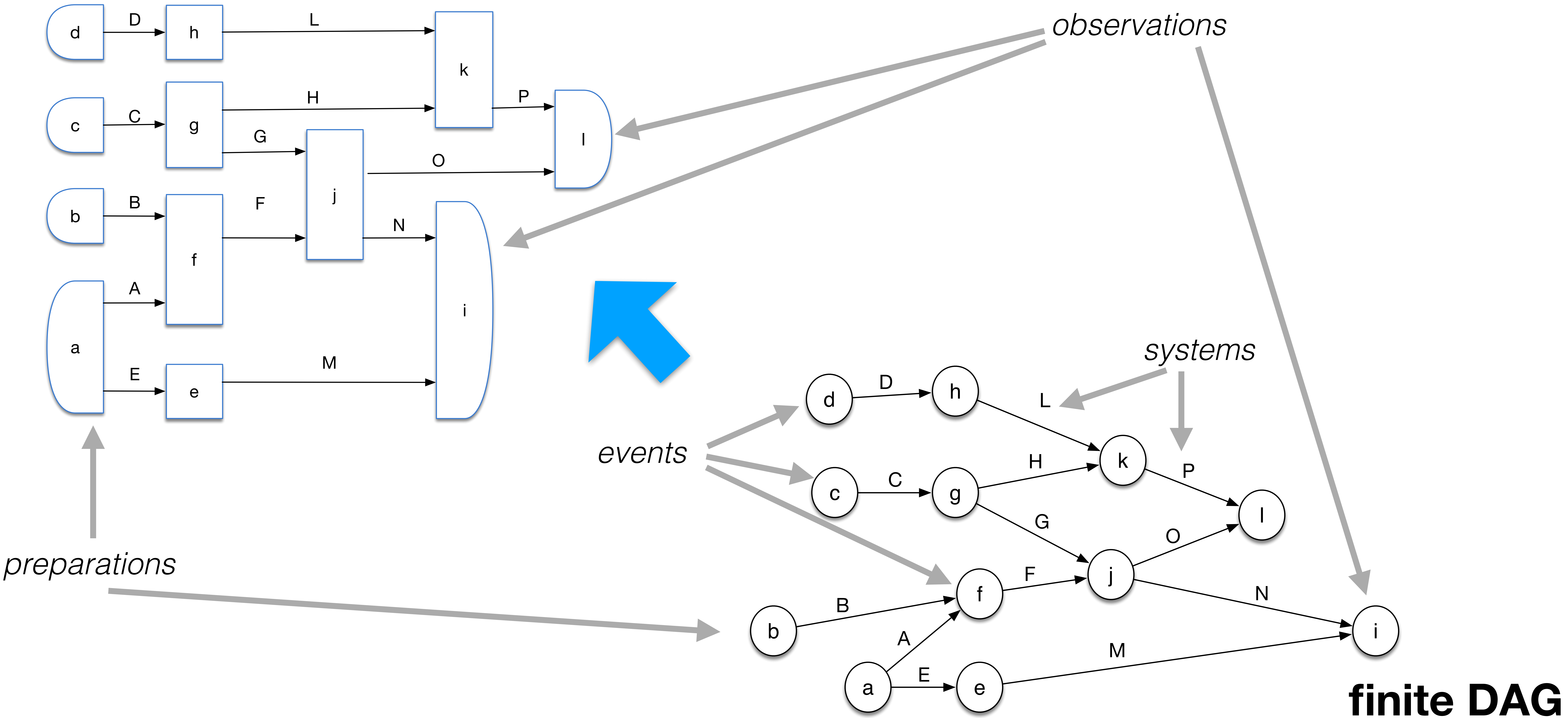
Systems are **theoretical**

OPT: methodologically fit, falsification-ready

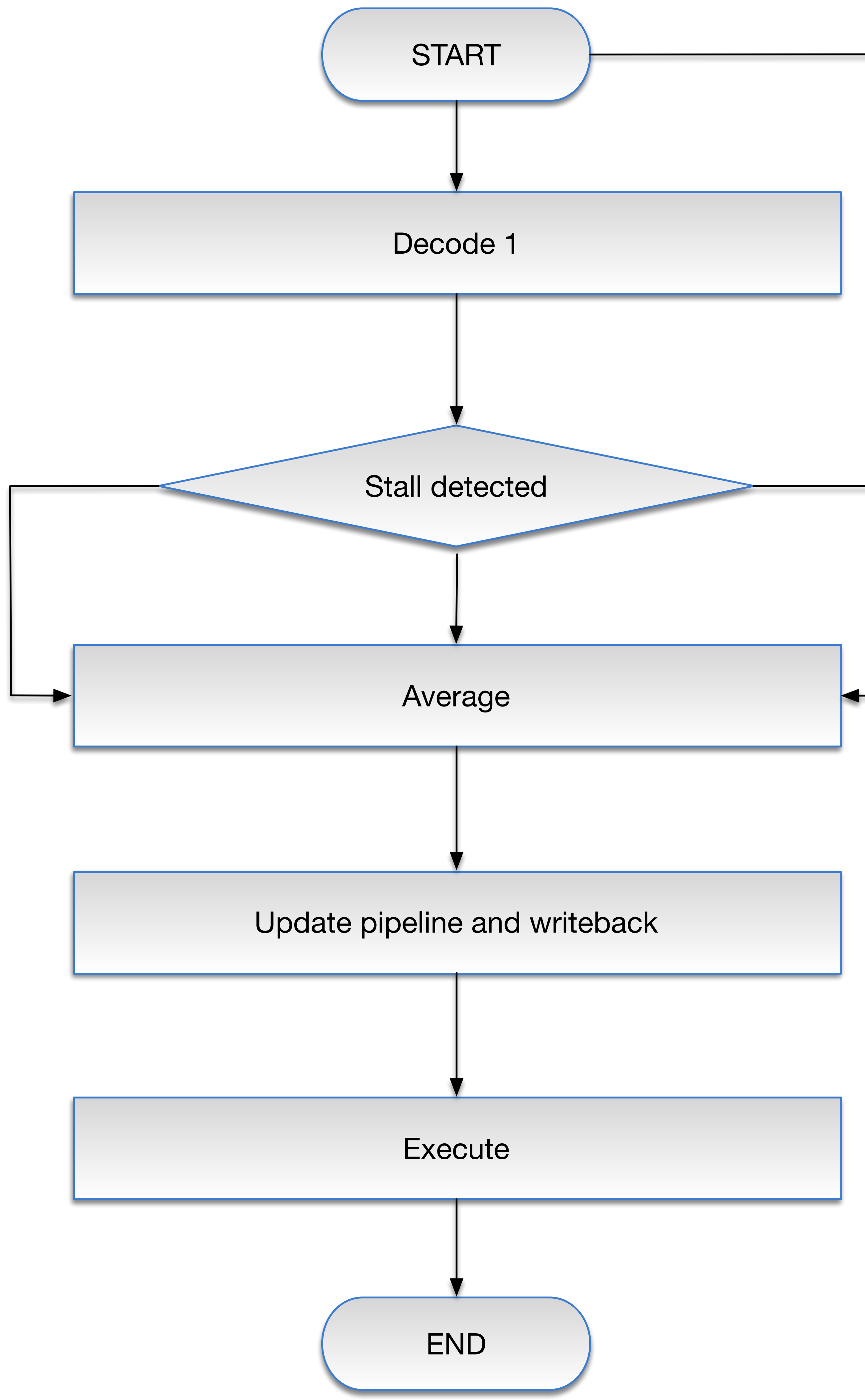
Goal of an OPT

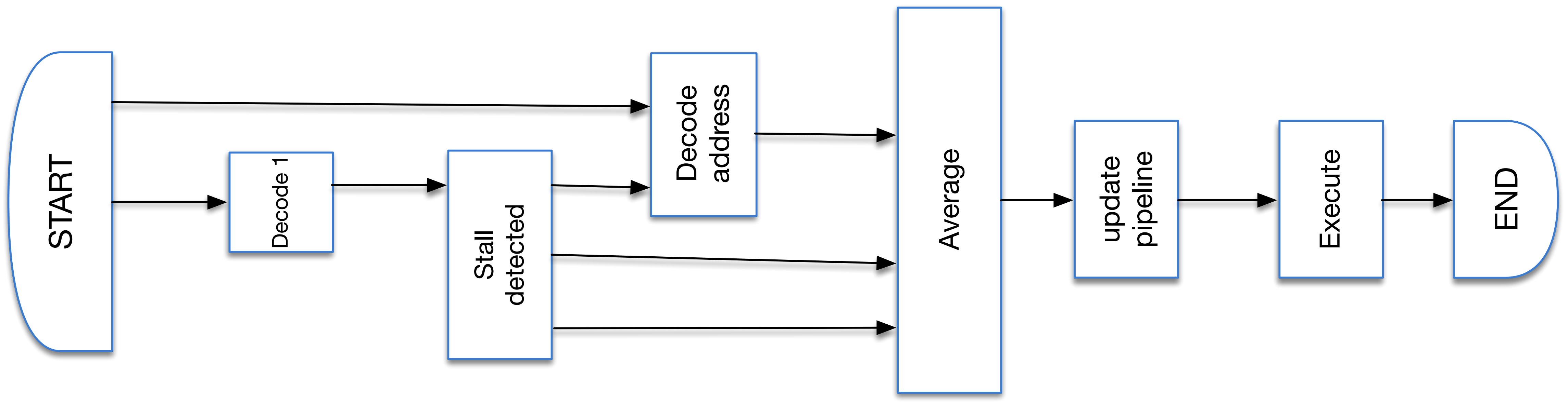
To provide a mathematical description of systems and events consistent with their composition rules, allowing to evaluate their joint probability distribution depending on the graph of connections

An OPT is an Information Theory



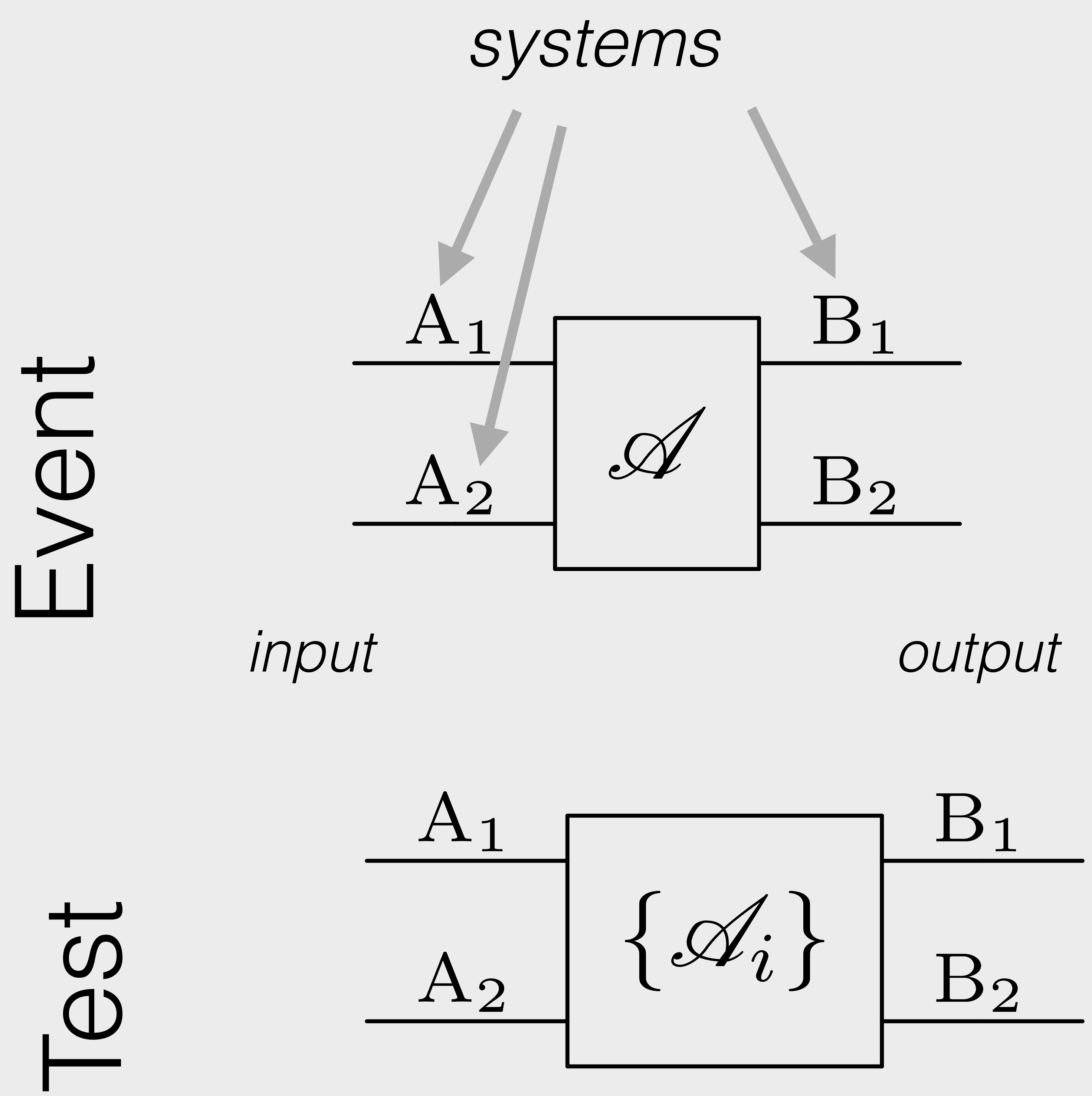
An OPT is an Information Theory





OPT framework

joint probabilities + connectivity



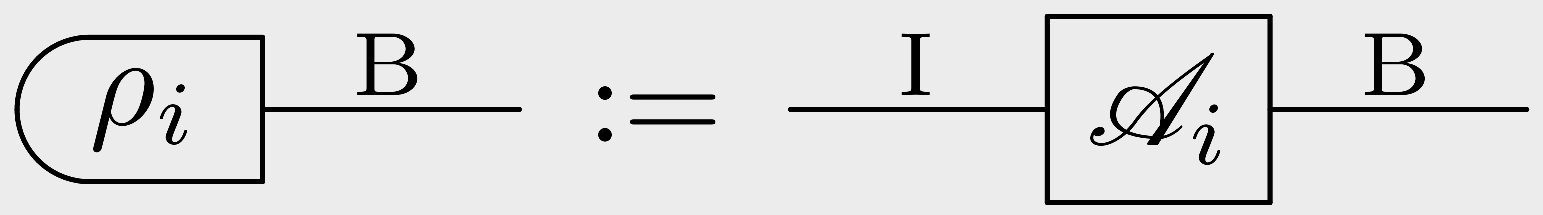
Marginal probability

$$\sum_{ik\ldots} p(i,j,k,\ldots|\textbf{circuit}) = p(j|\textbf{circuit})$$

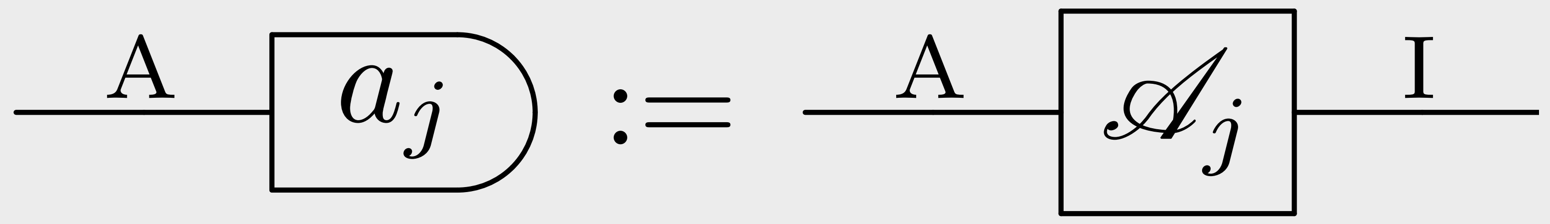
OPT framework

joint probabilities + connectivity

Trivial system

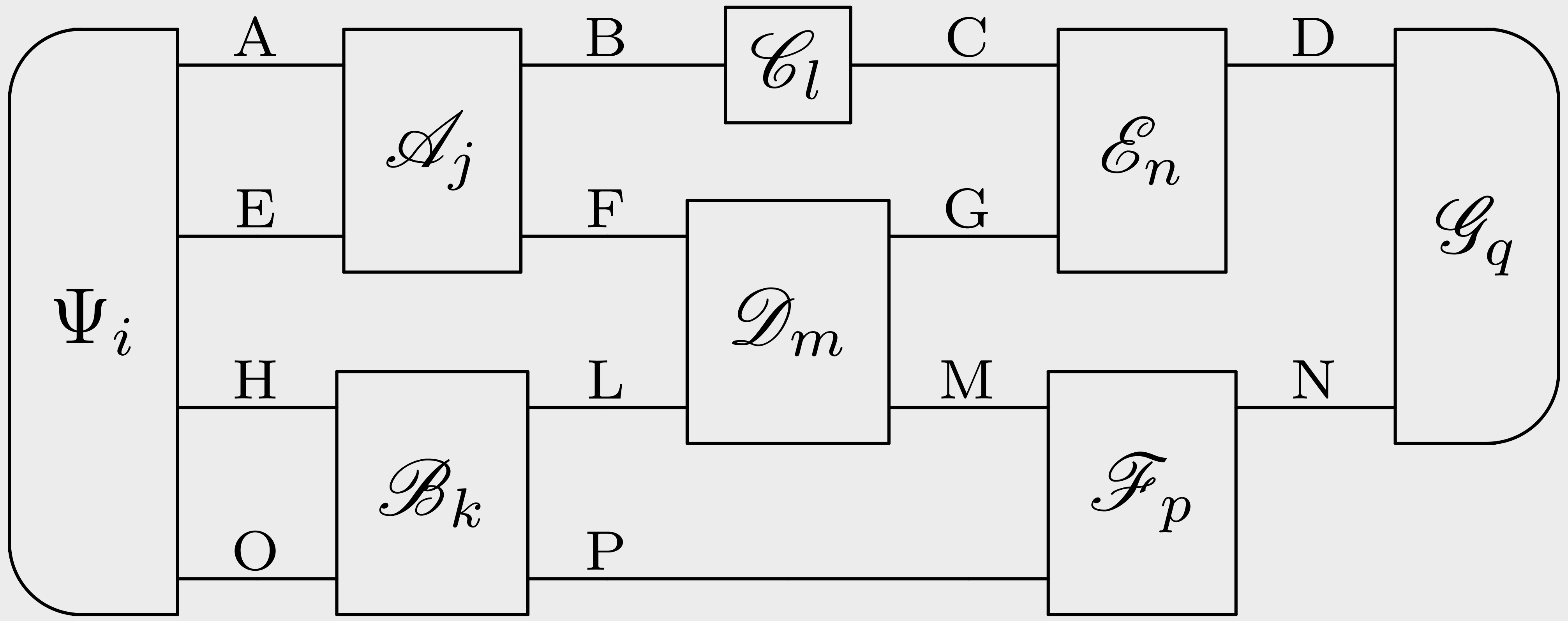


preparation



observation

$$p(i, j, k, l, m, n, p, q|\text{circuit})$$



Marginal probability

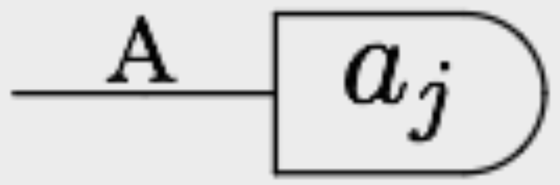
$$\sum_{ik\ldots} p(i, j, k, \ldots|\text{circuit}) = p(j|\text{circuit})$$

OPT framework

joint probabilities + connectivity

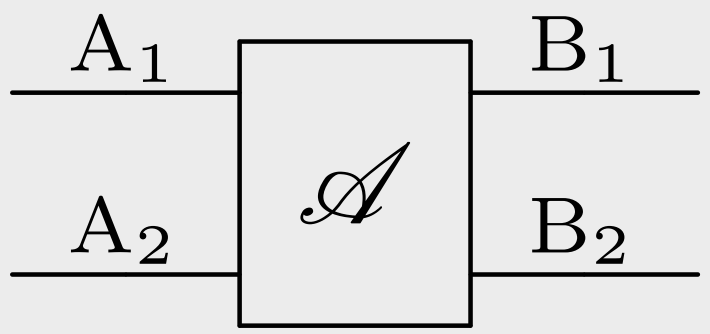
Probabilistic
equivalence classes

state

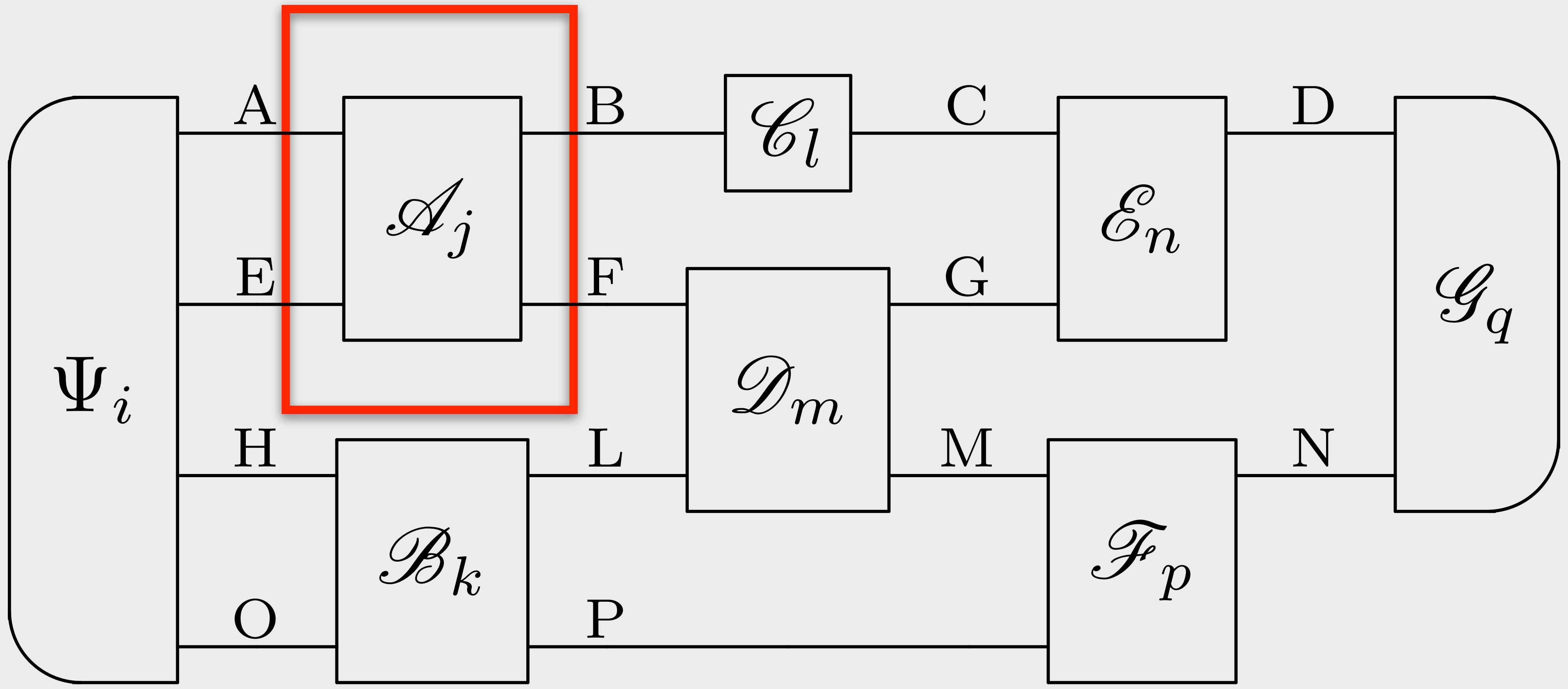


effect

transformation



$$p(i, j, k, l, m, n, p, q|\text{circuit})$$



OPT framework

joint probabilities + connectivity

Probabilistic
equivalence classes

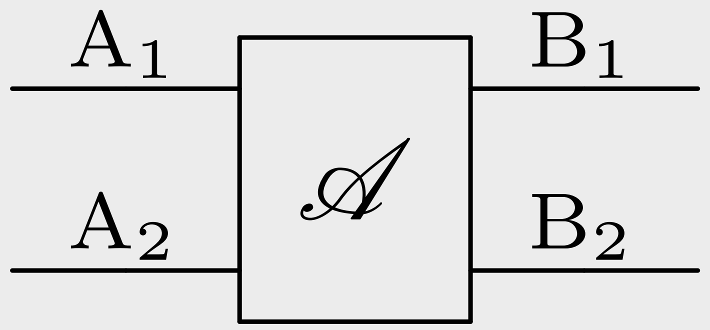
category theory:
transformations ➡ morphisms
systems ➡ objects

OPT: strict monoidal braided category

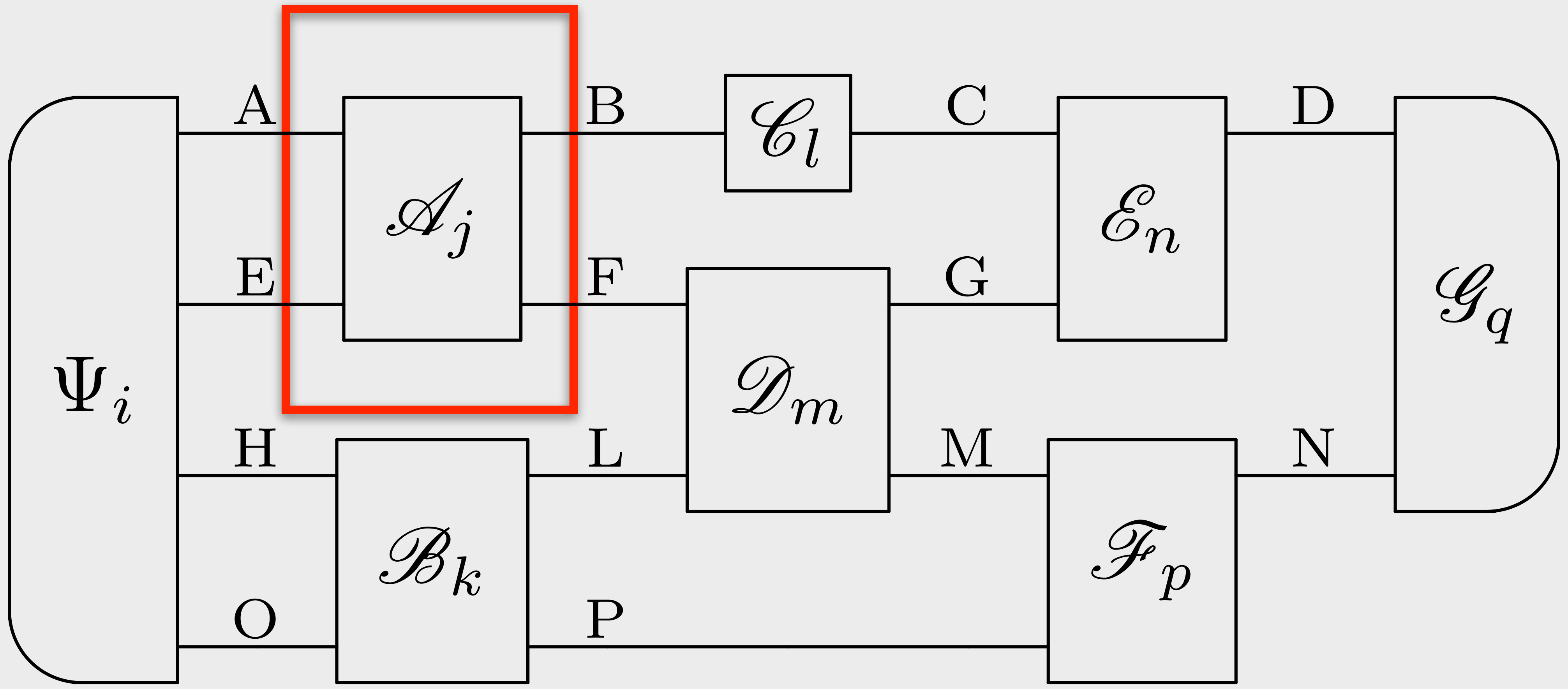
state

effect

transformation



$$p(i, j, k, l, m, n, p, q|\text{circuit})$$



OPT framework

Sequential composition (associative)

$$\overset{\mathrm{A}}{\text{——}}\boxed{\{\mathscr{A}_x\}_{x\in \mathrm{X}}}\overset{\mathrm{B}}{\text{——}}\boxed{\{\mathscr{B}_y\}_{y\in \mathrm{Y}}}\overset{\mathrm{C}}{\text{——}} \quad =: \quad \overset{\mathrm{A}}{\text{——}}\boxed{\{\mathscr{B}_x \circ \mathscr{A}_y\}_{(x,y)\in \mathrm{X}\times\mathrm{Y}}}\overset{\mathrm{C}}{\text{——}}$$

Identity test

$$\overset{\mathrm{A}}{\text{——}}\boxed{\mathscr{I}_{\mathrm{A}}}\overset{\mathrm{A}}{\text{——}}\boxed{\mathscr{C}}\overset{\mathrm{B}}{\text{——}} \quad = \quad \overset{\mathrm{A}}{\text{———}}\boxed{\mathscr{C}}\overset{\mathrm{B}}{\text{——}} \quad = \quad \overset{\mathrm{A}}{\text{——}}\boxed{\mathscr{C}}\overset{\mathrm{B}}{\text{——}}$$

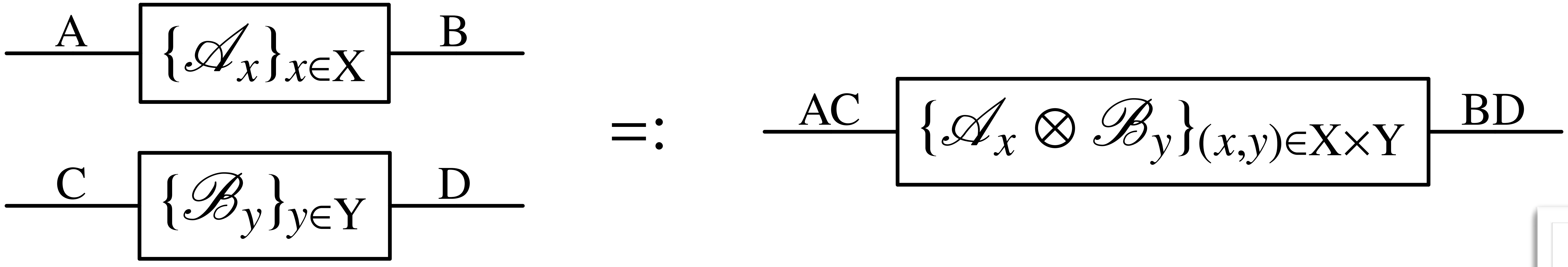
$$\overset{\mathrm{A}}{\text{——}}\boxed{\mathscr{D}}\overset{\mathrm{B}}{\text{——}}\boxed{\mathscr{I}_{\mathrm{B}}}\overset{\mathrm{B}}{\text{——}} \quad = \quad \overset{\mathrm{A}}{\text{——}}\boxed{\mathscr{D}}\overset{\mathrm{B}}{\text{———}} \quad = \quad \overset{\mathrm{A}}{\text{——}}\boxed{\mathscr{D}}\overset{\mathrm{B}}{\text{——}}$$

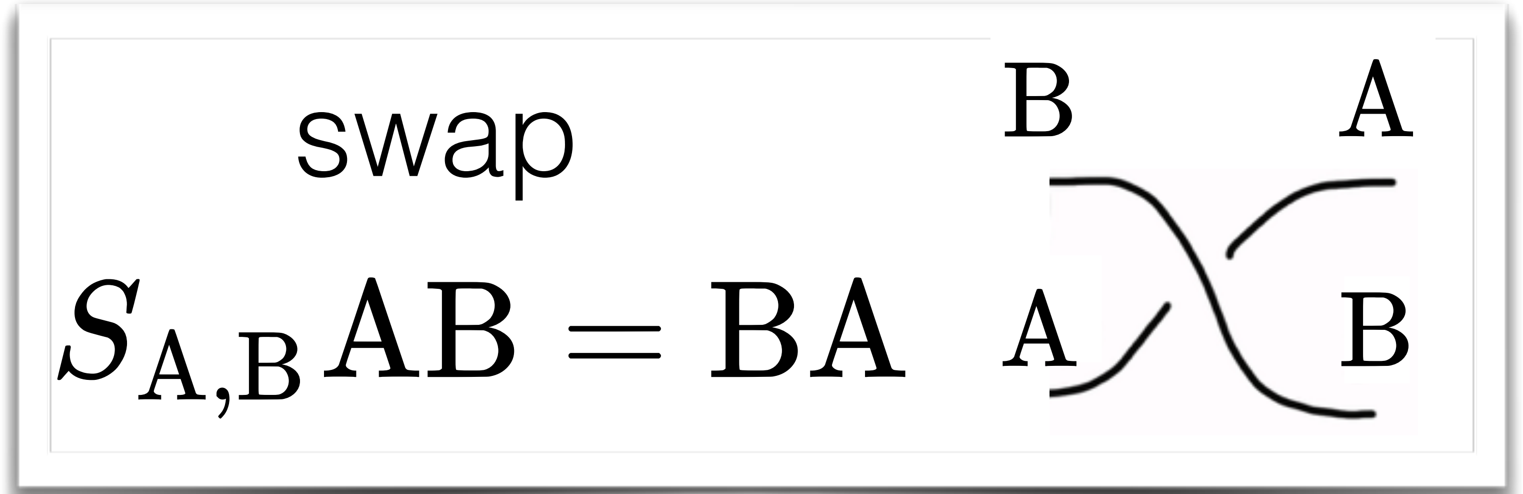
OPT framework

OPT: strict monoidal braided category

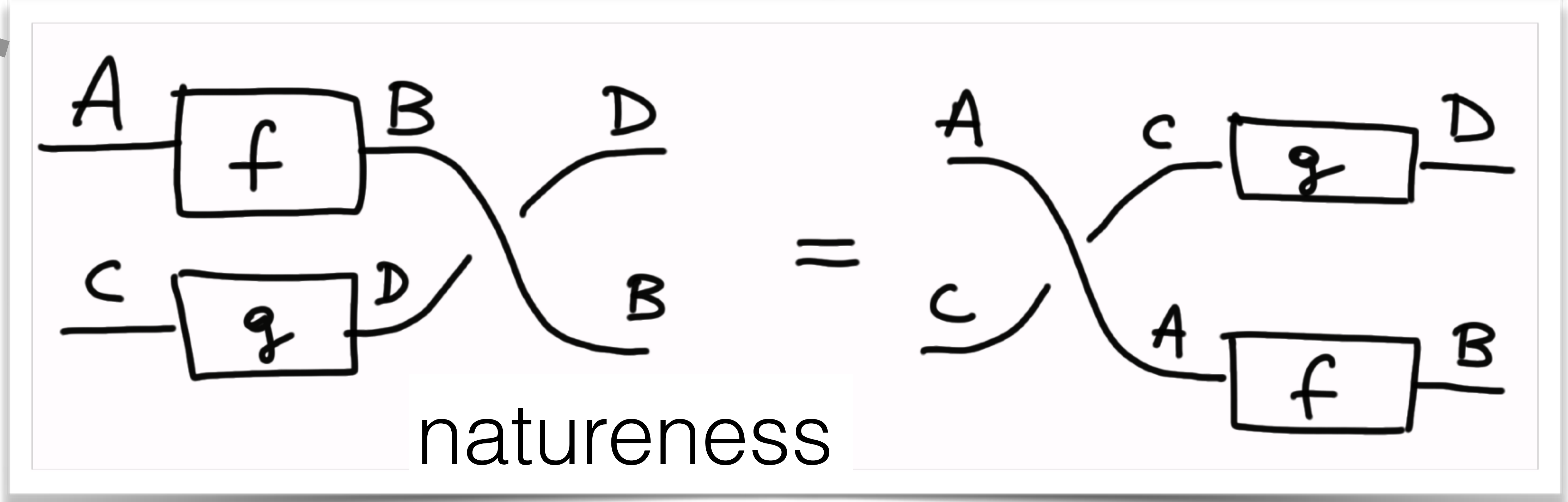
Quantum Theory: symmetric OPT

Parallel composition (associative)





$\mathrm{AB} \simeq \mathrm{BA} =: S_{\mathrm{A,B}}\mathrm{AB}$ (braided)



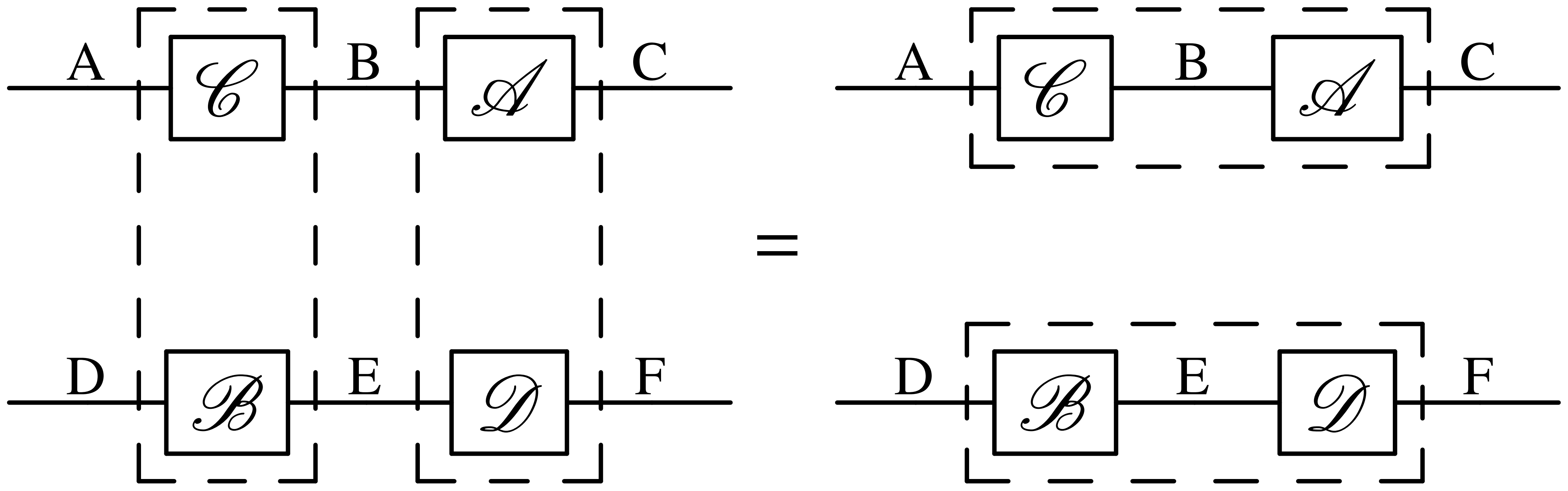
$\left.\begin{array}{l} \mathrm{AI} = \mathrm{IA} \\ \mathrm{A(BC)} = \mathrm{A(BC)} \end{array}\right\}$ (strict monoidal)

$\mathrm{(AB)C} \simeq \mathrm{A(BC)}$ (monoidal)

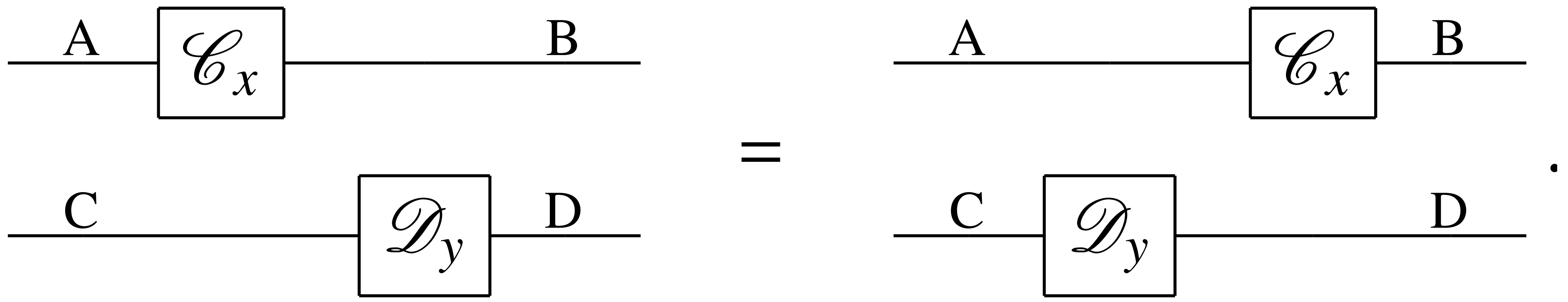
$S^{-1}_{\mathrm{A,B}} = S_{\mathrm{B,A}}$ (symmetric)

OPT framework

Sequential and parallel compositions commute



$$(\mathscr{A} \otimes \mathscr{D}) \circ (\mathscr{C} \otimes \mathscr{B}) = (\mathscr{A} \circ \mathscr{C}) \otimes (\mathscr{D} \circ \mathscr{B})$$



wire-stretching

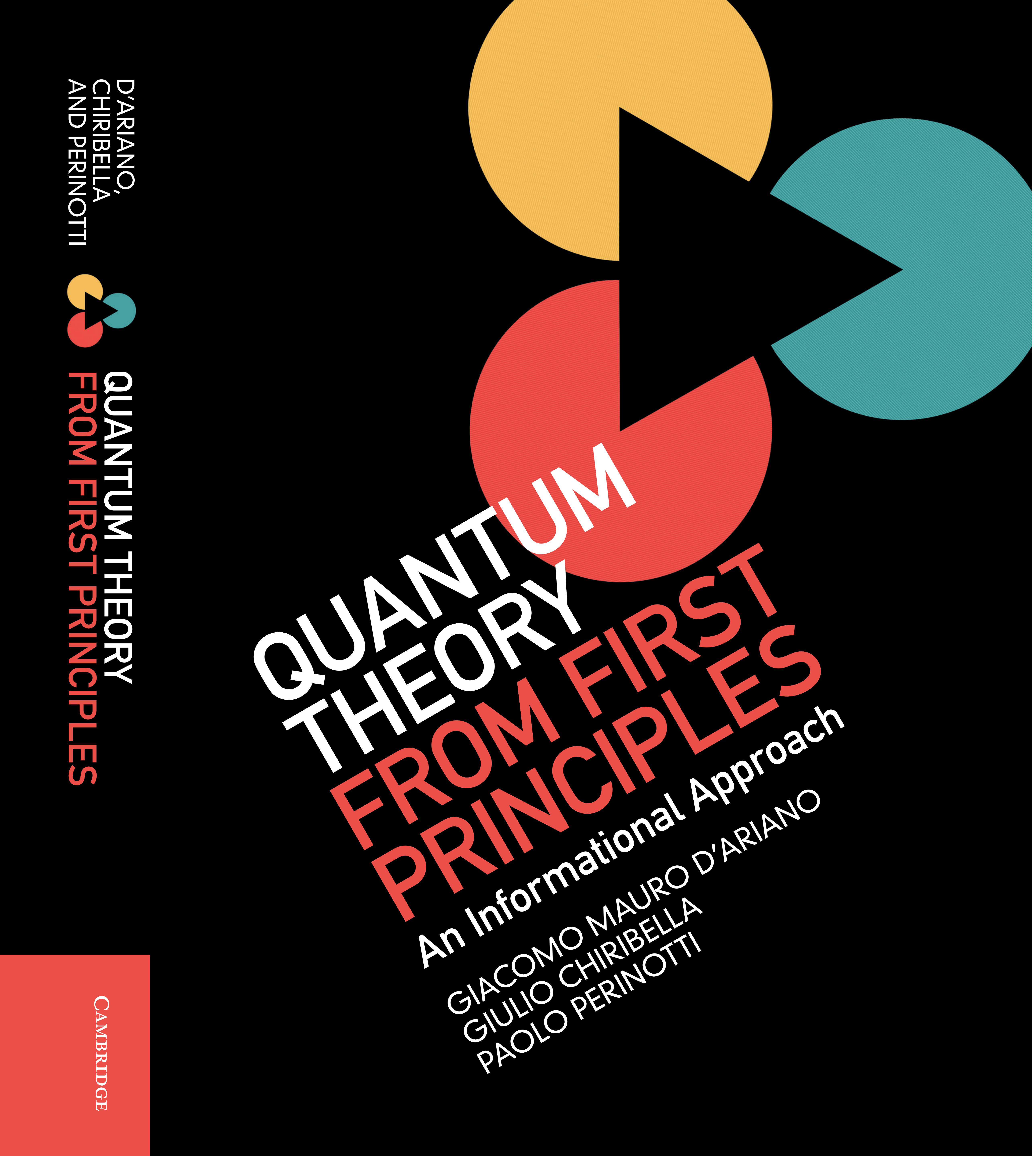
(foliations)

Quantum Theory as OPT

system	A	$\mathscr{H}_{\mathrm{A}}$	(1)
system composition	AB	$\mathscr{H}_{\mathrm{AB}} = \mathscr{H}_{\mathrm{A}} \otimes \mathscr{H}_{\mathrm{B}}$	
transformation	$\mathscr{T} \in \mathrm{Transf}(\mathrm{A} \to \mathrm{B})$	$\mathscr{T} \in \mathrm{CP}_{\leq}(\mathrm{T}(\mathscr{H}_{\mathrm{A}}) \to \mathrm{T}(\mathscr{H}_{\mathrm{B}}))$	(2)

Theorems

trivial system system	I	$\mathscr{H}_{\mathrm{I}} = \mathbb{C}$	
deterministic transformation	$\mathscr{T} \in \mathrm{Transf}_1(\mathrm{A} \to \mathrm{B})$	$\mathscr{T} \in \mathrm{CP}_{=}(\mathrm{T}(\mathscr{H}_{\mathrm{A}}) \to \mathrm{T}(\mathscr{H}_{\mathrm{B}}))$	(2)
states	$\rho \in \mathrm{St}(\mathrm{A}) \equiv \mathrm{Transf}(\mathrm{I} \to \mathrm{A})$	$\rho \in \mathrm{T}^{+}_{\leq 1}(\mathscr{H}_{\mathrm{A}})$	(3)
	$\rho \in \mathrm{St}_1(\mathrm{A}) \equiv \mathrm{Transf}_1(\mathrm{I} \to \mathrm{A})$	$\rho \in \mathrm{T}^{+}_{=1}(\mathscr{H}_{\mathrm{A}})$	(3)
	$\rho \in \mathrm{St}(\mathrm{I}) \equiv \mathrm{Transf}(\mathrm{I} \to \mathrm{I})$	$\rho \in [0,1]$	
	$\rho \in \mathrm{St}_1(\mathrm{I}) \equiv \mathrm{Transf}(\mathrm{I} \to \mathrm{I})$	$\rho = 1$	
effects	$\varepsilon \in \mathrm{Eff}(\mathrm{A}) \equiv \mathrm{Transf}(\mathrm{A} \to \mathrm{I})$	$\varepsilon(\cdot) = \mathrm{Tr}_{\mathrm{A}}[\cdot E],\ 0 \leq E \leq I_A$	(4)
	$\varepsilon \in \mathrm{Eff}_1(\mathrm{A}) \equiv \mathrm{Transf}_1(\mathrm{A} \to \mathrm{I})$	$\varepsilon = \mathrm{Tr}_{\mathrm{A}}$	(4)



Principles for Quantum Theory

P1. Causality

P2. Local discriminability

P3. Purification

P4. Atomicity of composition

P5. Perfect distinguishability

P6. Lossless Compressibility

G. Chiribella, G. M. D'Ariano, P. Perinotti, *Probabilistic Theories with Purification* Phys. Rev. A **81** 062348 (2010)

G. Chiribella, G. M. D'Ariano, P. Perinotti, *Informational derivation of Quantum Theory* Phys. Rev A **84** 012311 (2011)

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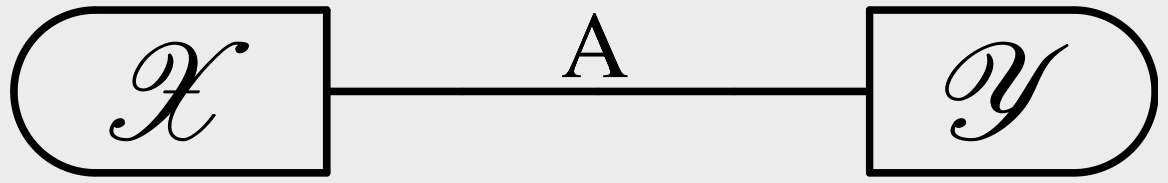
The probability of preparations is independent of the choice of observations

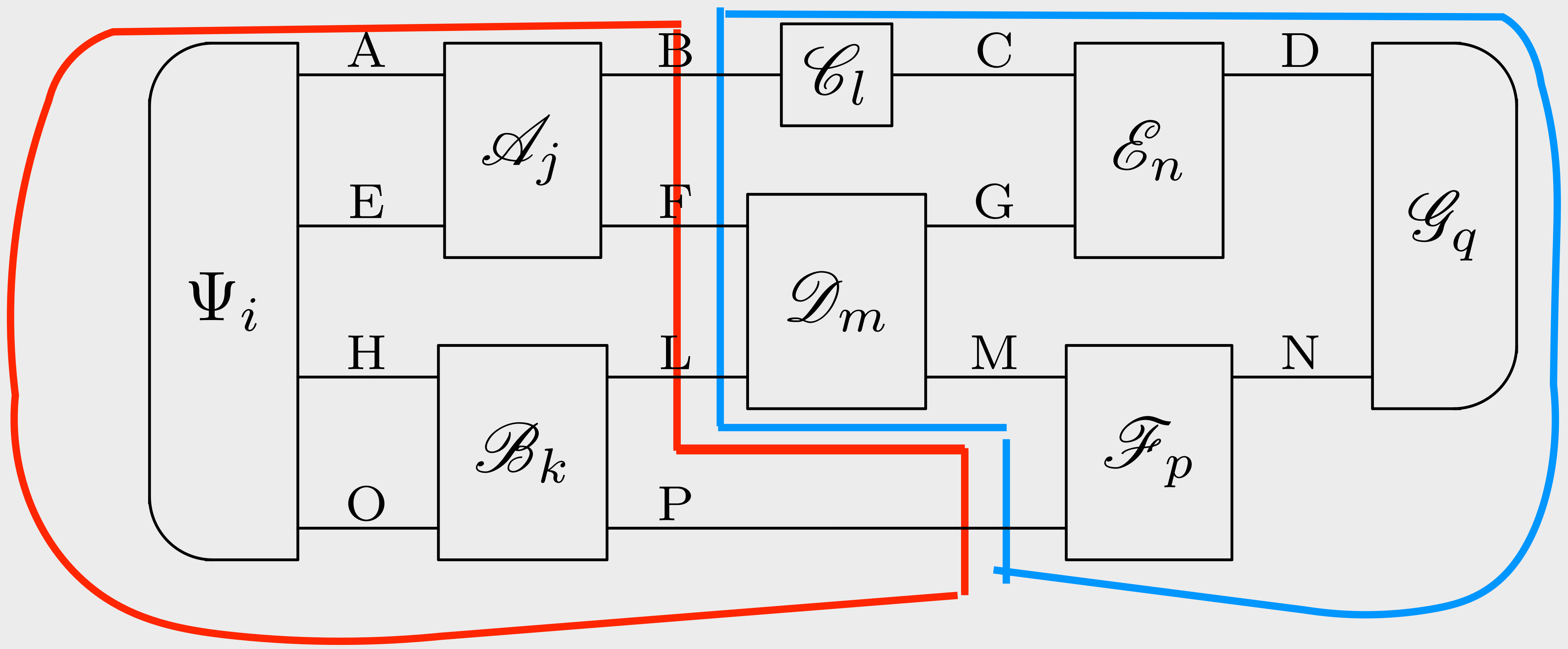


$$p(i, j|\mathscr{X}, \mathscr{Y}) := (a_j|\rho_i)$$

$$p(i|\mathscr{X}, \mathscr{Y}) = p(i|\mathscr{X}, \mathscr{Y}') = p(i|\mathscr{X})$$

Iff conditions: a) the deterministic effect is unique;
b) states are "normalizable"

$$p(i, j, k, l, m, n, p, q|\text{circuit})$$



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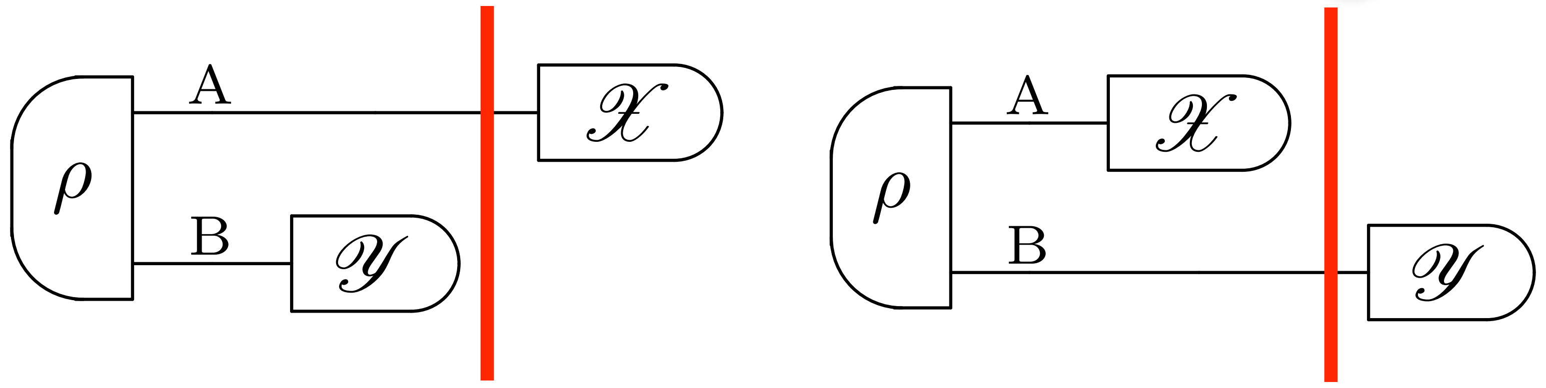
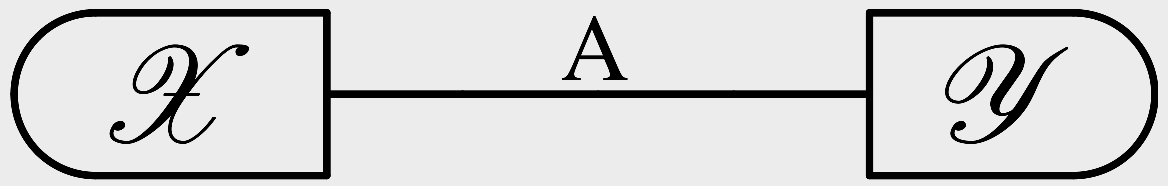
The probability of preparations is independent of the choice of observations

no signaling without interaction





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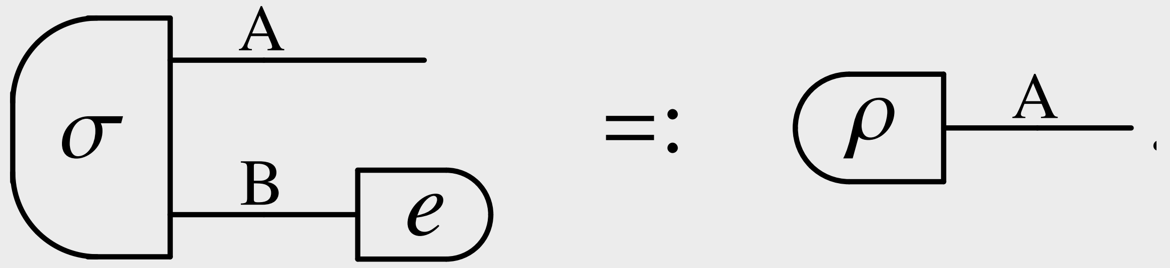
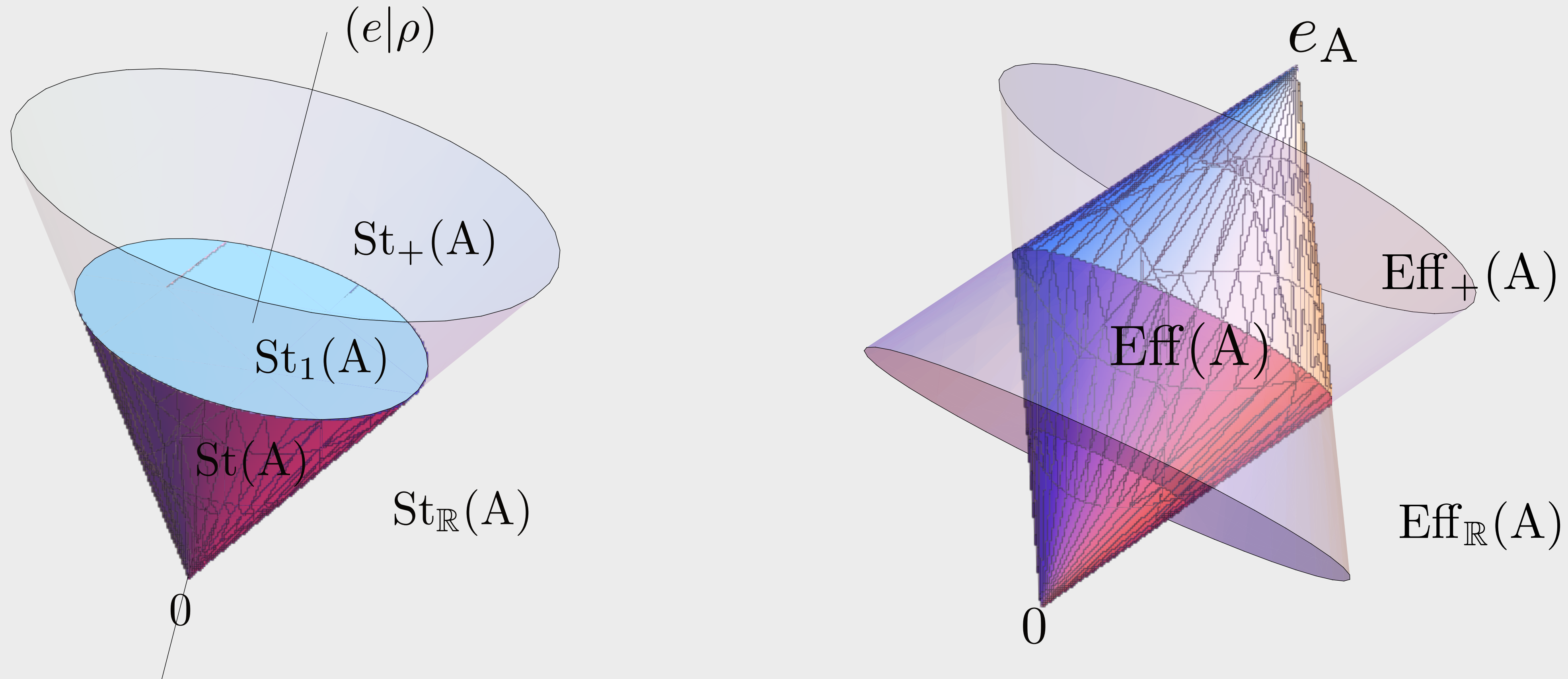


marginal state



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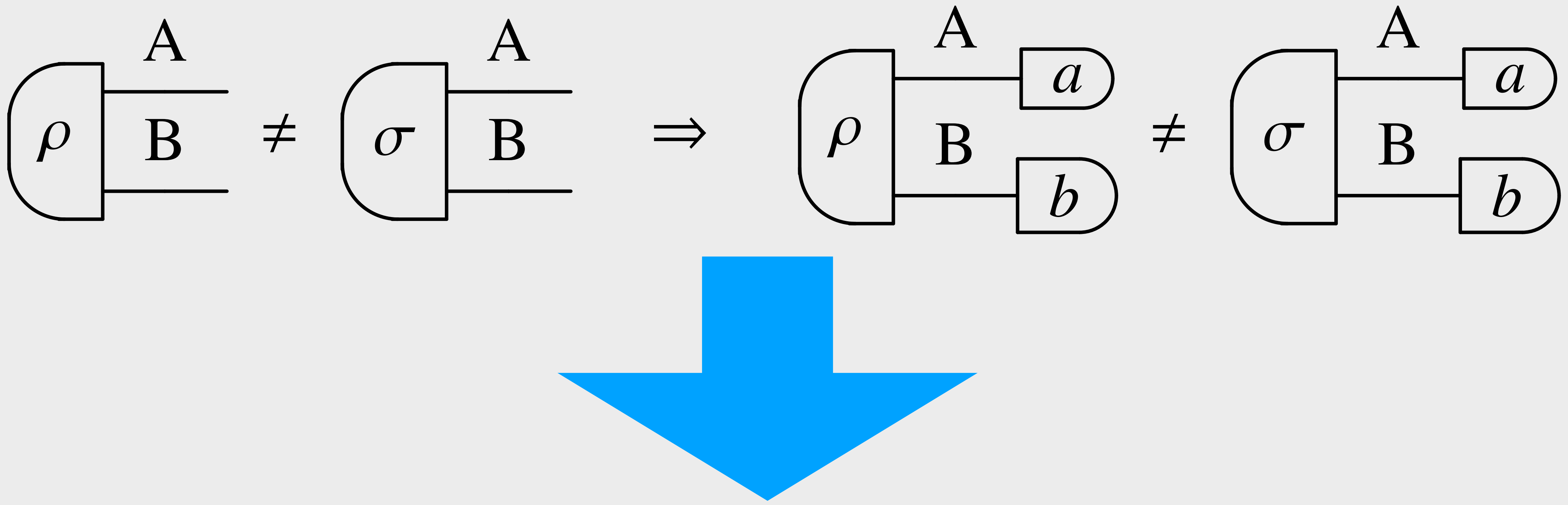
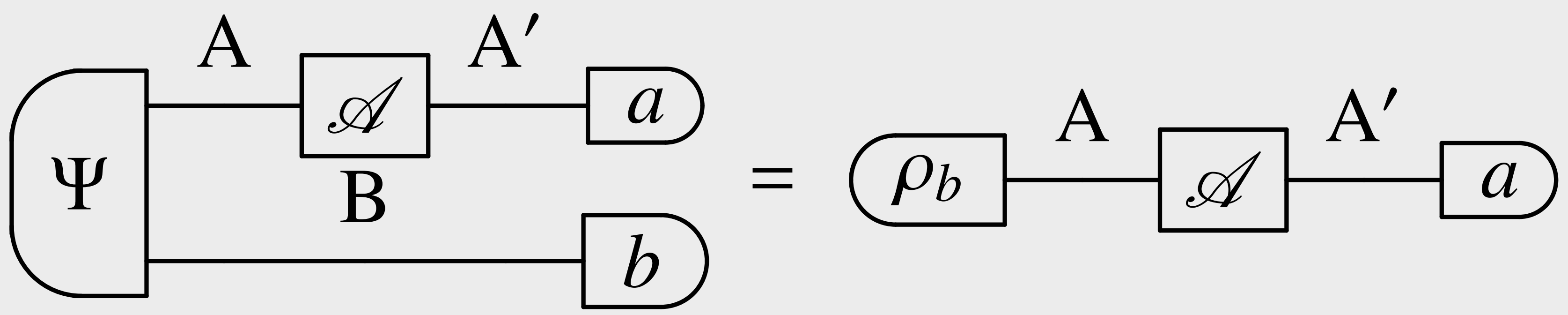
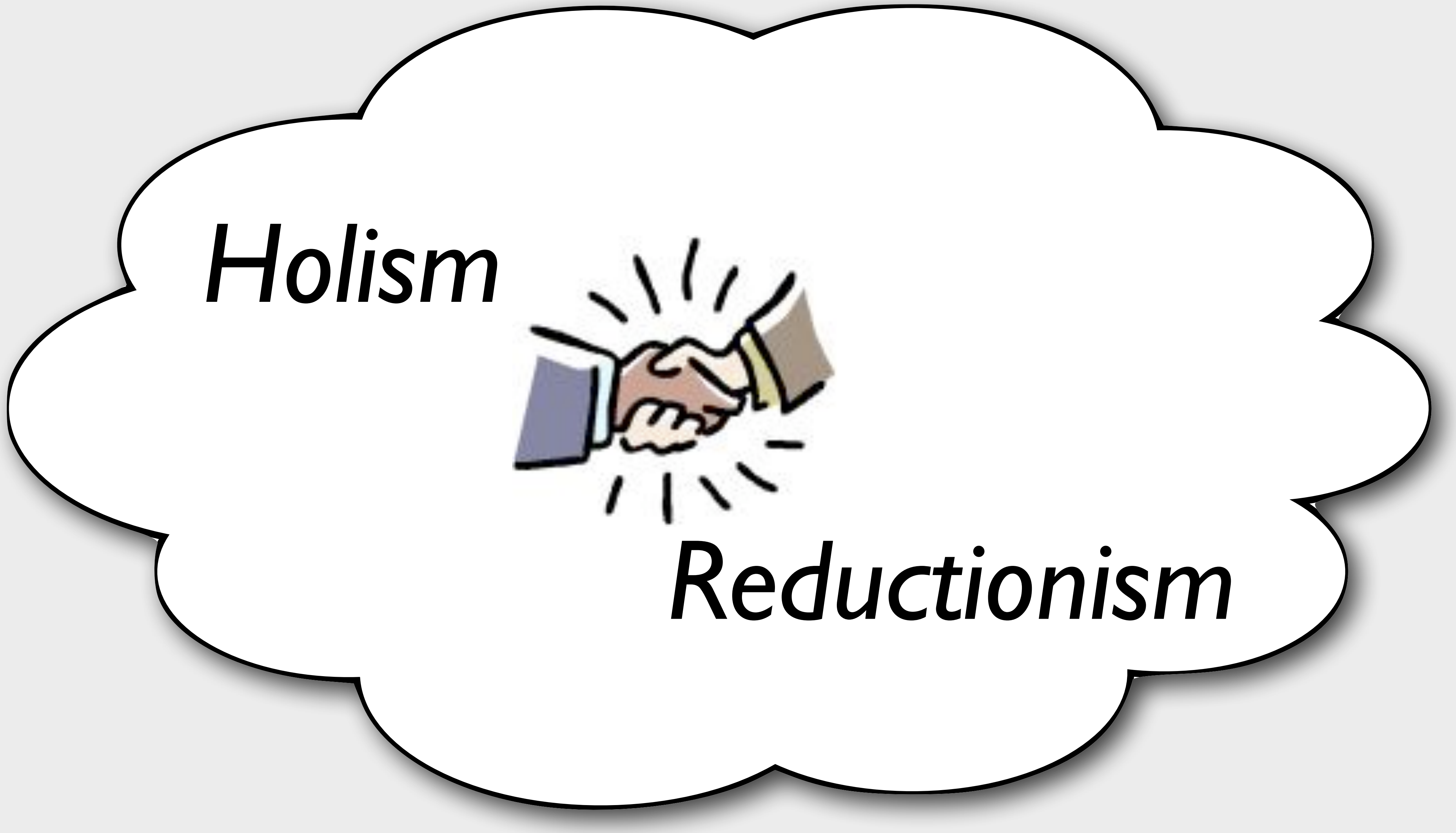
It is possible to discriminate any pair of states of composite systems using only local measurements.



Local characterization of transformations





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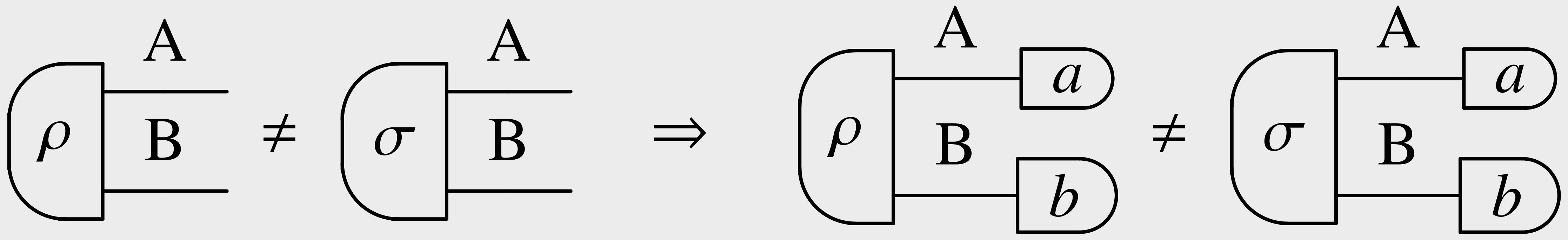
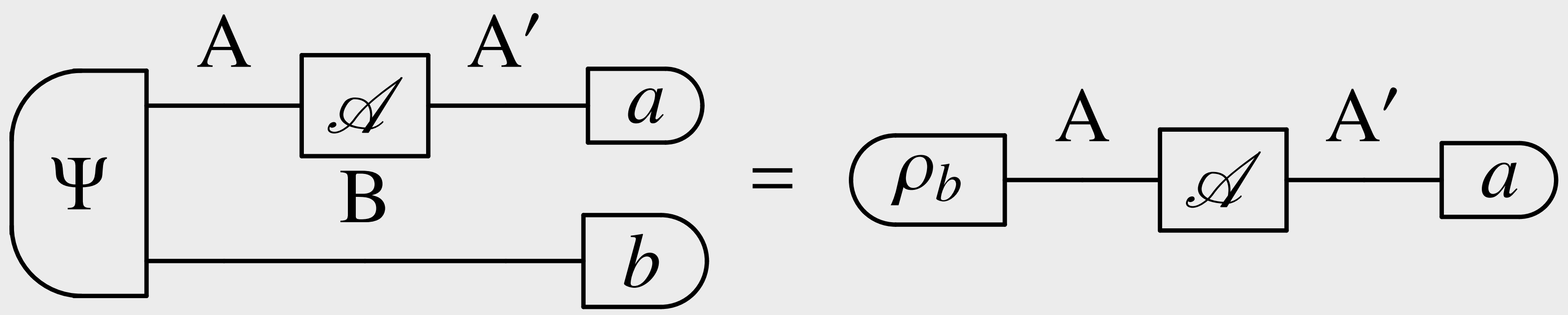
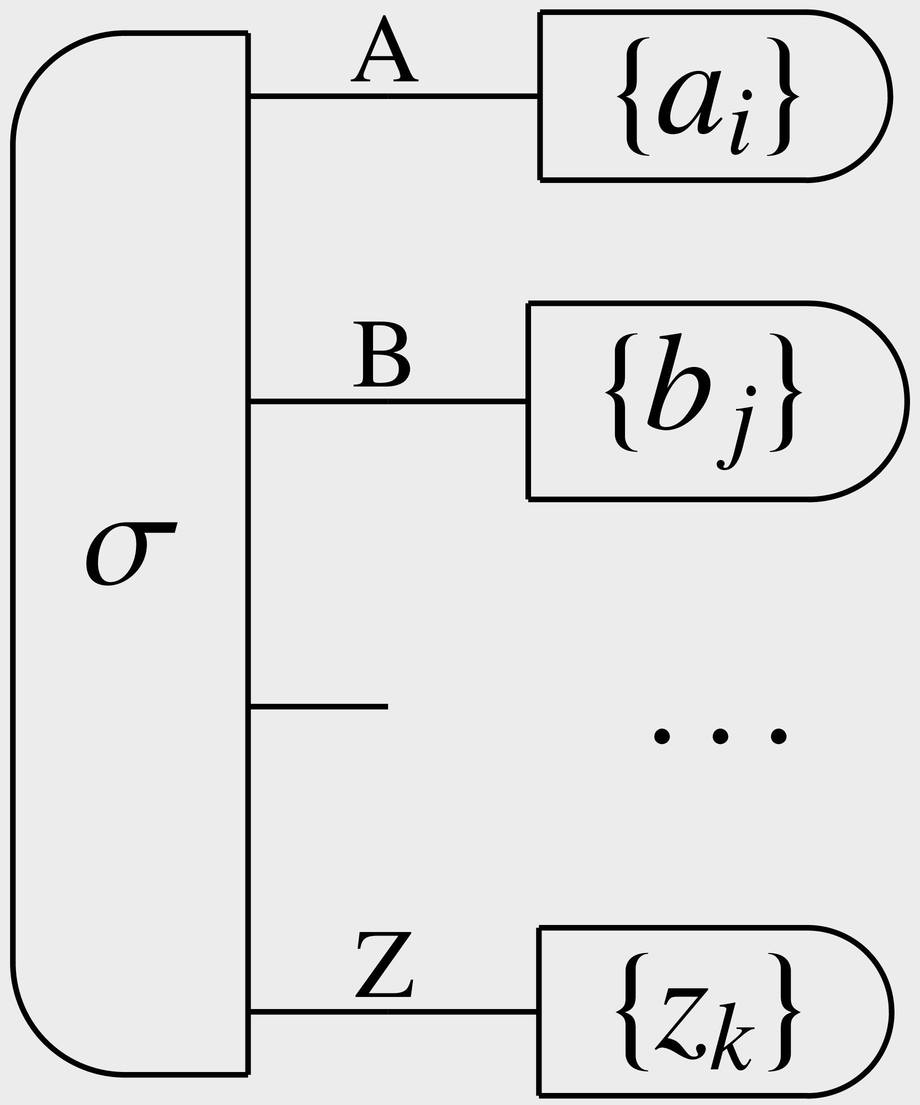
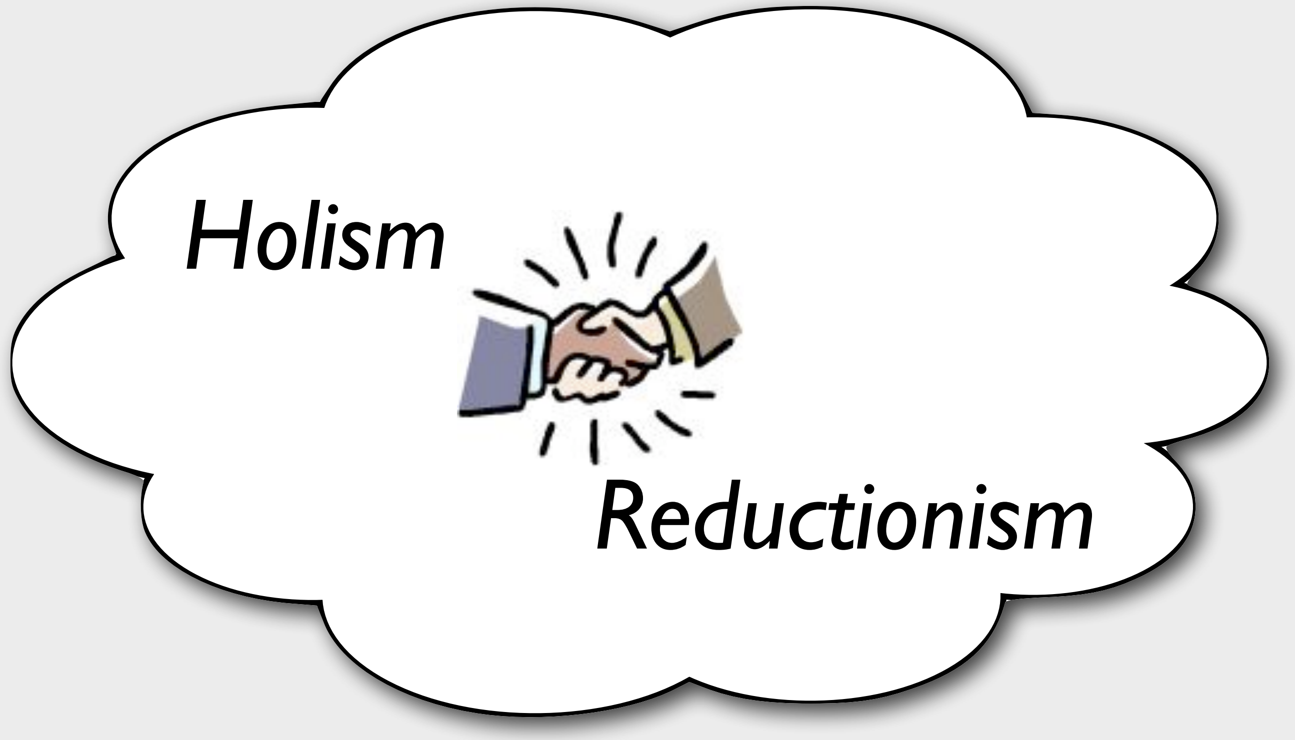
Origin of the complex tensor product



Local characterization of transformations







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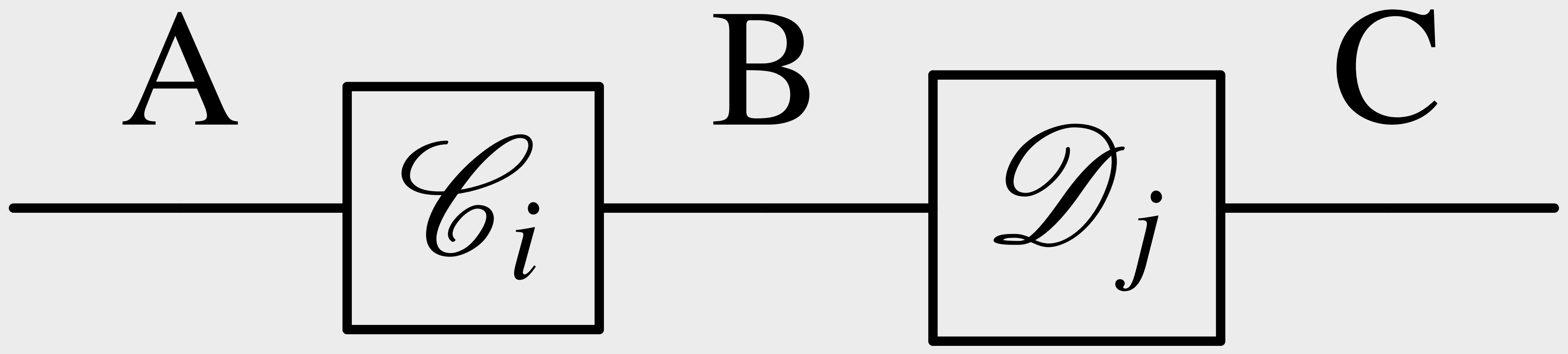
The composition of two atomic transformations is atomic

Complete information can be accessed on a step-by-step basis



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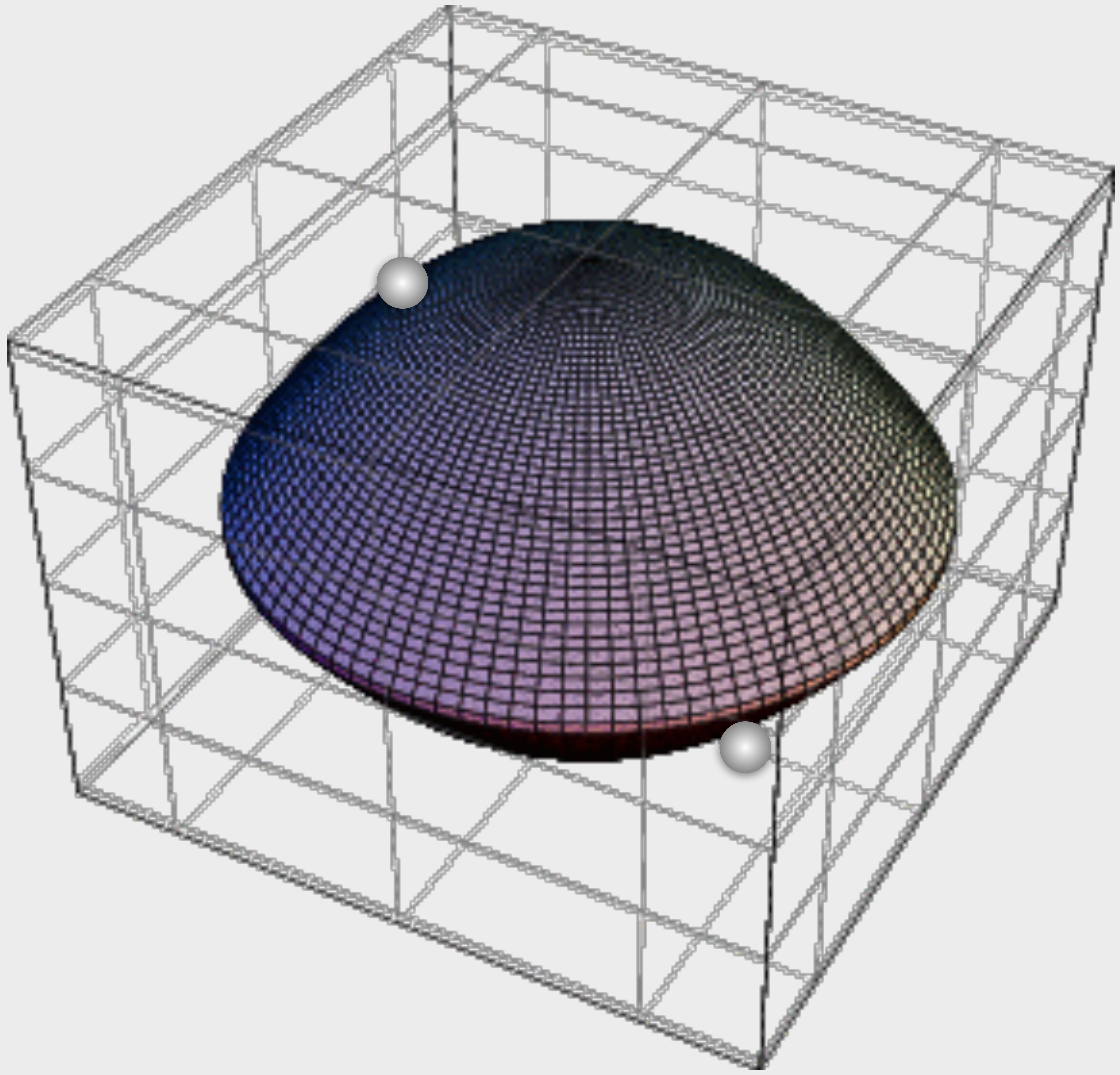
Every state that is not completely mixed (i.e. on the boundary of the convex) can be perfectly distinguished from some other state

Falsifiability of the theory



Principles for Quantum Theory

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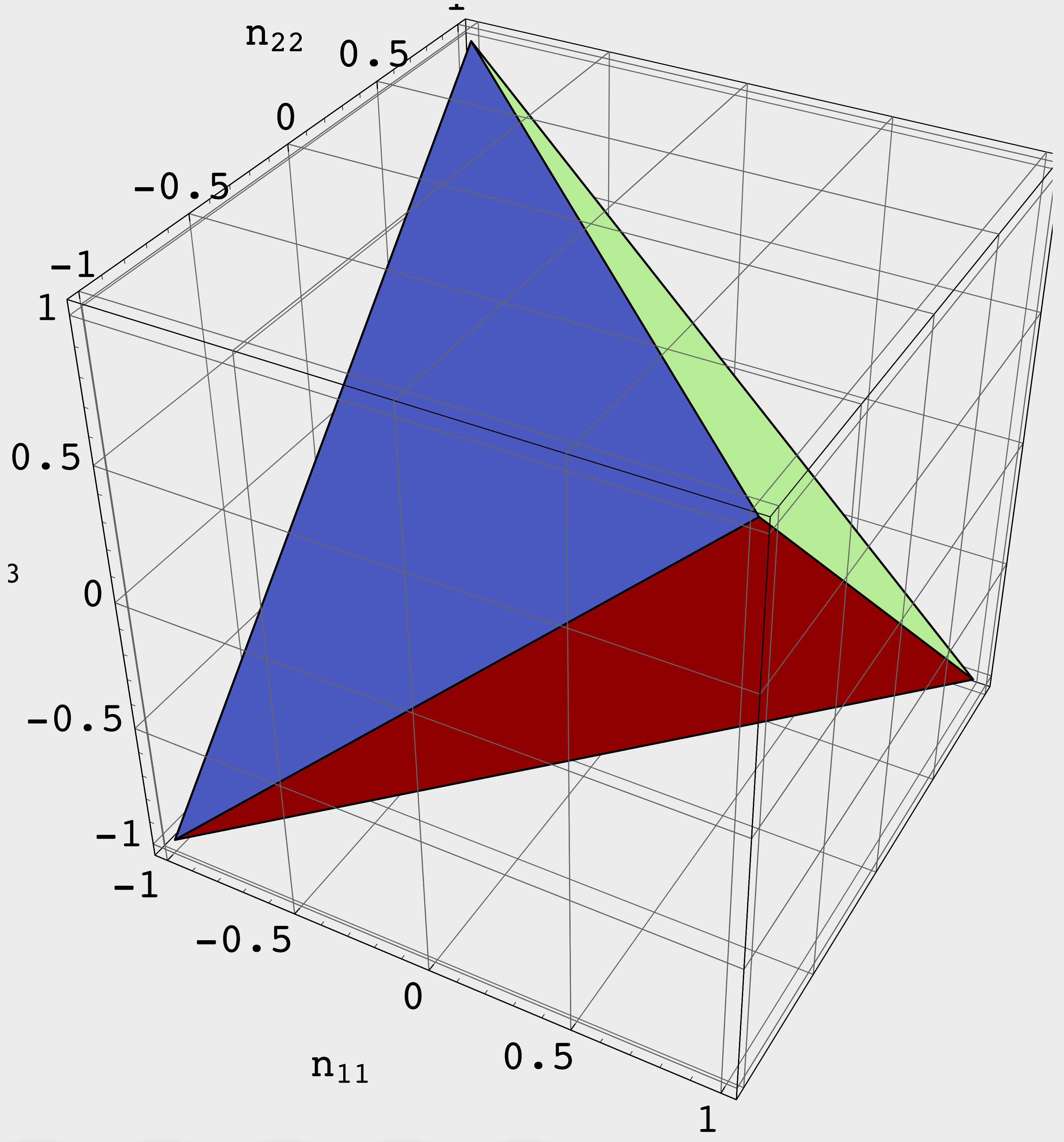
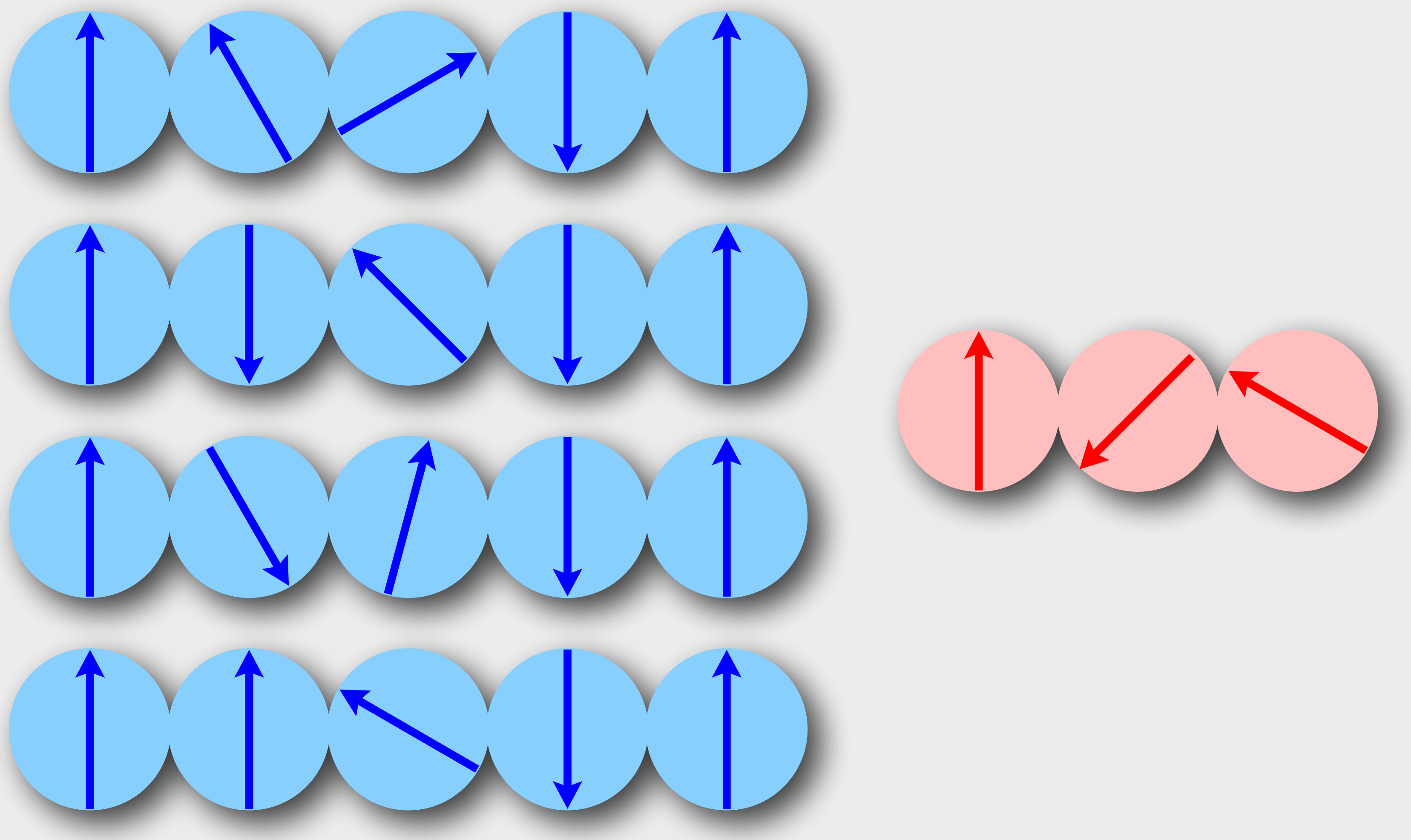
For states that are not completely mixed there exists an ideal compression scheme

Any face of the convex set of states is the convex set of states of some other system





Principles for Quantum Theory

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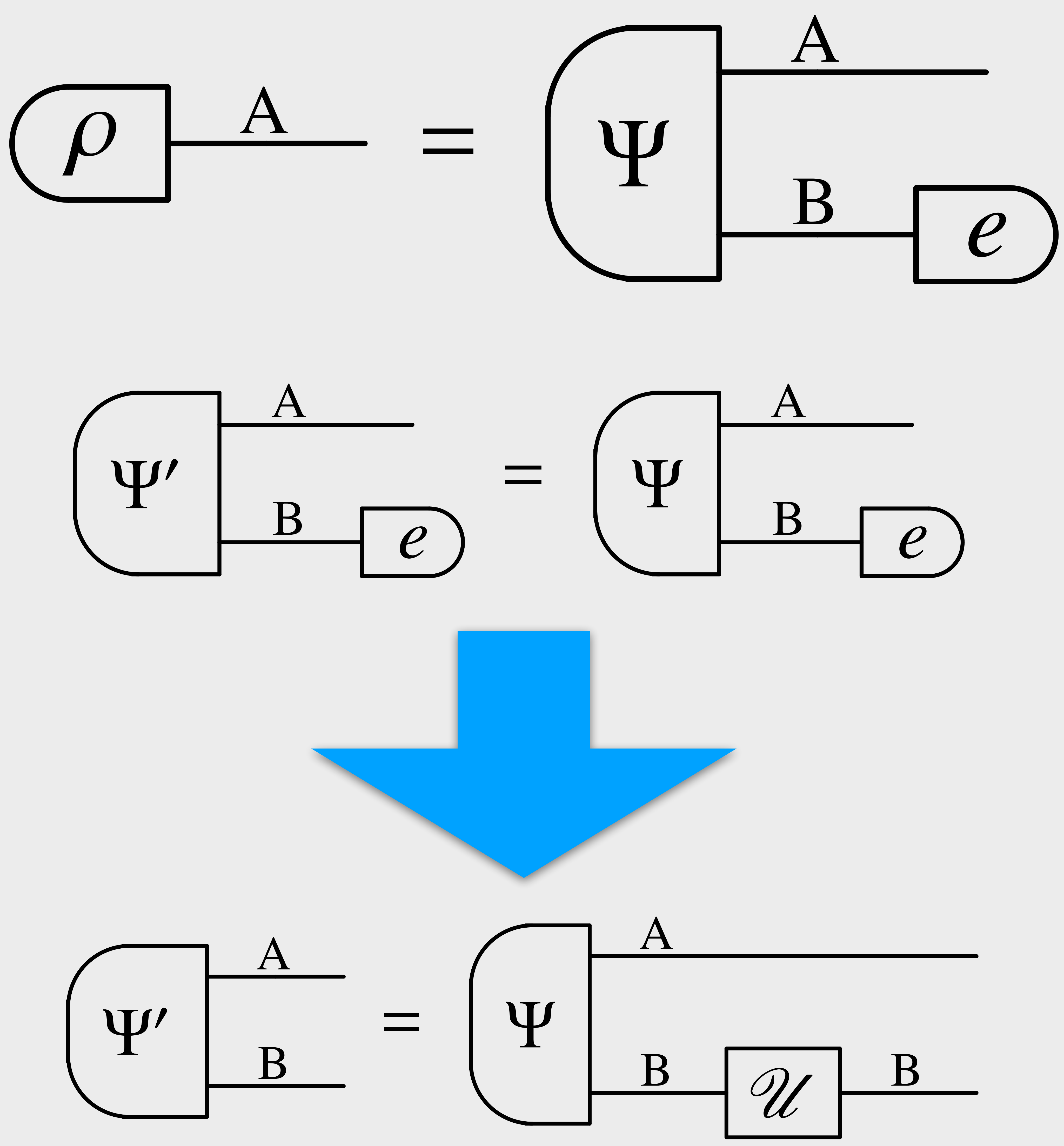
Every state has a purification.
For fixed purifying system, every two purifications of the same state are connected by a reversible transformation on the purifying system



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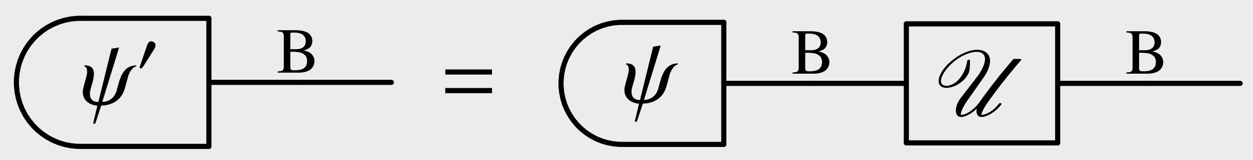
For fixed purifying system, every two purifications of the same state are connected by a reversible transformation on the purifying system

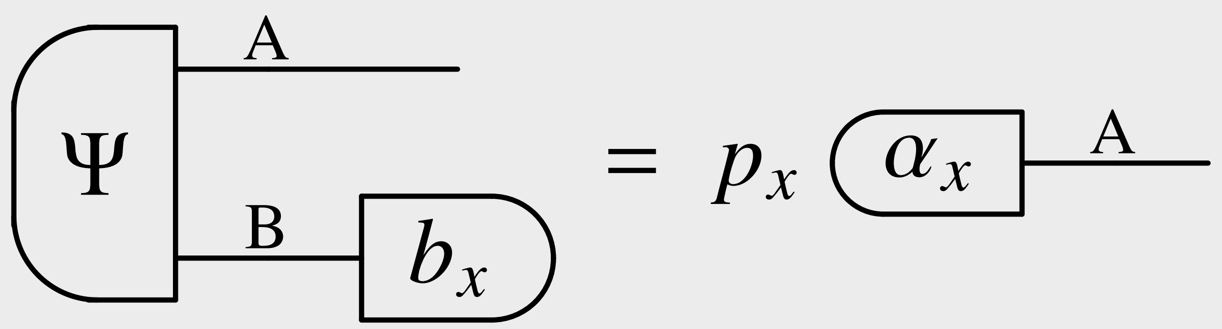
Consequences

1. Existence of entangled states:
the purification of a mixed state is an entangled state;
the marginal of a pure entangled state is a mixed state;

2. Every two normalized pure states of the same system are connected by a reversible transformation

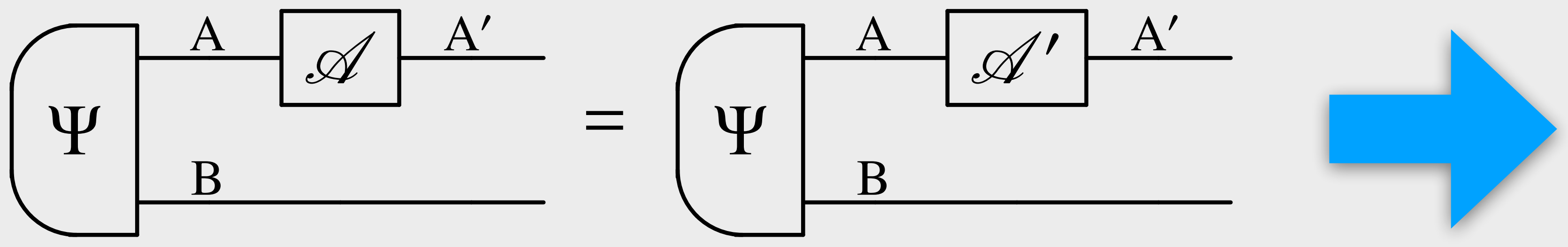


3. Steering: Let $\mathbf{\Psi}$ purification of ρ. Then for every ensemble decomposition $\rho=\sum_x p_x \alpha_x$ there exists a measurement $\{b_x\}$, such that



$\forall x \in X$

4. Process tomography (faithful state):



$\mathscr{A}\rho = \mathscr{A}'\rho \quad \forall\rho$

5. No information without disturbance

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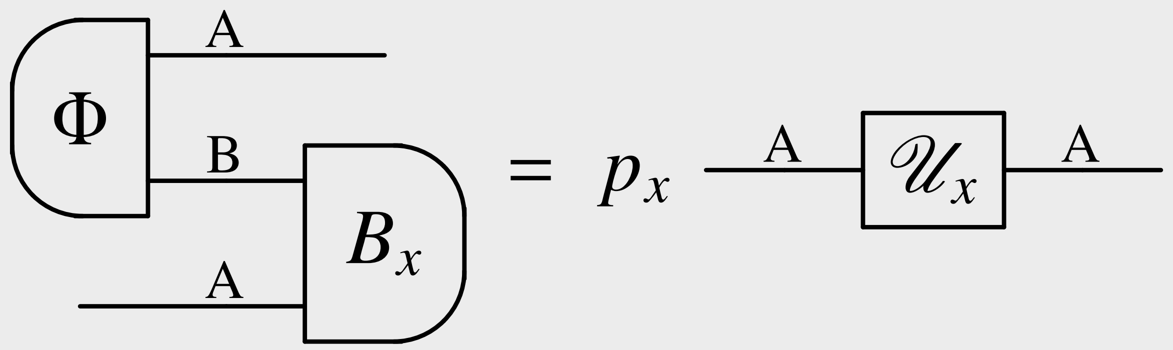
For fixed purifying system, every two purifications of the same state are connected by a reversible transformation on the purifying system

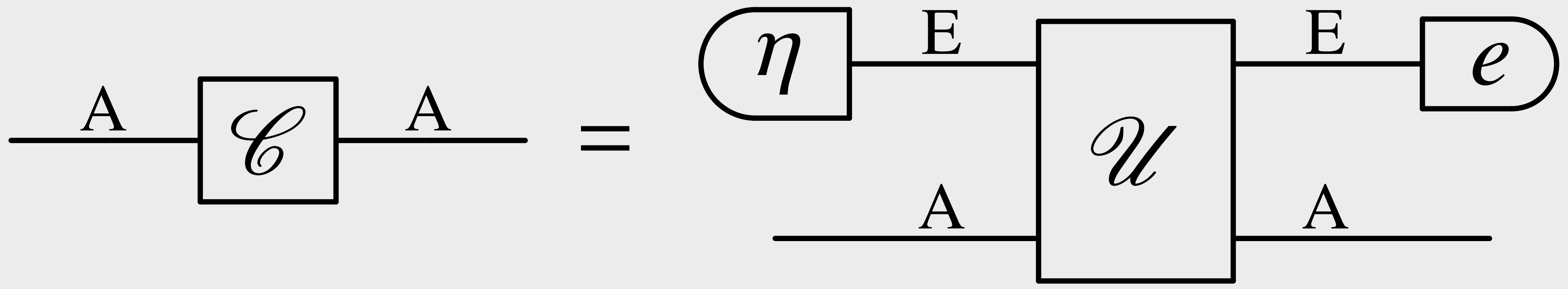
Consequences

6. Teleportation

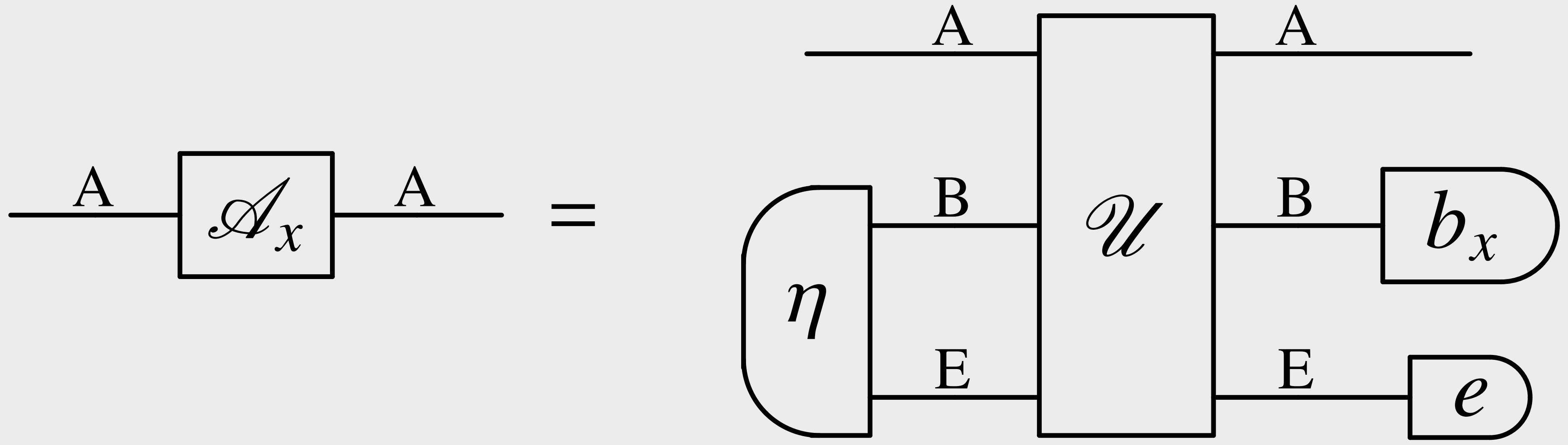


$\forall x \in$ X

7. Reversible dilation of "channels"



8. Reversible dilation of "instruments"



$\forall x \in$ X

9. State-transformation cone isomorphism

10. Reversible transform. for a system make a compact Lie group

Other OPTs

	Caus.	Perf. disc.	Loc. discr.	n-loc. discr.	At. par. comp.	At. seq. comp.	Compr.	∃ Purification	∃! Purification	NIWD
QT	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
CT	✓	✓	✓	✓	✓	✓	✓	✗	✗	✗
QBIT	✓	✓	✓	✓	✓	✓	✗	✓	✓	✓
FQT	✓	✓	✗	✓	✓	✓	✗	✓	✓	✓
RQT	✓	✓	✗	✓	✓	✓	✓	✓	✓	✓
NSQT	?	?	✗	✗	?	?	?	?	?	?
PR	✓	?	✓	✓	✓	?	✗	✗	✗	✓
DPR	✓	?	✓	✓	✓	?	✗	✗	✗	✓
HPR	✓	?	✓	✓	✓	✓	✓	✓	✓	✓
FOCT	✗	?	✓	✓	✓	?	?	✗	✗	?
FOQT	✗	?	?	✓	?	?	?	?	?	?
NLCT	✓	✓	✗	✓	✗	?	✓	✗	✗	✗
NLQT	?	?	?	✓	?	?	?	?	?	?

QT: Quantum theory

CT: Classical theory

QBIT: Qubit theory

FQT: Fermionic quantum theory

RQT: Real quantum theory

NSQT: Number superselected quantum theory

PR: PR-boxes theory

DPR: Dual PR-boxes theory

HPR: Hybrid PR-boxes theory

FOCT: First order classical theory

FOQT: First order quantum theory

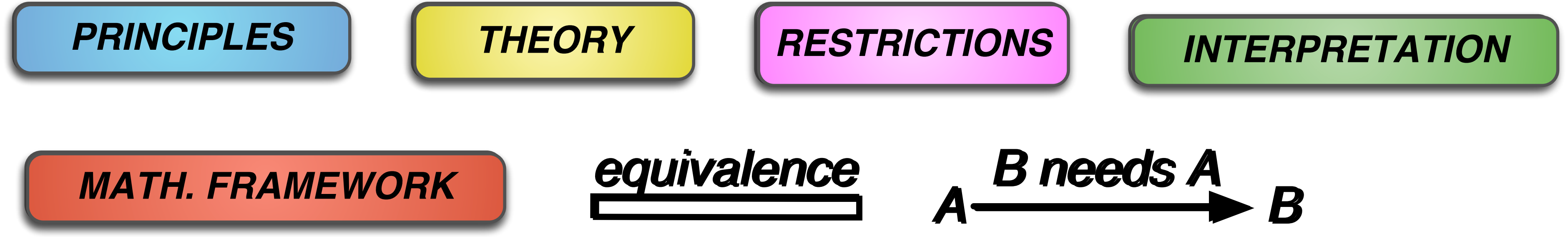
NLCT: Non-local classical theory

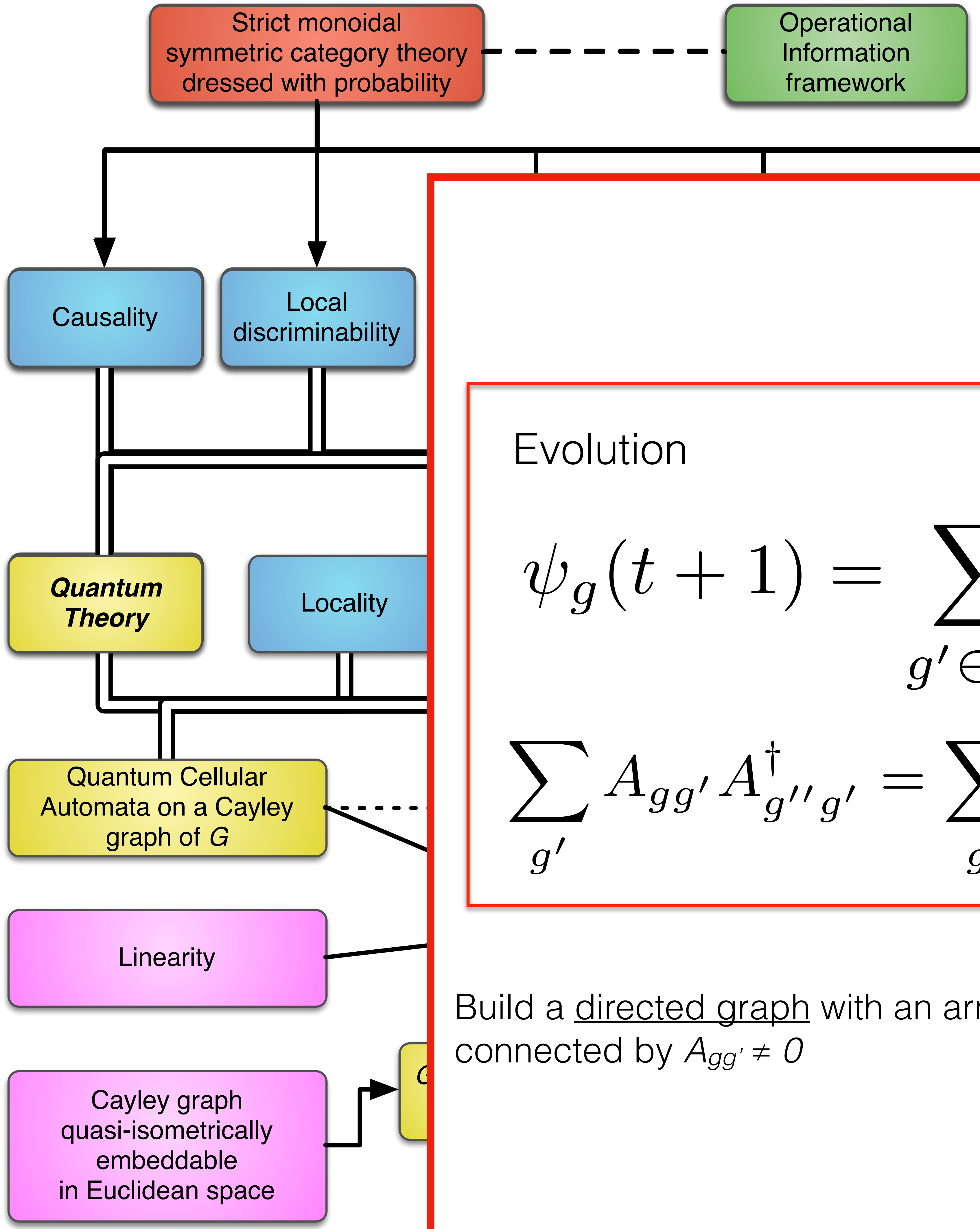
NLQT: Non-local quantum theory

“HOW TO GET THE “MECHANICS?”

QUANTUM FIELD THEORY: an ultra-short account

PRINCIPLES
THEORY
RESTRICTIONS
INTERPRETATION
MATH. FRAMEWORK
equivalence
B needs A
A
B
Info-theoretical principles for Quantum Field Theory
Strict monoidal symmetric category theory dressed with probability
Operational Information framework
Causality
Local discriminability
Purification
Atomicity of composition
Perfect discriminability
Ideal compressibility
Quantum Theory
Locality
Homogeneity
Isotropy
Unitarity
Quantum Cellular Automata on a Cayley graph of G
Linearity
Quantum Walk on Cayley graph of G
Cayley graph quasi-isometrically embeddable in Euclidean space
G virtually Abelian
Quantum Walk on Cayley graph of Abelian G
Relativity Principle without space-time
m>0: deformed De Sitter
Particle as Poincaré invariant
m=0: deformed Lorentz
GR heuristics
Relativistic limit
Free Quantum Field Theory
LTM Standards
$m \in S^1$
discrete proper time



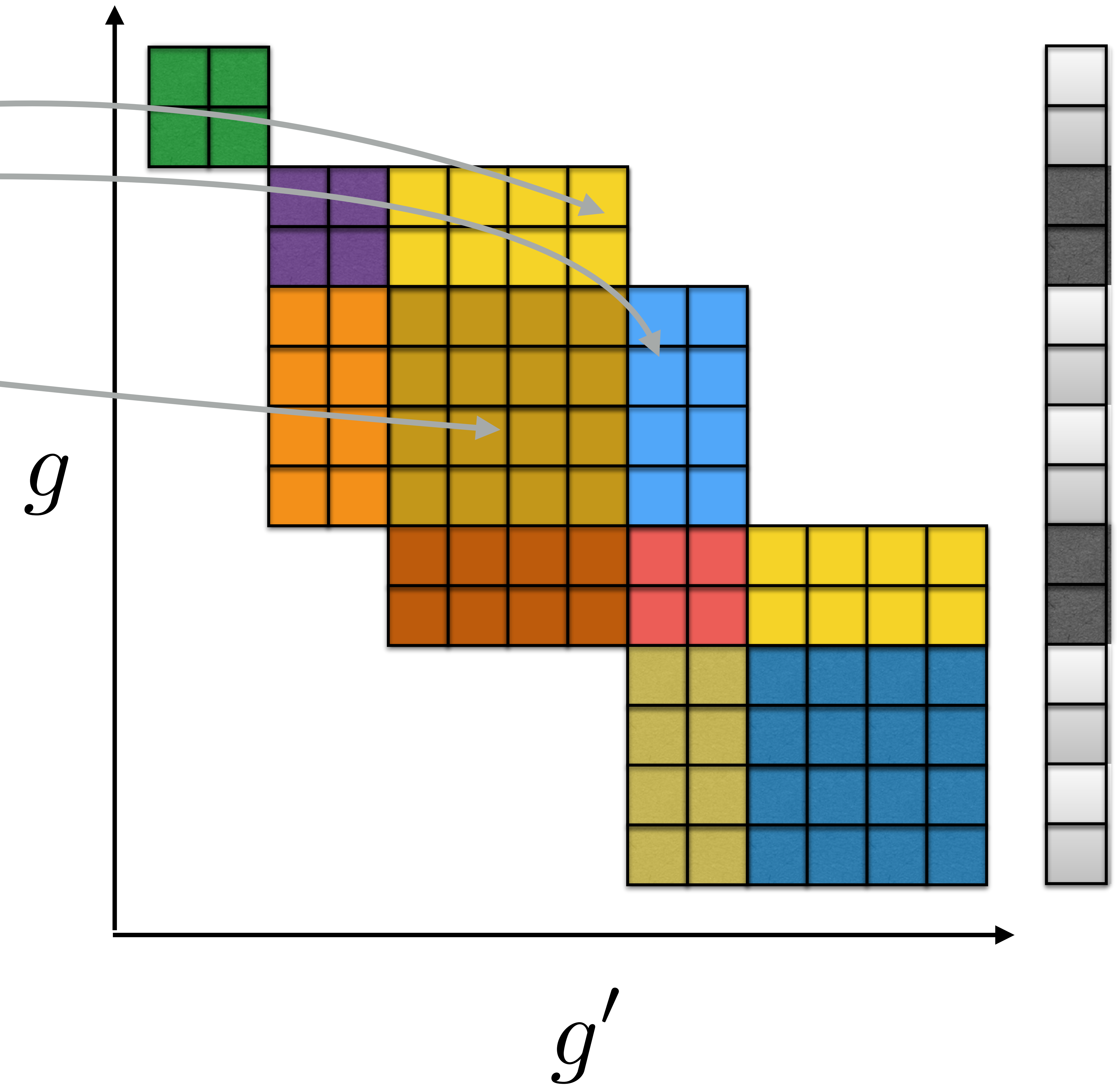


$$\mathcal{H} = \oplus_{g\in G}\mathbb{C}^{s_g} \quad |G| \leqslant \aleph,\ s_g \in \mathbb{N}$$

Evolution

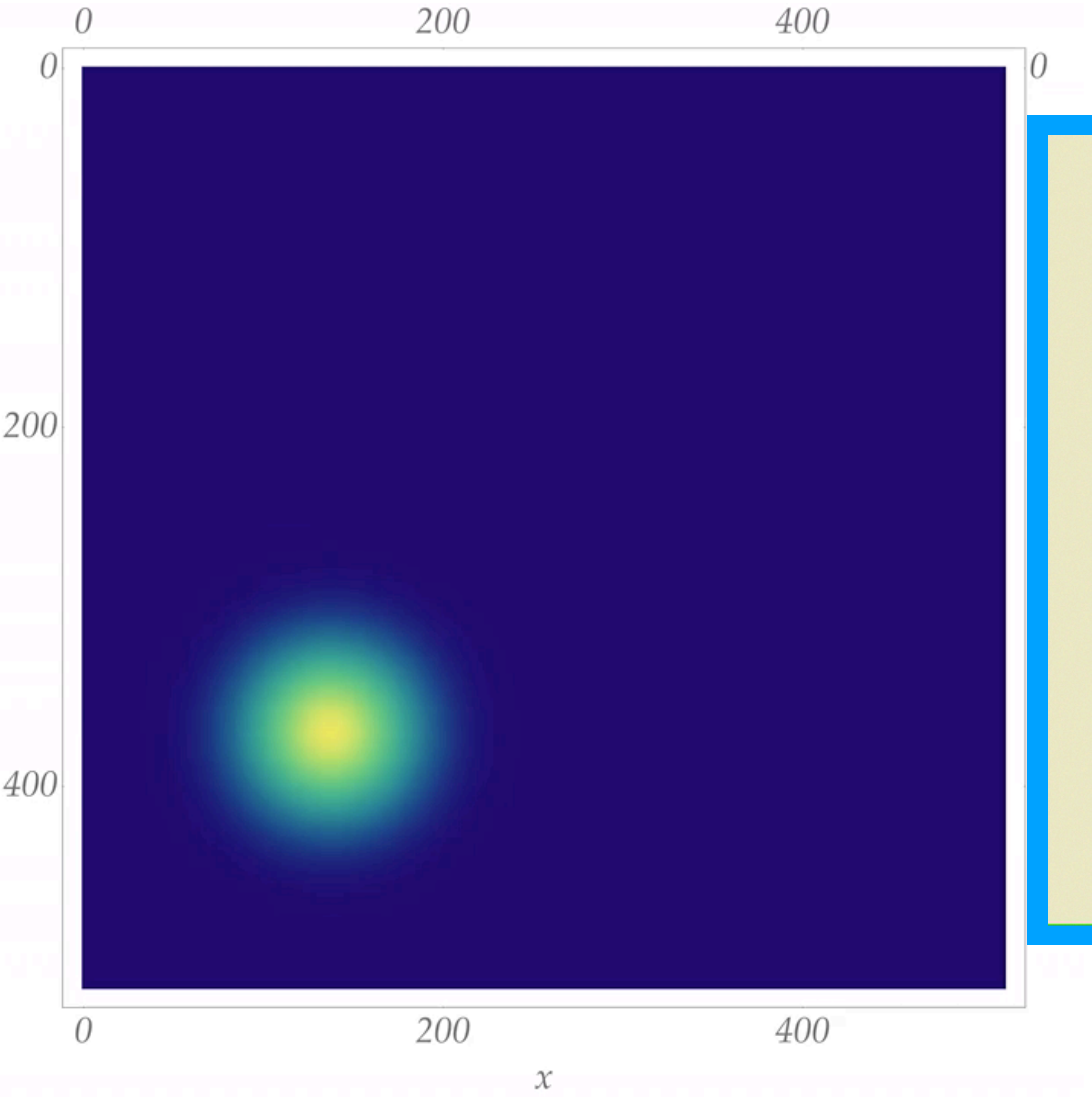
$$\psi_g(t+1) = \sum_{g'\in S_g} A_{gg'}\psi_{g'}(t)$$

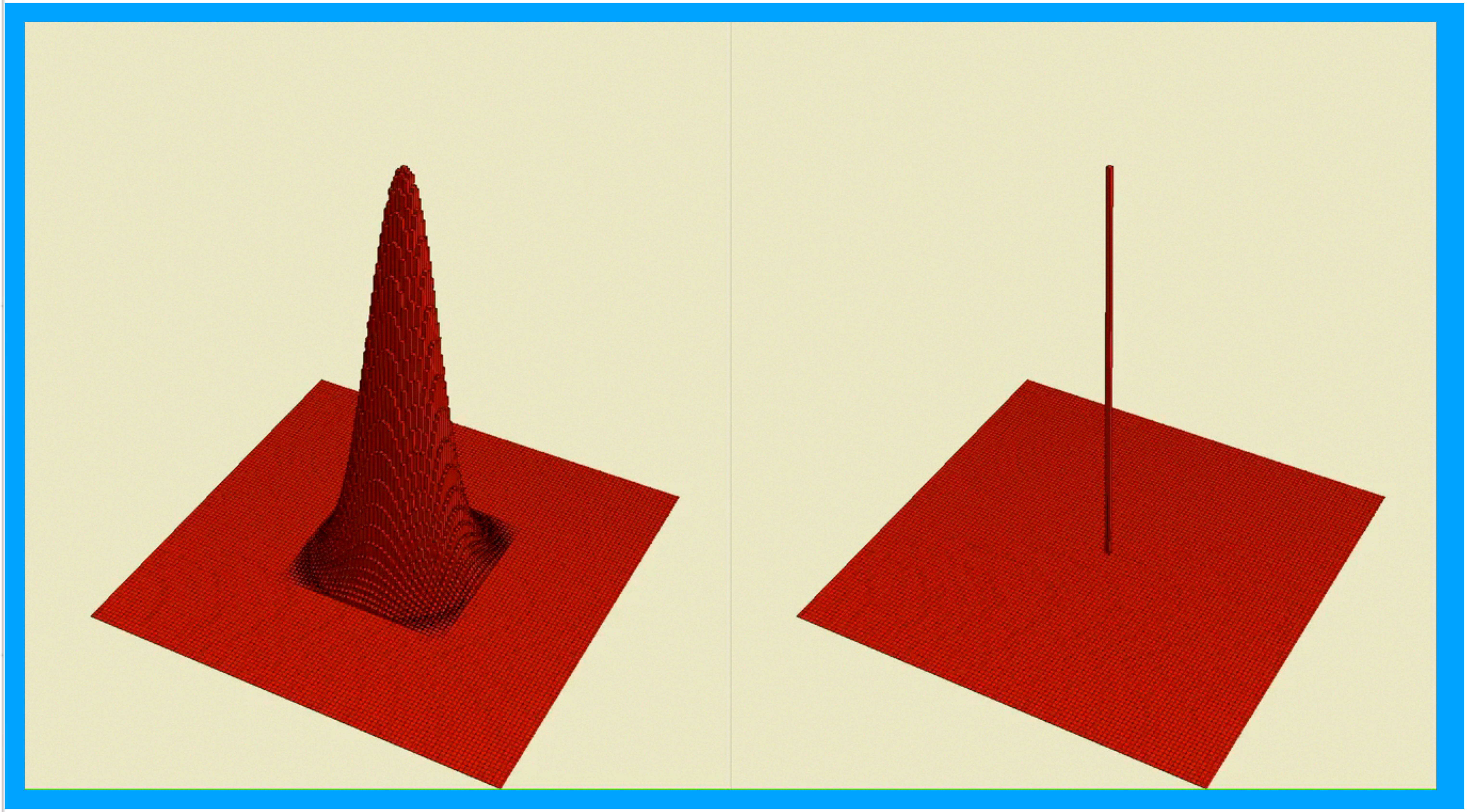
$$\sum_{g'} A_{gg'}A^\dagger_{g''g'} = \sum_{g'} A^\dagger_{gg'}A_{g''g'} = \delta_{gg''}I_{s_g}$$



Build a directed graph with an arrow from g to g' *wherever they* are connected by $A_{gg'} \neq 0$

Info-theoretical principles for Quantum Field Theory





“NO PURIFICATION ONTOLOGY”

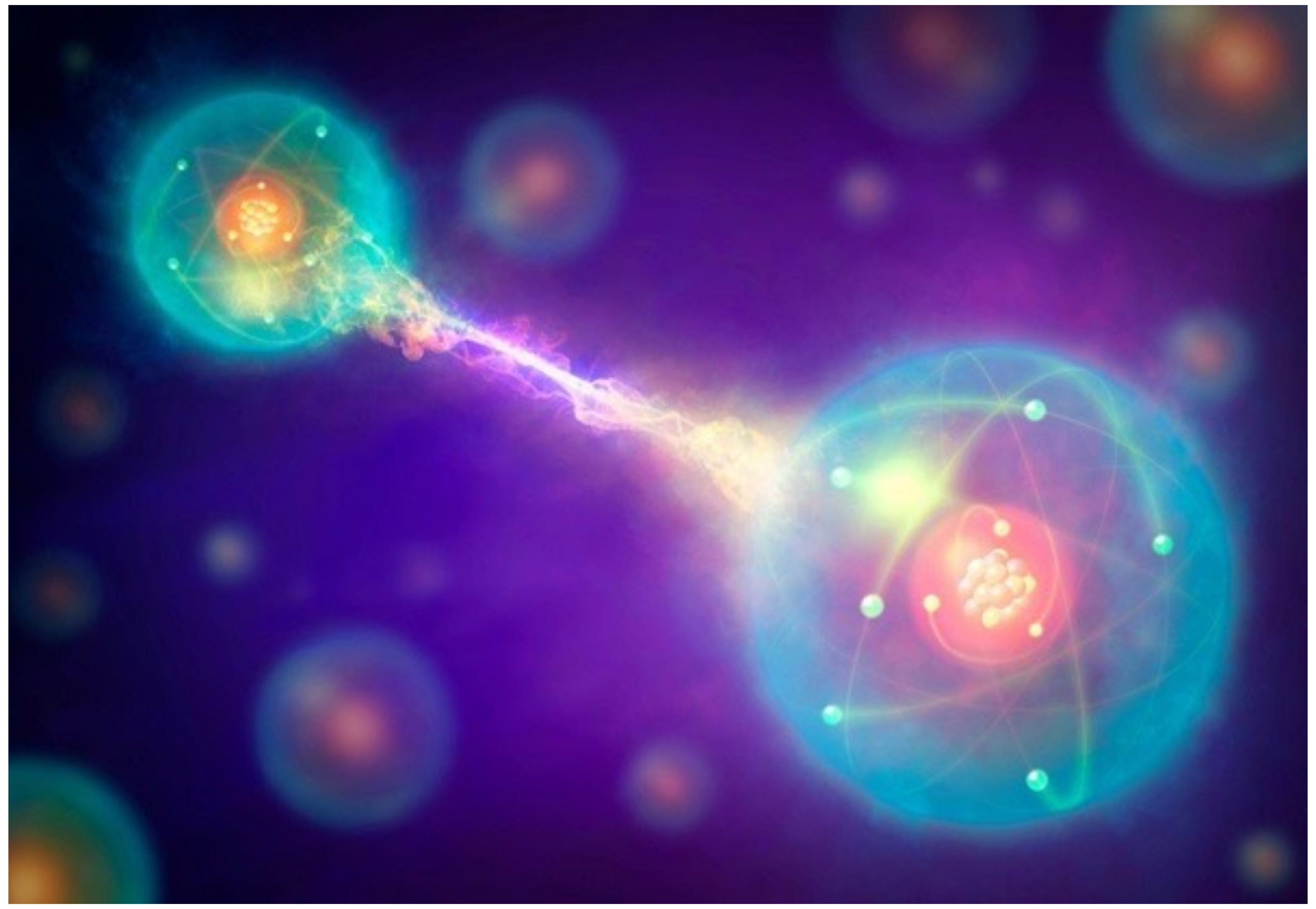
NO PARADOXES!

Quantum Theory: no purification ontology

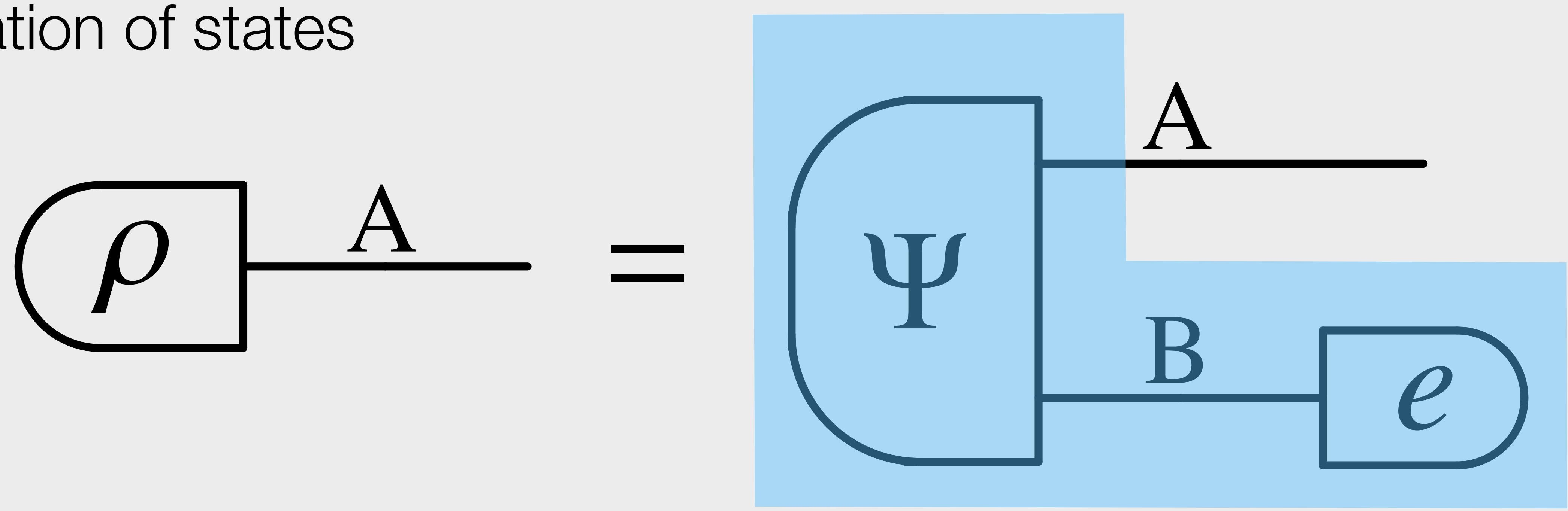
Unfalsifiable ontologies!

P3. Purification

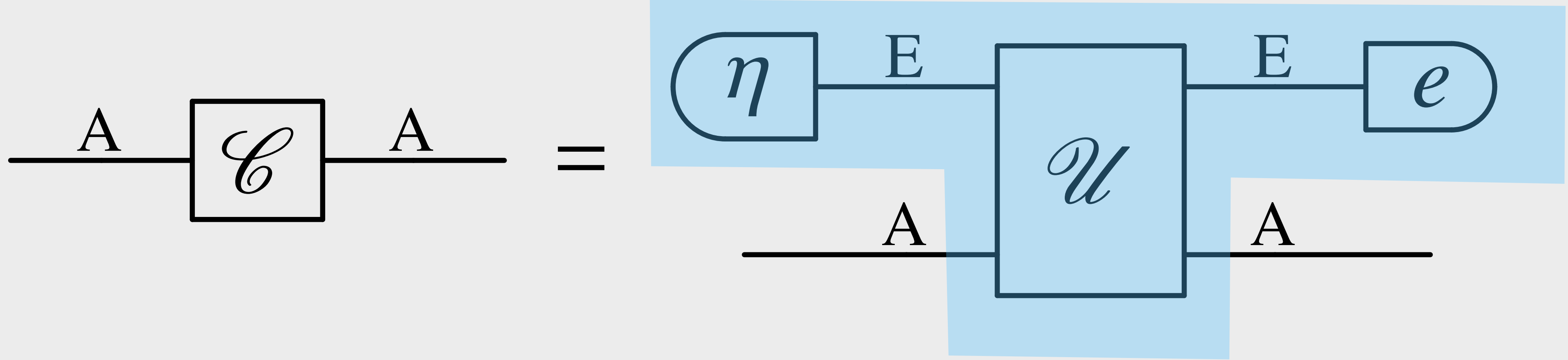
1. Isolated systems don't need to be in a pure state!
2. Isolated systems don't need to undergo unitary transformations!



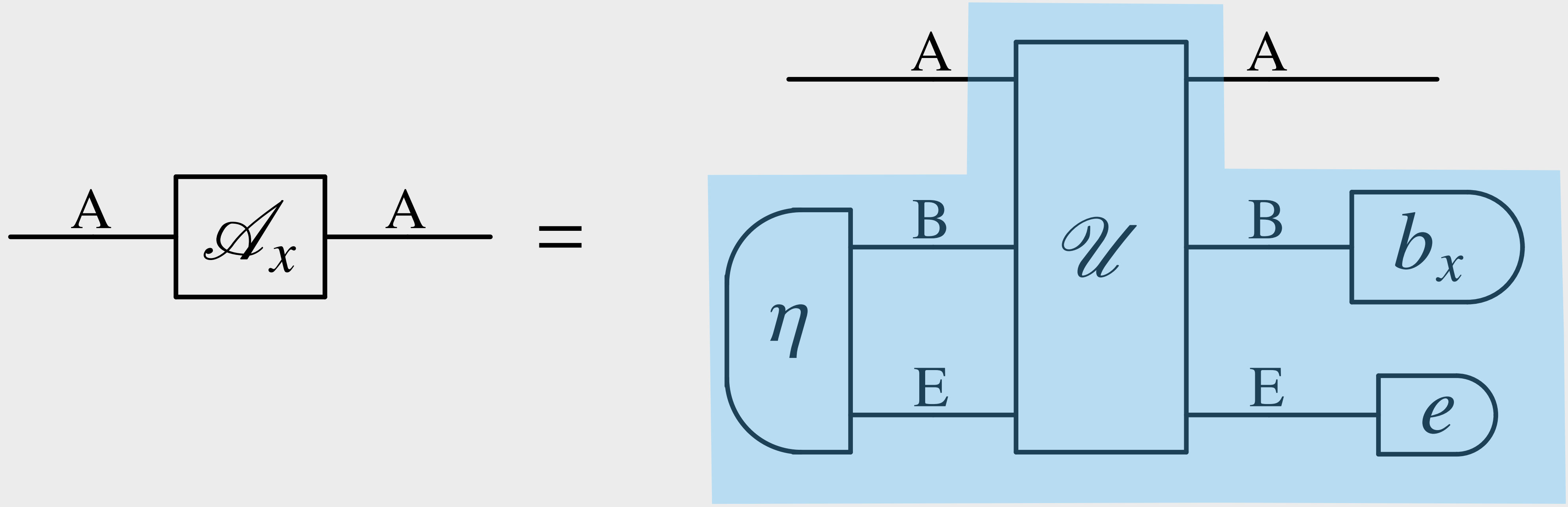
Purification of states



Unitary purification of channels



Unitary dilation of quantum instruments



$\forall x \in X$

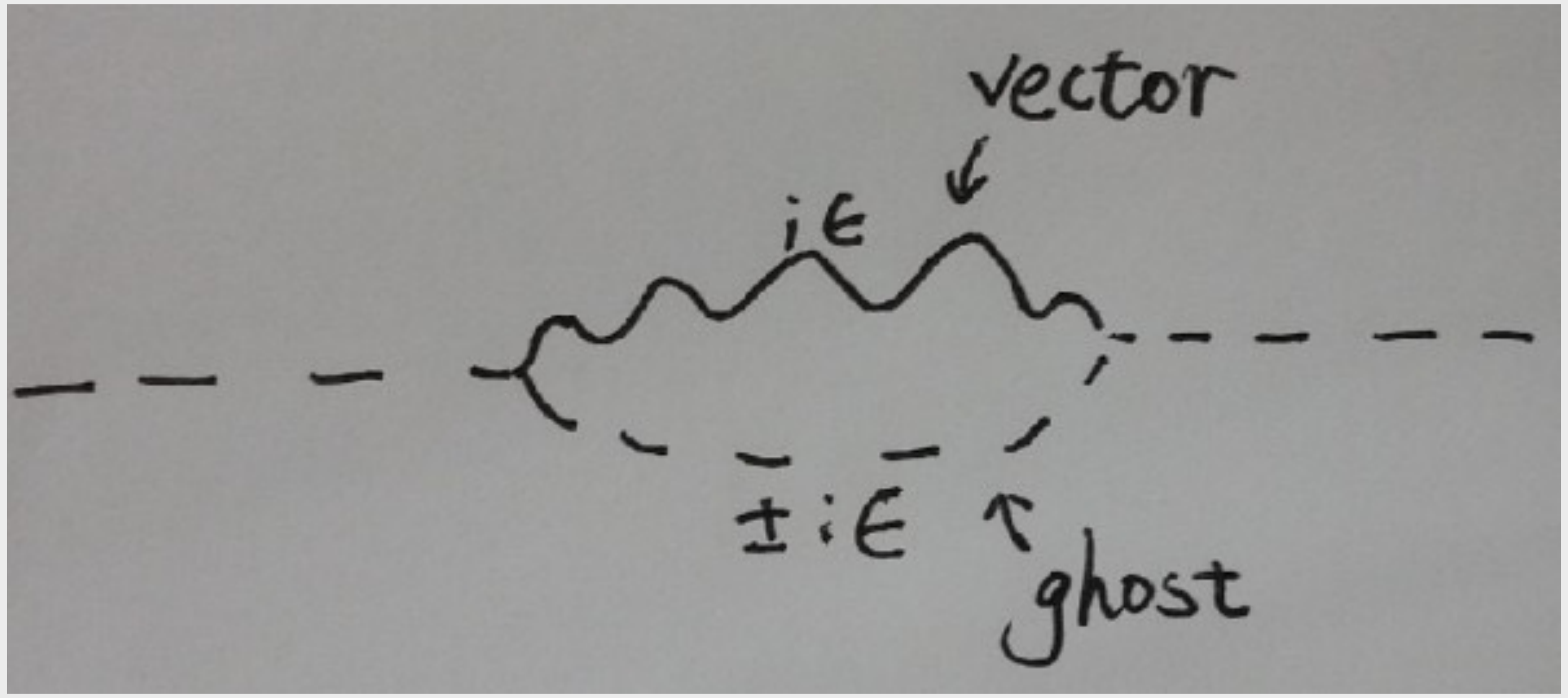
Quantum Theory: no purification ontology

P3. $\boxed{\text{Purification}}$

1. Isolated systems don't need to be in a pure state
2. Isolated systems don't need to undergo unitary transformations

The necessity for Faddeev–Popov ghosts follows from the requirement that quantum field theories yield unambiguous, non-singular solutions. This is not possible in the path integral formulation when a gauge symmetry is present since there is no procedure for selecting among physically equivalent solutions related by gauge transformation. The path integrals overcount field configurations corresponding to the same physical state; the measure of the path integrals contains a factor which does not allow obtaining various results directly from the action.
It is possible, however, to modify the action, such that methods such as Feynman diagrams will be applicable by adding ghost fields which break the gauge symmetry. **The ghost fields do not correspond to any real particles in external states: they appear as virtual particles in Feynman diagrams – or as the absence of gauge configurations. However, they are a necessary computational tool to preserve unitarity.**

Unitarity in quantum field theory?

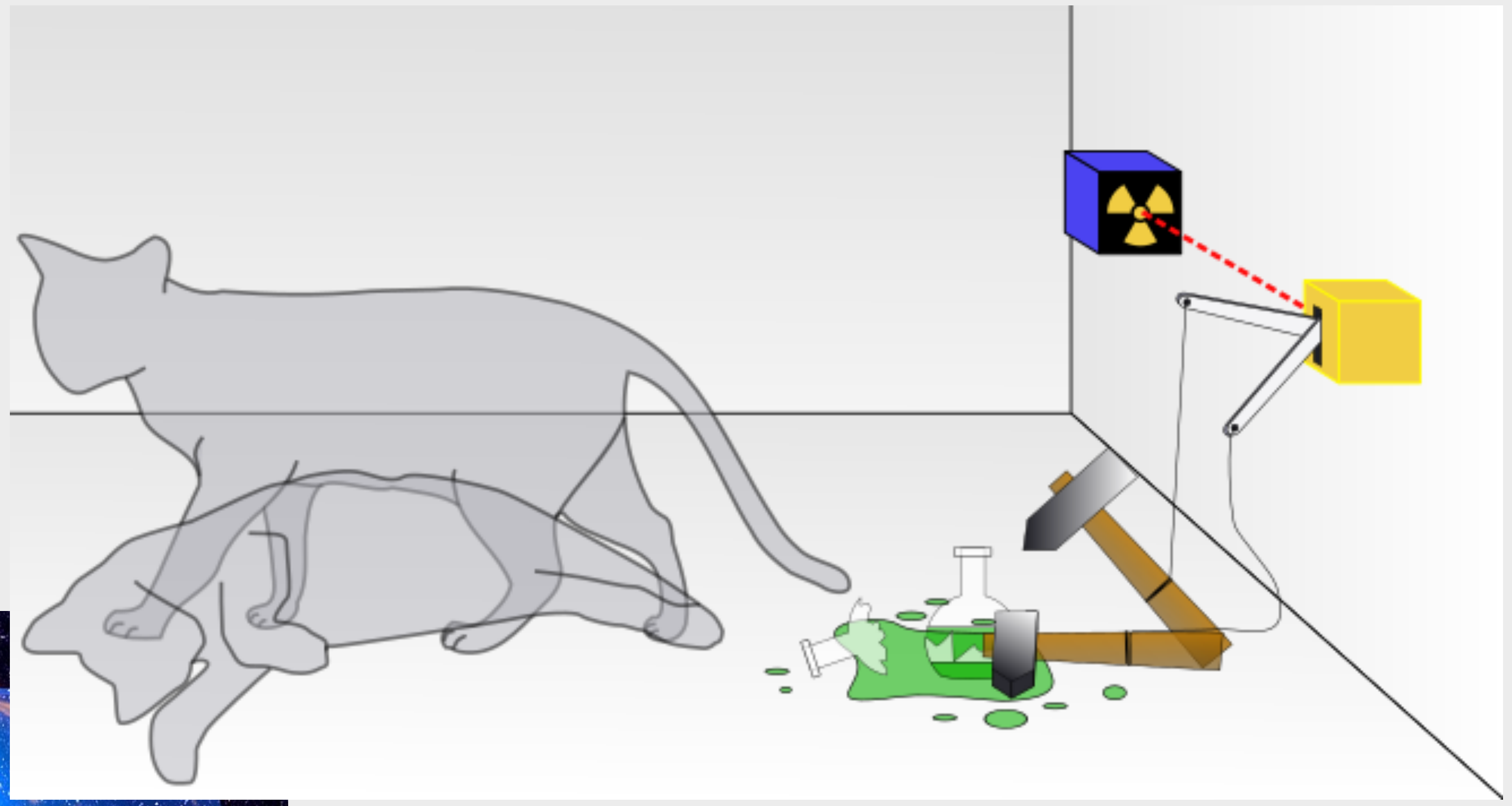


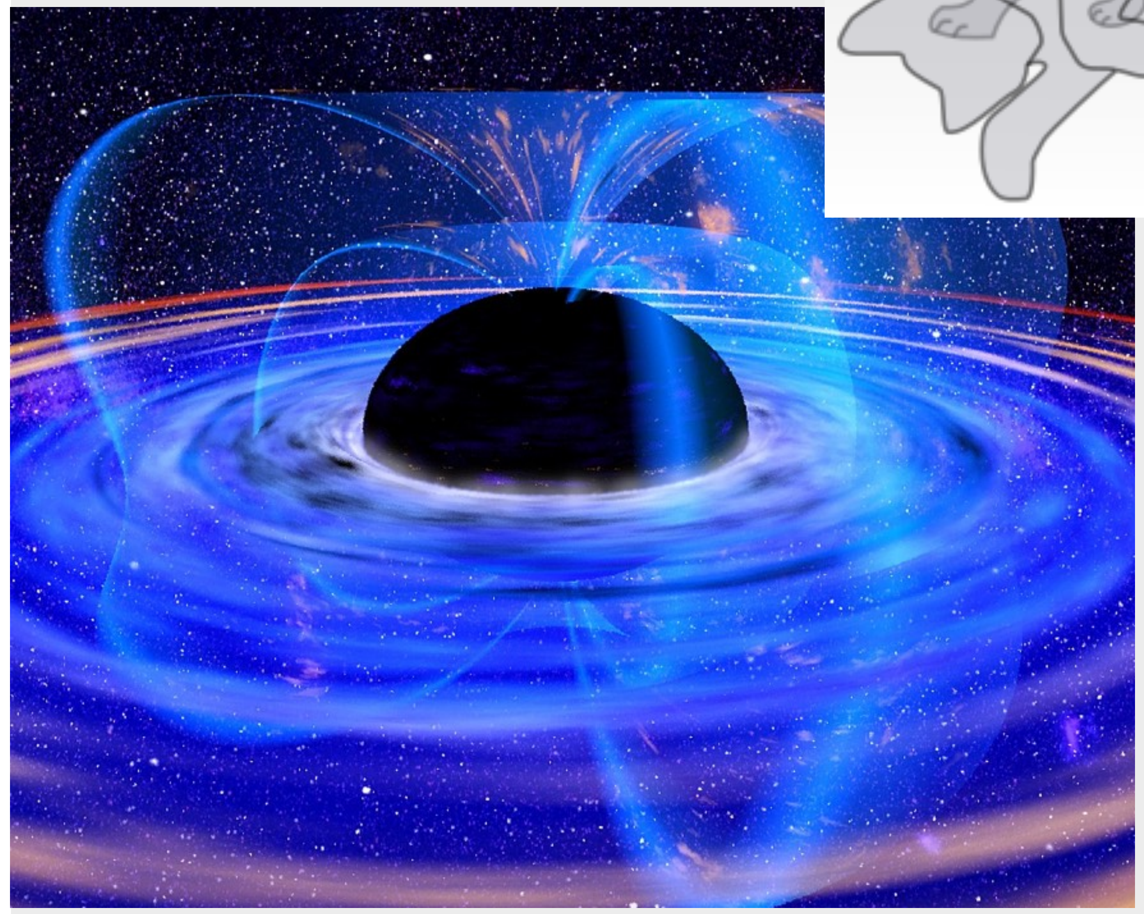
Quantum Theory: no purification ontology

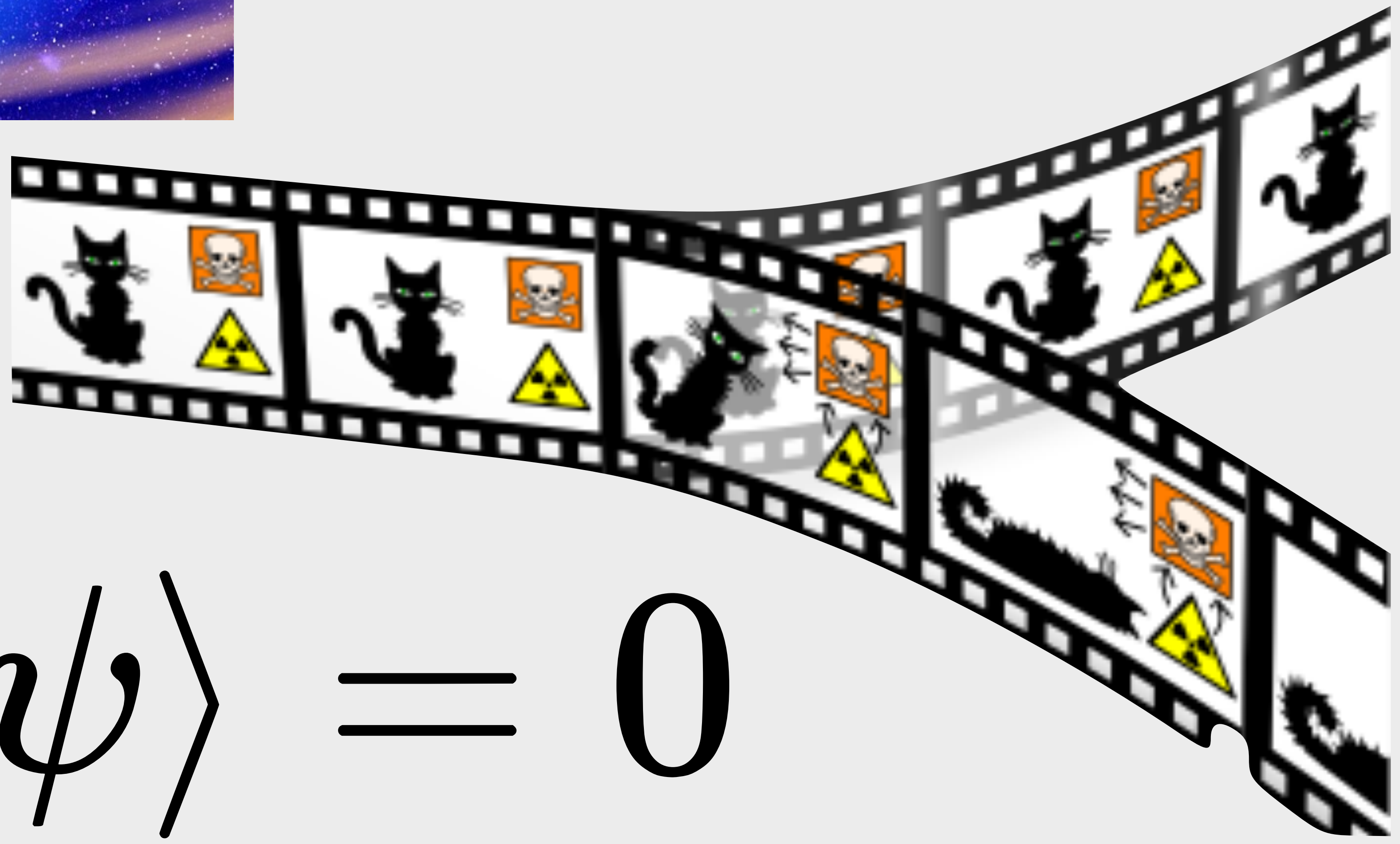
No paradoxes, and more …

P3. Purification

1. Isolated systems don't need to be in a pure state
2. Isolated systems don't need to undergo unitary transformations







$$H(x)|\psi\rangle = 0$$

This is more or less what I wanted to say

THANK YOU!

A Quantum-Digital Universe, Grant ID: 43796

Quantum Causal Structures, Grant ID: 60609

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Follow **project on Researchgate**: *The algorithmic paradigm: deriving the whole physics from information-theoretical principles.*

REVIEW

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