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La teoria quantistica è una teoria dell’informazione

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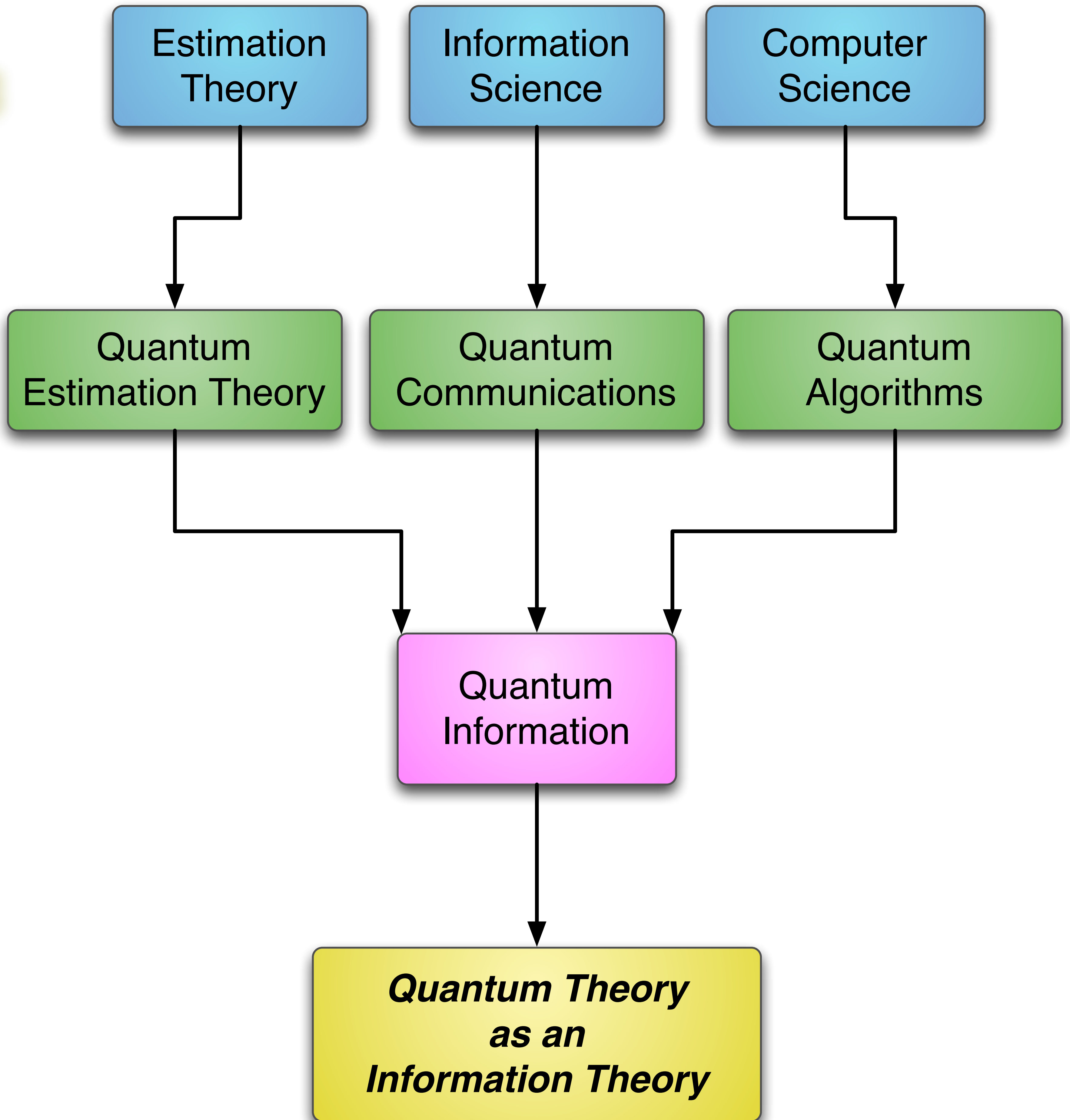
Venerdì 1 Luglio 2016


Estimation Theory
Information Science
Computer Science
Quantum Estimation Theory
Quantum Communications
Quantum Algorithms
Quantum Information
Quantum Theory as an Information Theory

Quantum Theory is an Information Theory

Selected for a Viewpoint in *Physics*

PHYSICAL REVIEW A **84**, 012311 (2011)

Informational derivation of quantum theory

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We derive quantum theory from purely informational principles. Five elementary axioms—causality, perfect distinguishability, ideal compression, local distinguishability, and pure conditioning—define a broad class of theories of information processing that can be regarded as standard. One postulate—purification—singles out quantum theory within this class.

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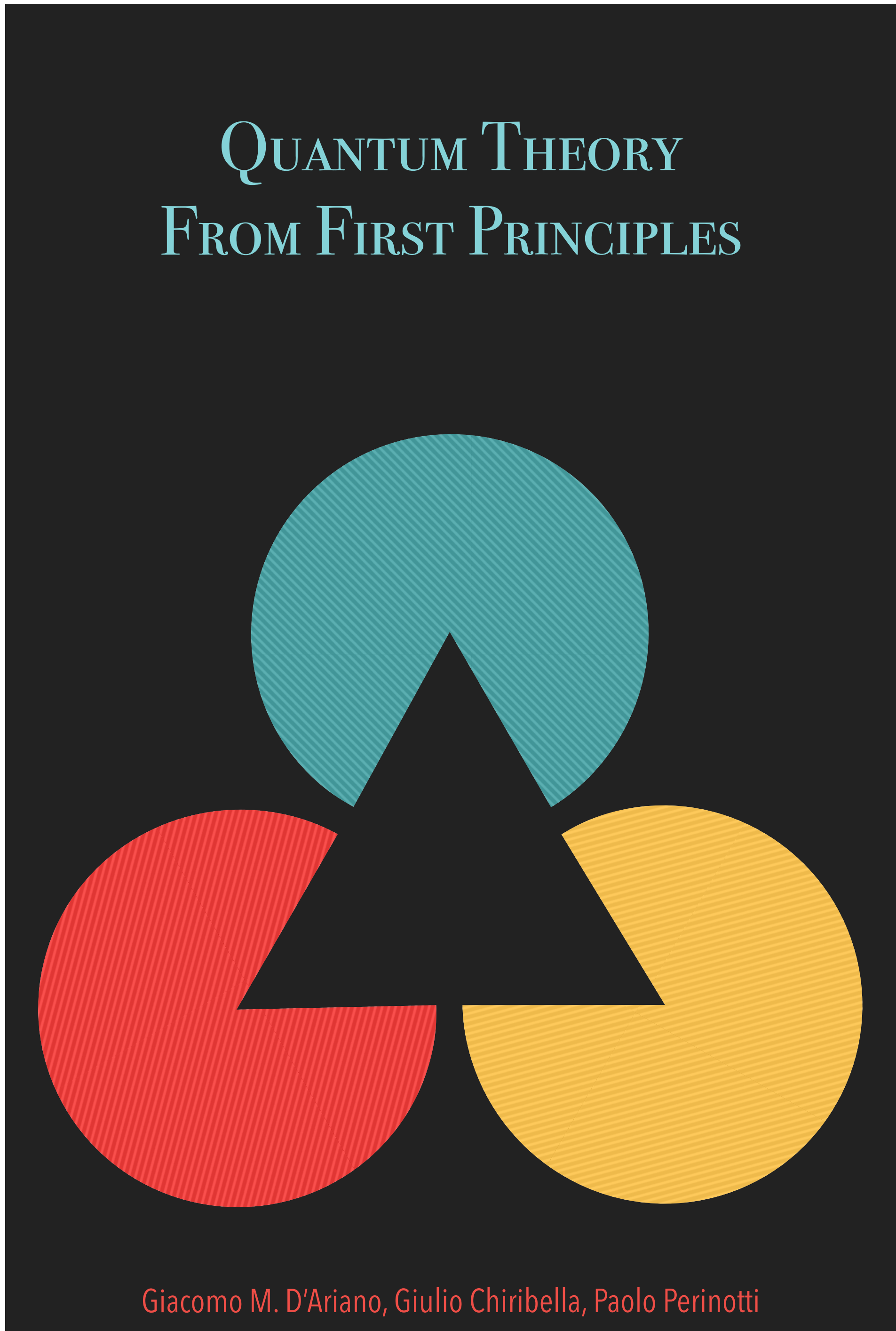


Operational Probabilistic Theory

The framework

Logic ⊂ Probability ⊂ OPT

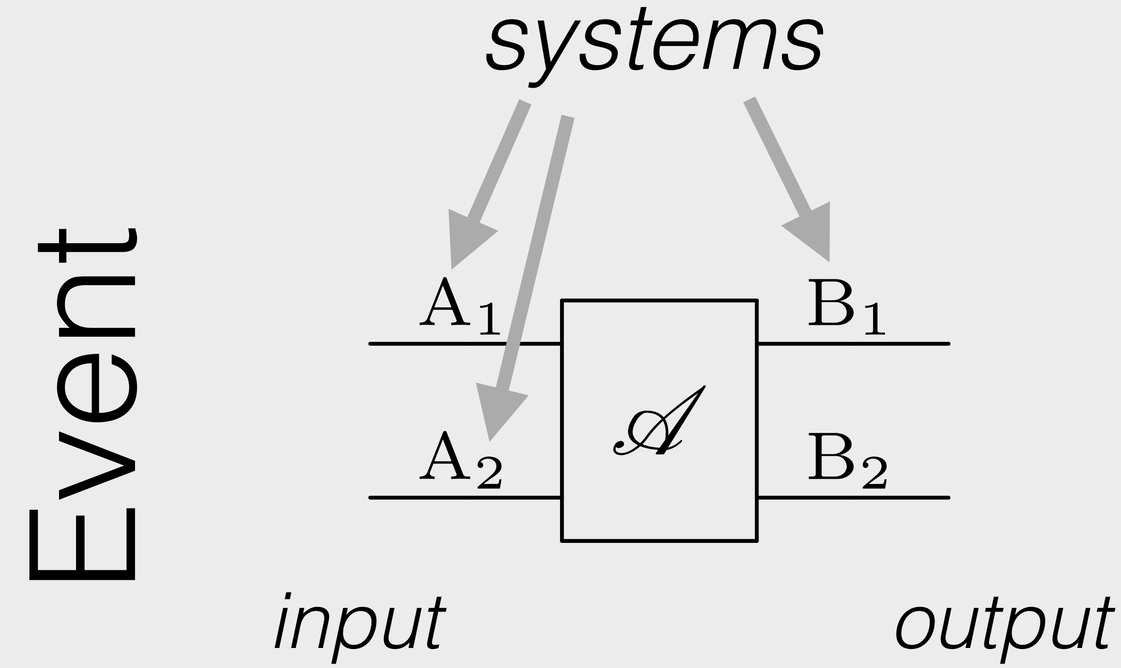
joint probabilities + connectivity

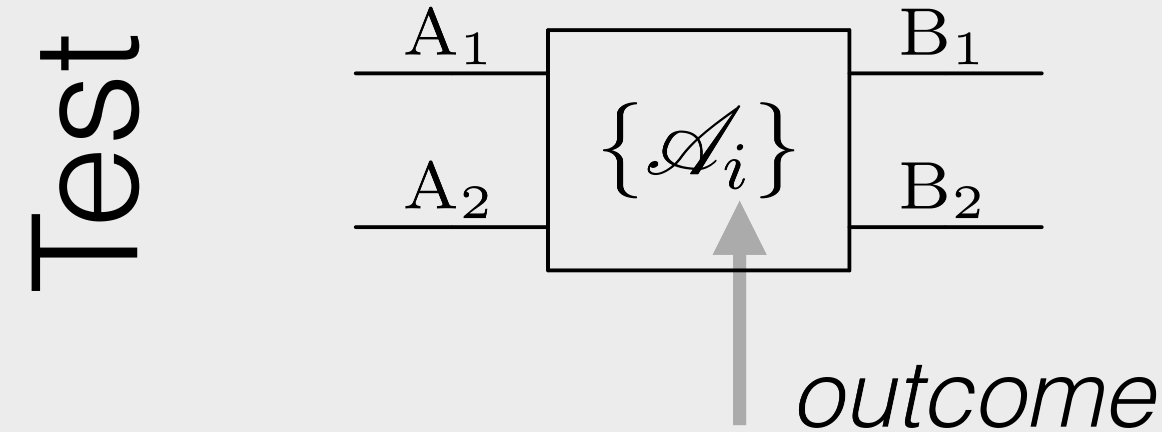
$$p(i, j, k, \ldots | \text{circuit})$$

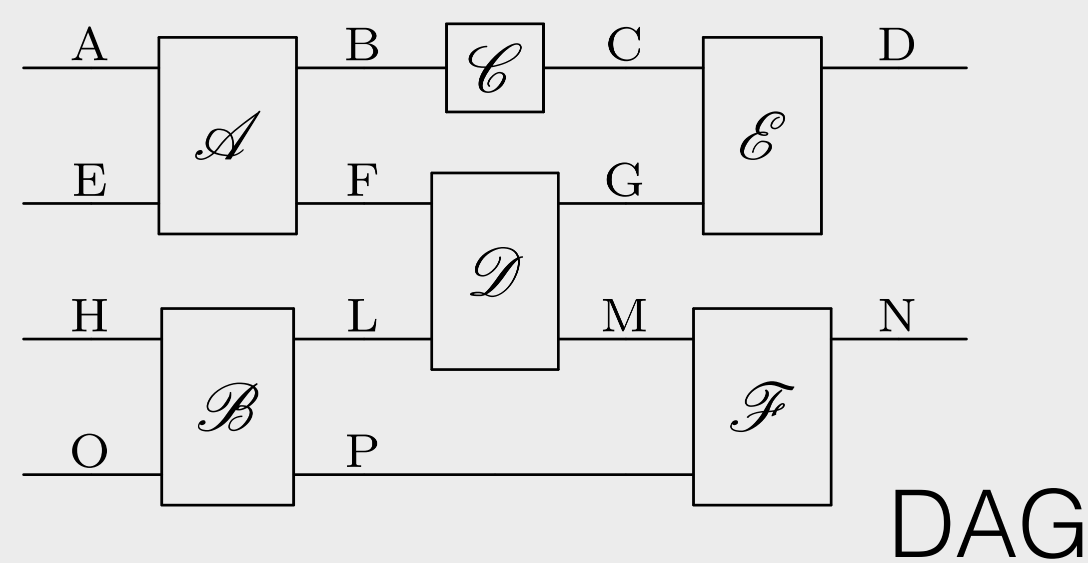
Marginal probability

$$\sum_{i,k,\ldots} p(i, j, k, \ldots | \text{circuit}) =$$

$$p(j | \text{circuit})$$







Operational Probabilistic Theory

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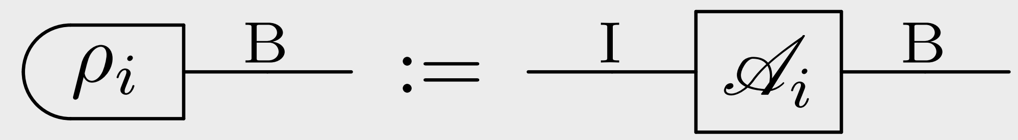
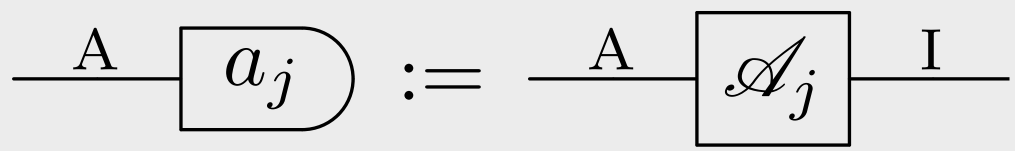
$$p(i, j, k, \ldots | \text{circuit})$$

Notice: the probability of a "preparation" generally depends on the circuit at its output.

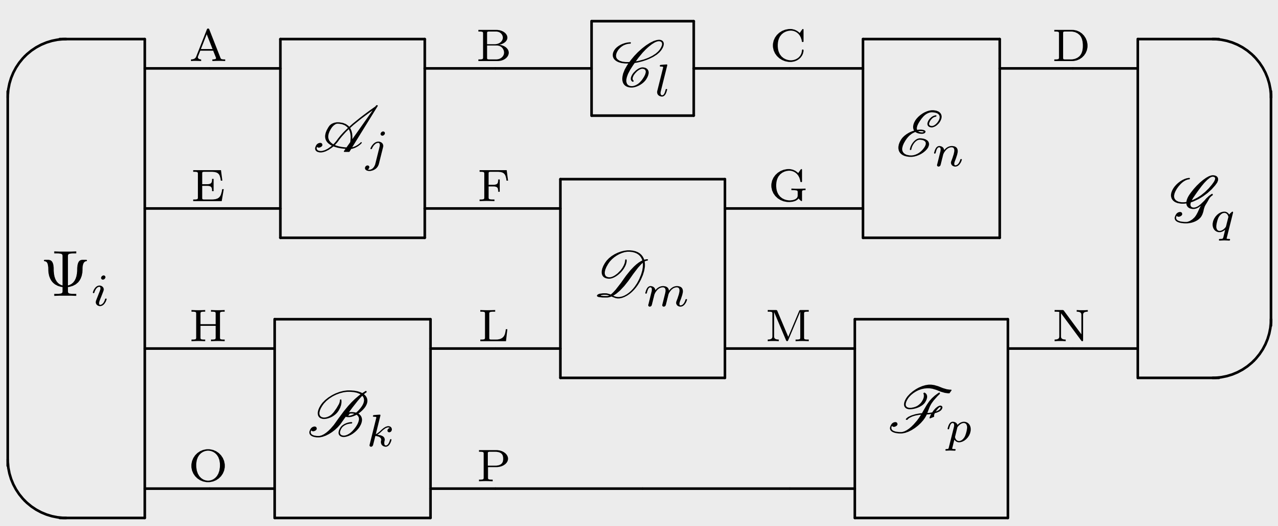


preparation



observation

$$p(i, j, k, l, m, n, p, q | \text{circuit})$$



Operational Probabilistic Theory

The framework

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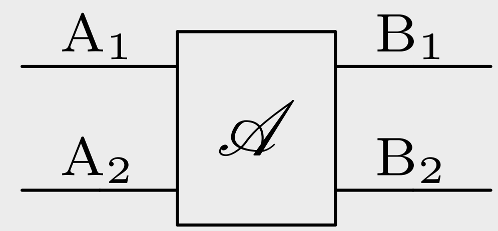
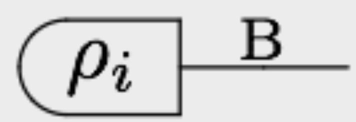
joint probabilities + connectivity

Probabilistic equivalence classes

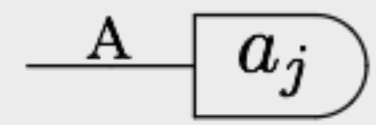
Notice: the probability of a transformation generally depends on the circuit at its output!!



transformation

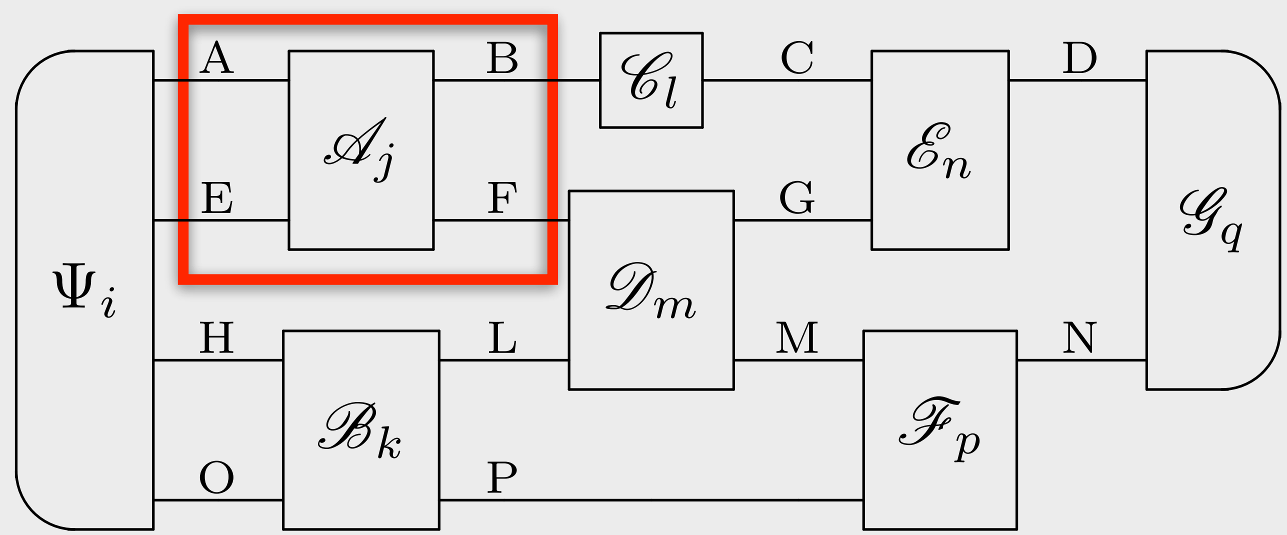


state



effect

$$p(i, j, k, l, m, n, p, q|\text{circuit})$$



Operational Probabilistic Theory

The framework

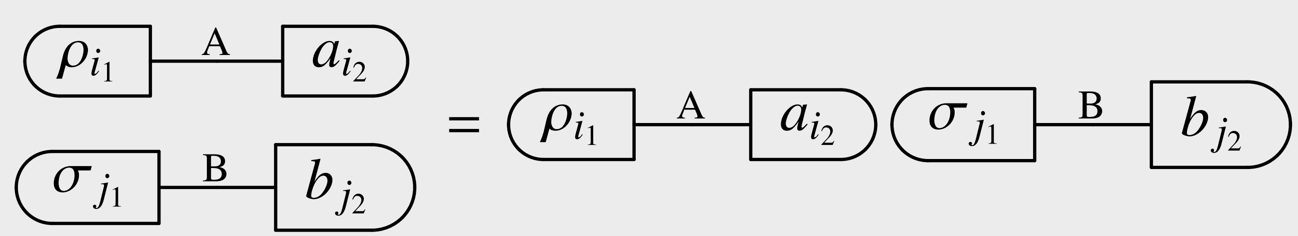
Logic ⊂ Probability ⊂ OPT

joint probabilities + connectivity

Probabilistic equivalence classes

Strictly symmetric monoidal category theory

Multiplication of closed circuits



$$= p(i_1, i_2)\, q(j_1, j_2)$$

Operational Probabilistic Theory

Sequential composition (associative)

$$\overset{\mathrm{A}}{\text{---}}\boxed{\{\mathscr{A}_x\}_{x\in \mathrm{X}}}\overset{\mathrm{B}}{\text{---}}\boxed{\{\mathscr{B}_y\}_{y\in \mathrm{Y}}}\overset{\mathrm{C}}{\text{---}} \quad =: \quad \overset{\mathrm{A}}{\text{---}}\boxed{\{\mathscr{B}_x \circ \mathscr{A}_y\}_{(x,y)\in \mathrm{X}\times\mathrm{Y}}}\overset{\mathrm{C}}{\text{---}}$$

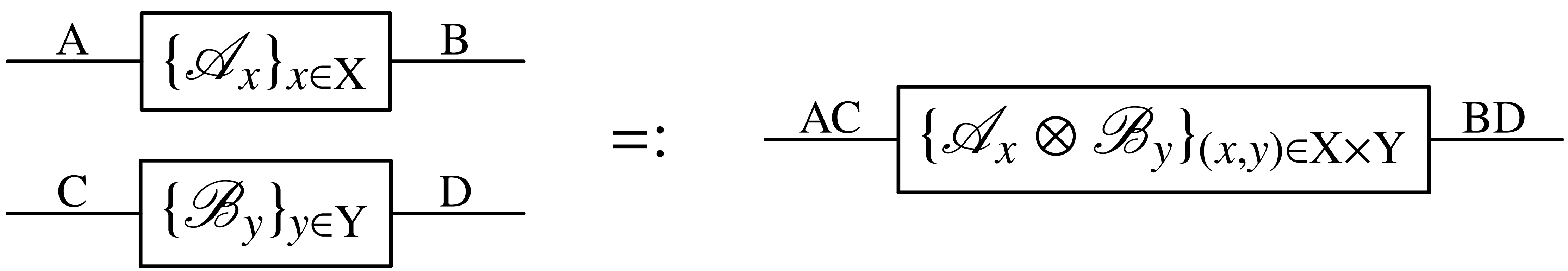
Identity test

$$\overset{\mathrm{A}}{\text{---}}\boxed{\mathscr{I}_{\mathrm{A}}}\overset{\mathrm{A}}{\text{---}}\boxed{\mathscr{C}}\overset{\mathrm{B}}{\text{---}} \quad = \quad \overset{\mathrm{A}}{\text{---}}\boxed{\mathscr{C}}\overset{\mathrm{B}}{\text{---}}$$

$$\overset{\mathrm{B}}{\text{---}}\boxed{\mathscr{D}}\overset{\mathrm{A}}{\text{---}}\boxed{\mathscr{I}_{\mathrm{A}}}\overset{\mathrm{A}}{\text{---}} \quad = \quad \overset{\mathrm{B}}{\text{---}}\boxed{\mathscr{D}}\overset{\mathrm{A}}{\text{---}}$$

Operational Probabilistic Theory

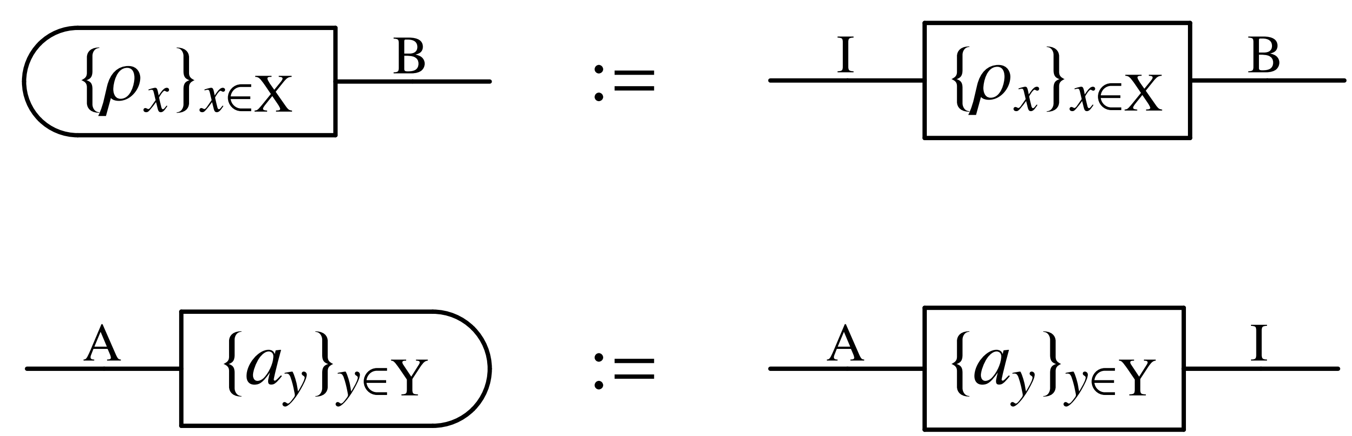
Parallel composition (associative)



$$\mathrm{AB} = \mathrm{BA}$$

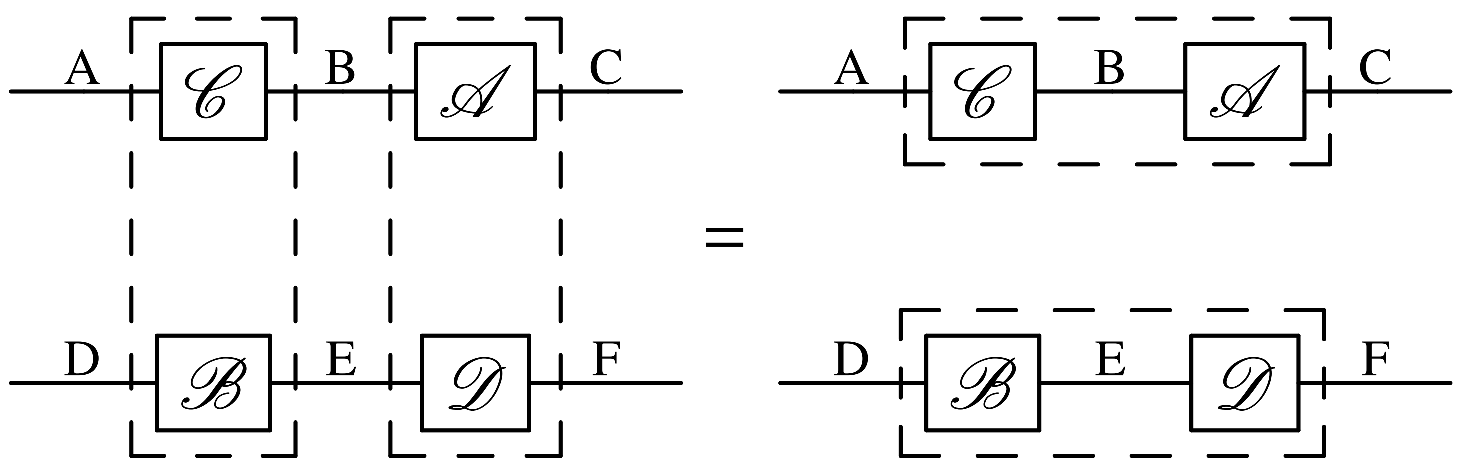
$$\mathrm{AI} = \mathrm{IA} = \mathrm{A}$$

$$\mathrm{A(BC)} = \mathrm{(AB)C}$$

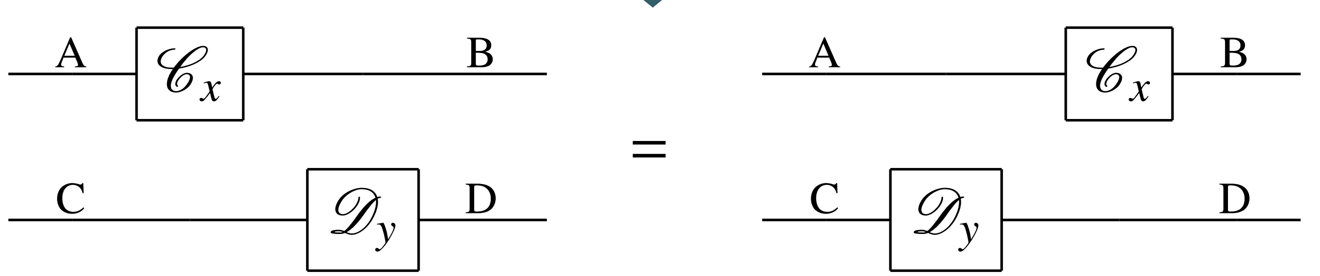


Operational Probabilistic Theory

Sequential and parallel compositions commute



$$(\mathscr{A} \otimes \mathscr{D}) \circ (\mathscr{C} \otimes \mathscr{B}) = (\mathscr{A} \circ \mathscr{C}) \otimes (\mathscr{D} \circ \mathscr{B})$$



wire-stretching

(foliations)

Operational Probabilistic Theory

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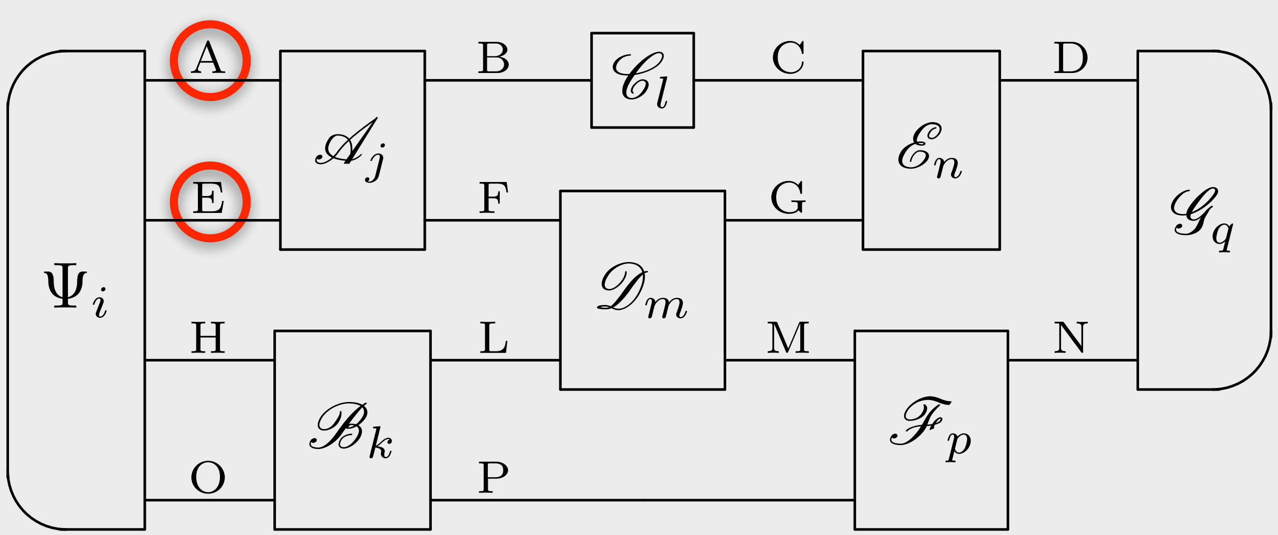
$$p(i, j, k, \ldots | \text{circuit})$$

independent systems

$$p(i, j, k, l, m, n, p, q | \text{circuit})$$



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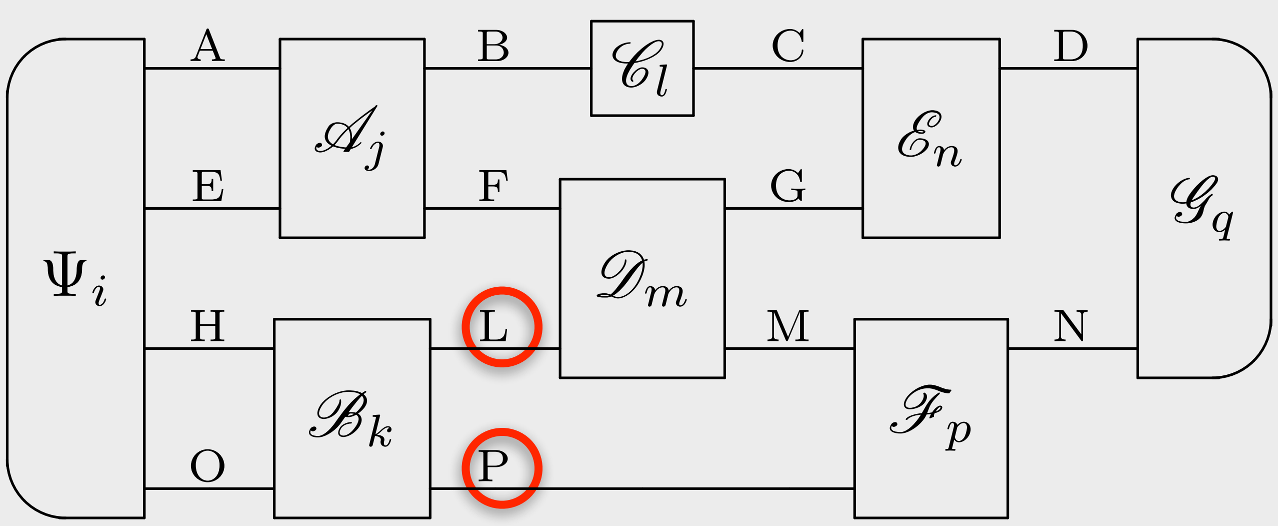

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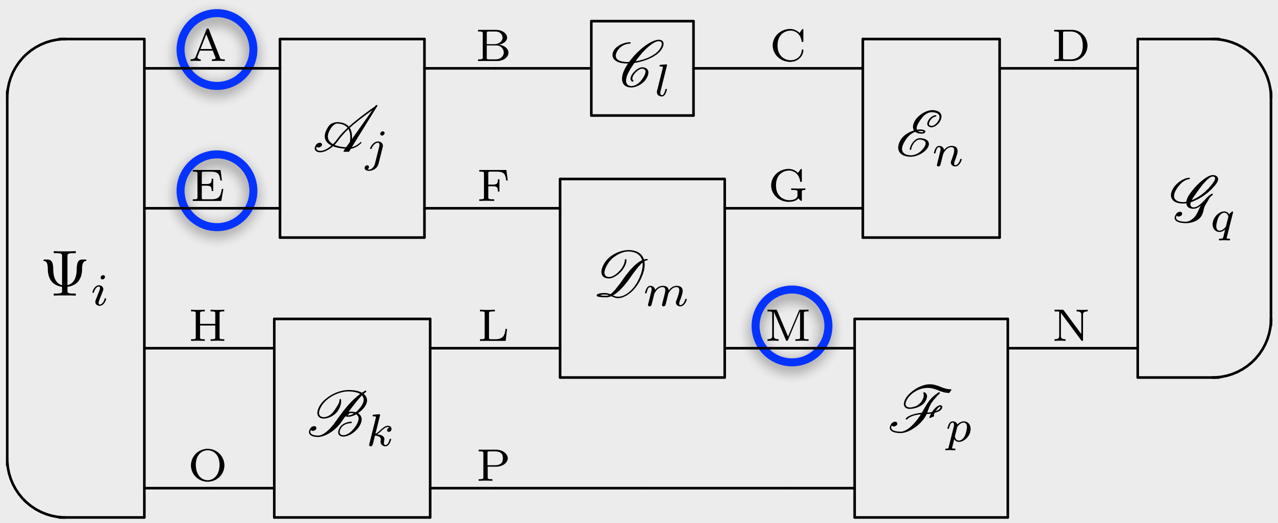

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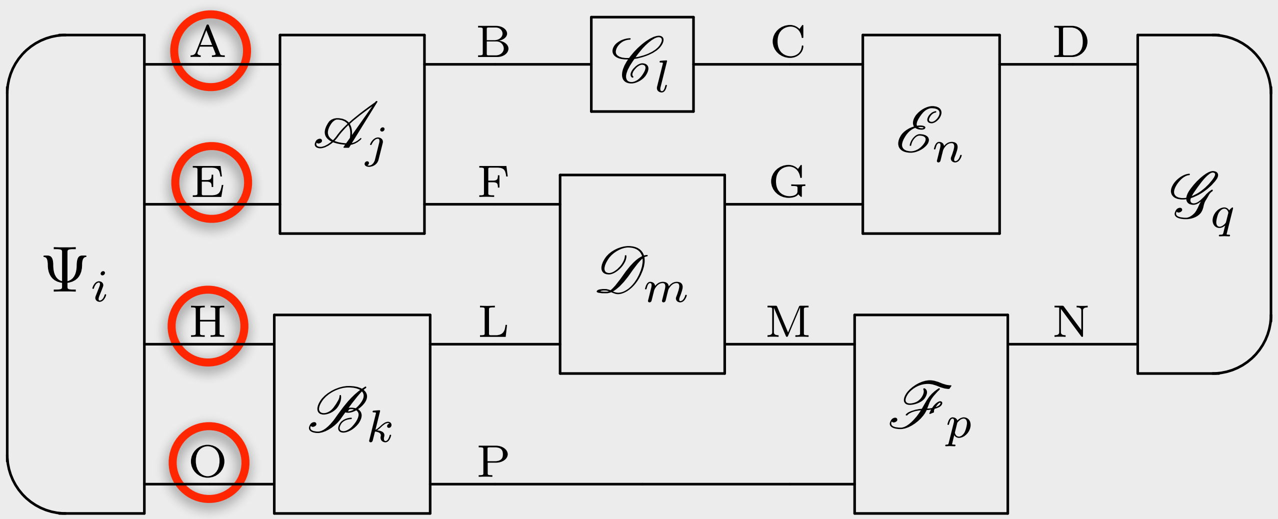
Maximal set of
independent systems
= *“leaf”*

$$p(i, j, k, l, m, n, p, q | \text{circuit})$$



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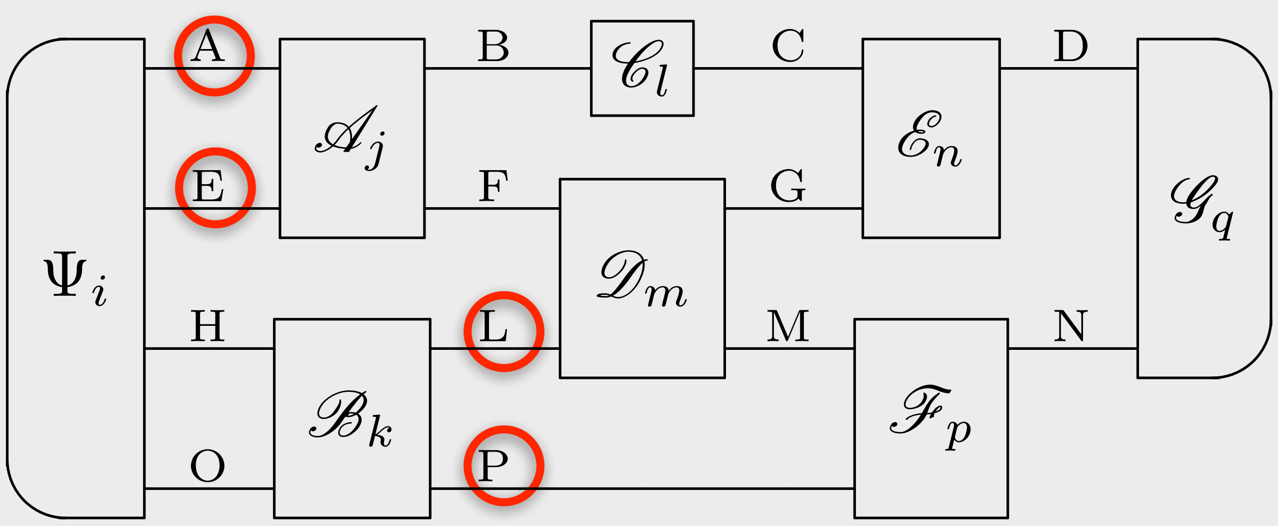

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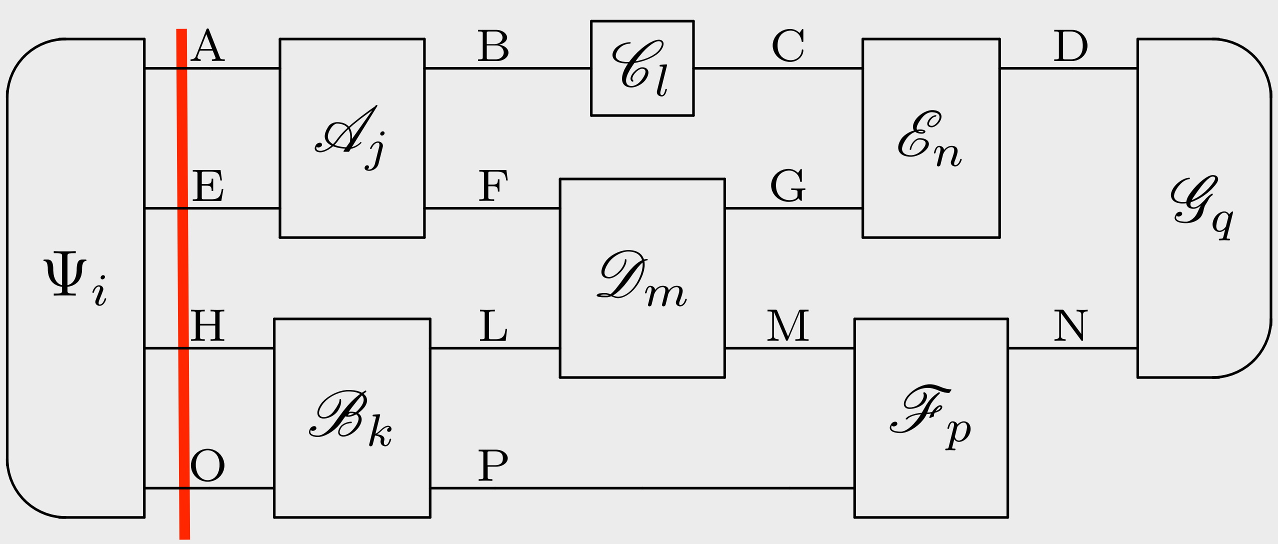

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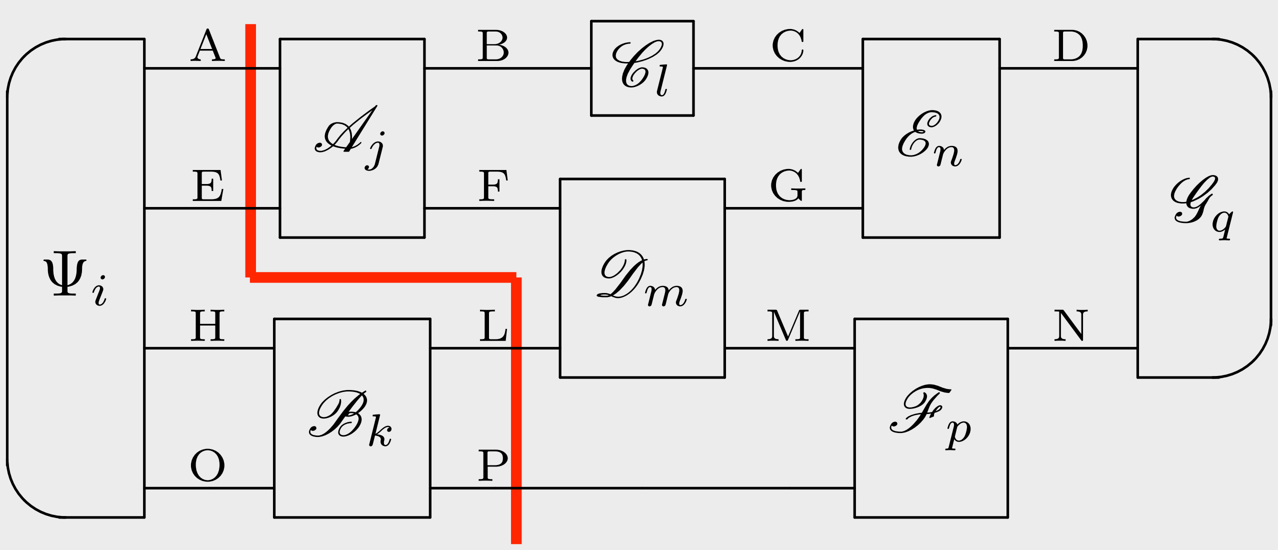

Operational Probabilistic Theory

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Logic ⊂ Probability ⊂ OPT

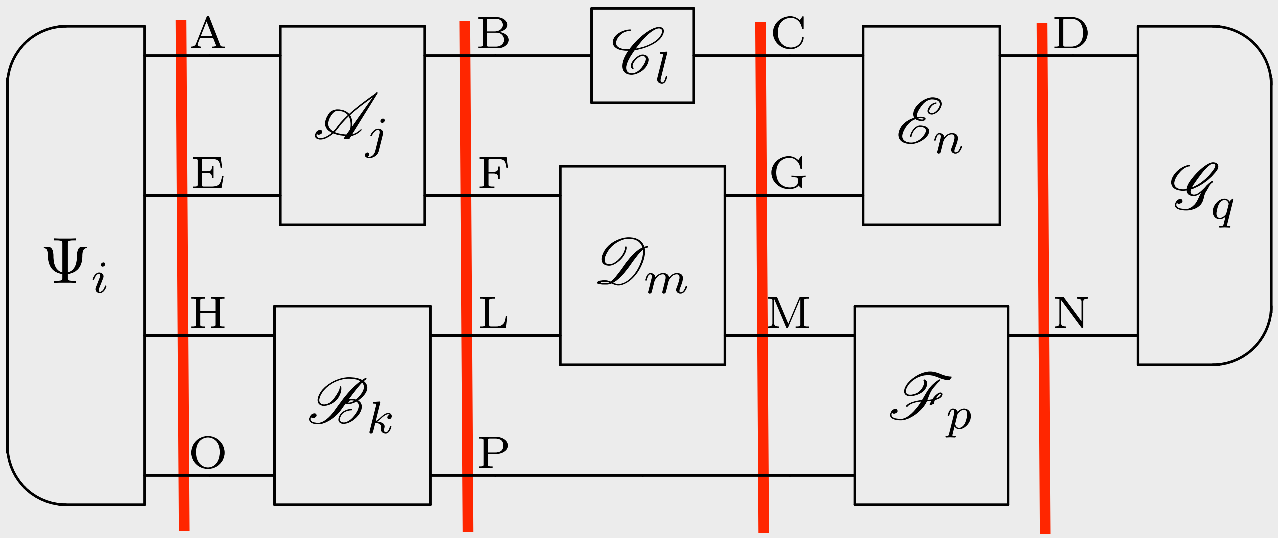
joint probabilities + connectivity

$$p(i, j, k, ...|\text{circuit})$$

Maximal set of independent systems = *"leaf"*

Foliation

$$p(i, j, k, l, m, n, p, q|\text{circuit})$$



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Foliation

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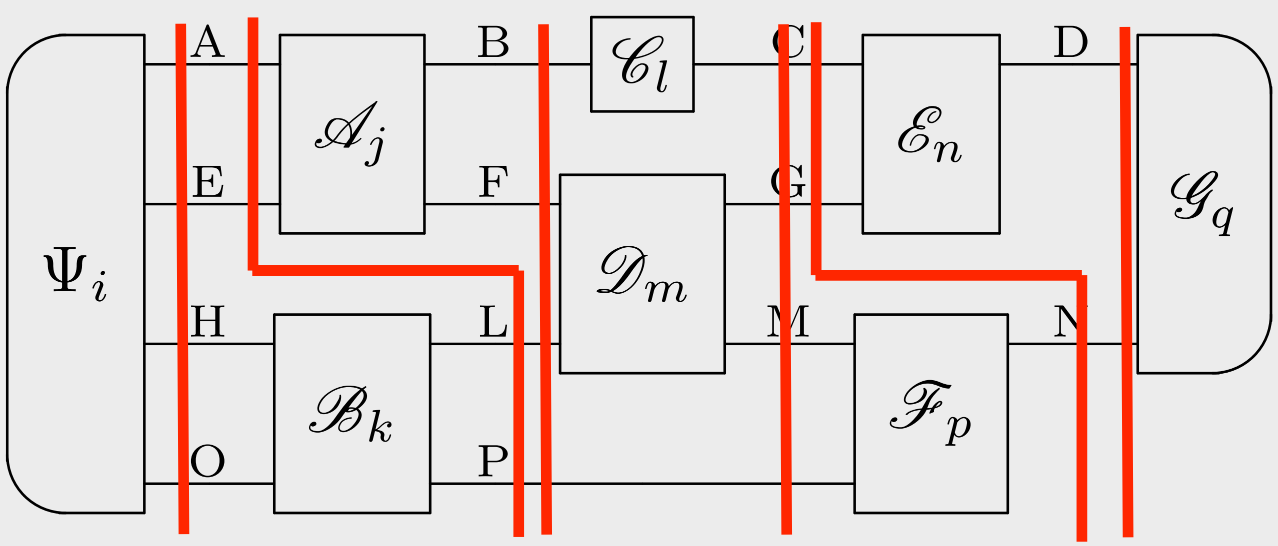


Operational Probabilistic Theory

States are functionals for effects

States are separating for effects

Effects are functionals on states

Effects are separating for states

Embedding in real vector spaces

St(A), $\mathrm{St}_1(\mathrm{A})$, $\mathrm{St}_{\mathbb{R}}(\mathrm{A})$

Eff(A), $\mathrm{Eff}_1(\mathrm{A})$, $\mathrm{Eff}_{\mathbb{R}}(\mathrm{A})$

Dimension D_{A}

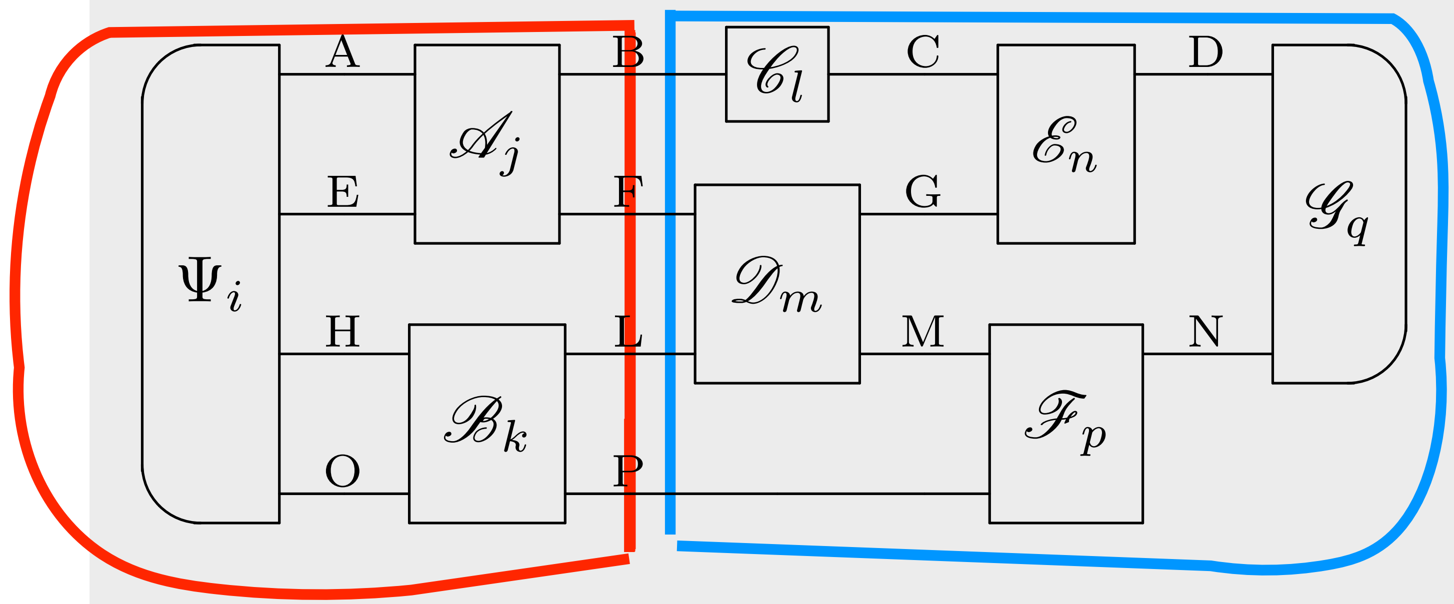
$$\boxed{\begin{array}{l}\mathrm{Eff}_{\mathbb{R}}(\mathrm{A}) = \mathrm{St}_{\mathbb{R}}(\mathrm{A})^{\vee} \\ \\ \mathrm{St}_{\mathbb{R}}(\mathrm{A}) = \mathrm{Eff}_{\mathbb{R}}(\mathrm{A})^{\vee}\end{array}}$$

Paring notation:

$\rho \in \mathrm{St}(\mathrm{A}), a \in \mathrm{Eff}(\mathrm{A})$, $\quad (\rho \xrightarrow{\mathrm{A}} a) = (a|\rho)$

$$(\Psi_i, \mathscr{A}_j, \mathscr{B}_k) \xrightarrow{\mathrm{BFLP}} (\mathscr{D}_m, \mathscr{F}_p, \mathscr{C}_l, \mathscr{E}_n, \mathscr{G}_q)$$

$$p(i, j, k, l, m, n, p, q|\text{circuit})$$



Operational Probabilistic Theory

$$\{\mathscr{T}_i\}_{i\in\{\underbrace{i_1, i_2, \ldots, i_n}_{j_1}, \underbrace{i_{n+1}, i_{n+2}, \ldots}_{j_2}, \underbrace{\ldots}_{\cdots}\}}$$

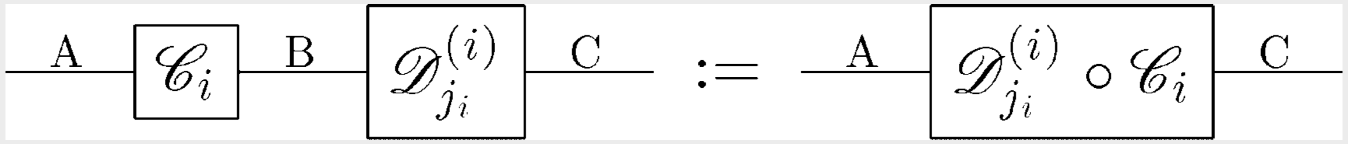
Coarse-graining $\downarrow$ $\uparrow$ Refinement

$$\{\hat{\mathscr{T}}_j\}_{j\in\{j_1,j_2,\ldots\}}$$

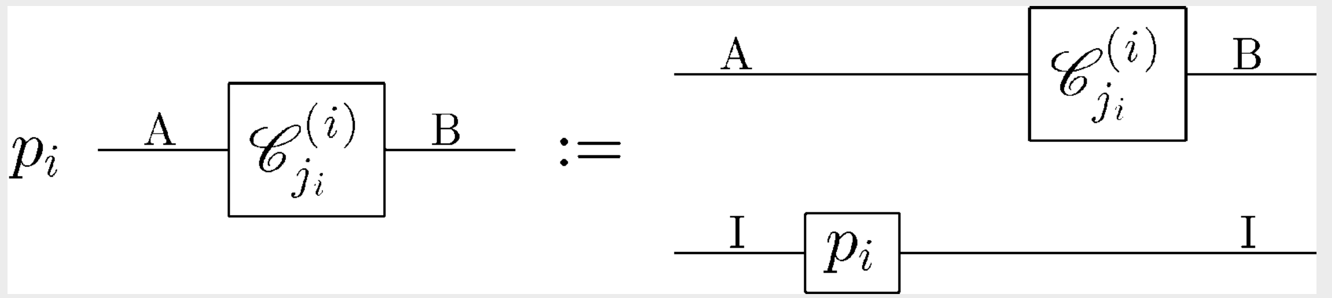
$$\hat{\mathscr{T}}_S = \sum_{i\in S} \mathscr{T}_i$$

Partial ordering

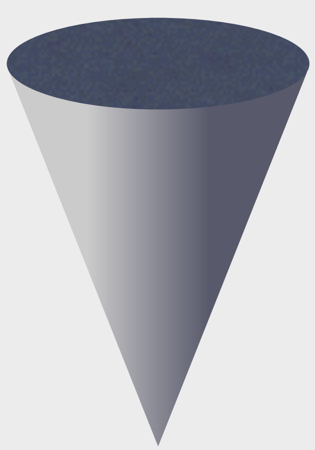
Conditioned test (needs causality)



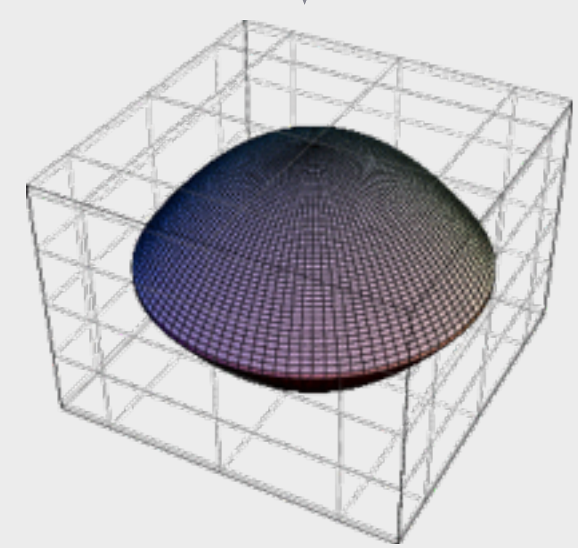
Circuit multiplication: randomize tests



Cone structure



Convex structure



Principles for Quantum Theory

$\{\rho_0, \rho_1\} \subseteq \mathsf{St}(\mathrm{A})$ preparation test

$\{a_0, a_1\}$ observation test

success probability of discrimination

$$p_{\mathrm{succ}} = (a_0|\rho_0) + (a_1|\rho_1)$$
$$= (a|\rho_0) + (a_1|\rho_1 - \rho_0)$$
$$= (a|\rho_1) + (a_0|\rho_0 - \rho_1)$$
$$= \frac{1}{2}[1 + (a_1 - a_0|\rho_1 - \rho_0)]$$

$$a := a_0 + a_1$$

Metric

$$p_{\mathrm{succ}}^{(\mathrm{opt})} = \frac{1}{2}[1 + ||\rho_1 - \rho_0||]$$

$$||\delta|| := \sup_{\{a_0, a_1\}} (a_0 - a_1|\delta)$$

$$||\delta|| = \sup_{a_0 \in \mathsf{Eff}(\mathrm{A})} (a_0|\delta) - \inf_{a_1 \in \mathsf{Eff}(\mathrm{A})} (a_1|\delta)$$

monotonicity

$$\mathscr{C} \in \mathsf{Transf}_1(\mathrm{A}, \mathrm{B})$$

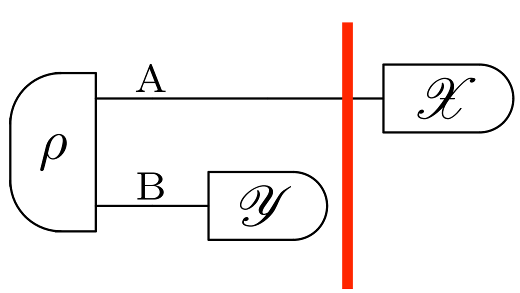
$$||\mathscr{C}\delta||_{\mathrm{B}} \leq ||\delta||_{\mathrm{A}}$$

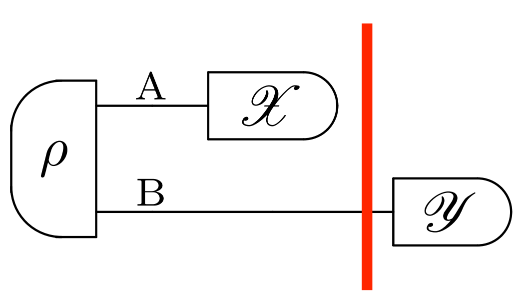
Principles for Quantum Theory

P1. Causality
P2. Local discriminability
P3. Purification
P4. Atomicity of composition
P5. Perfect distinguishability
P6. Lossless Compressibility

The probability of preparations is independent of the choice of observations

no signaling without interaction

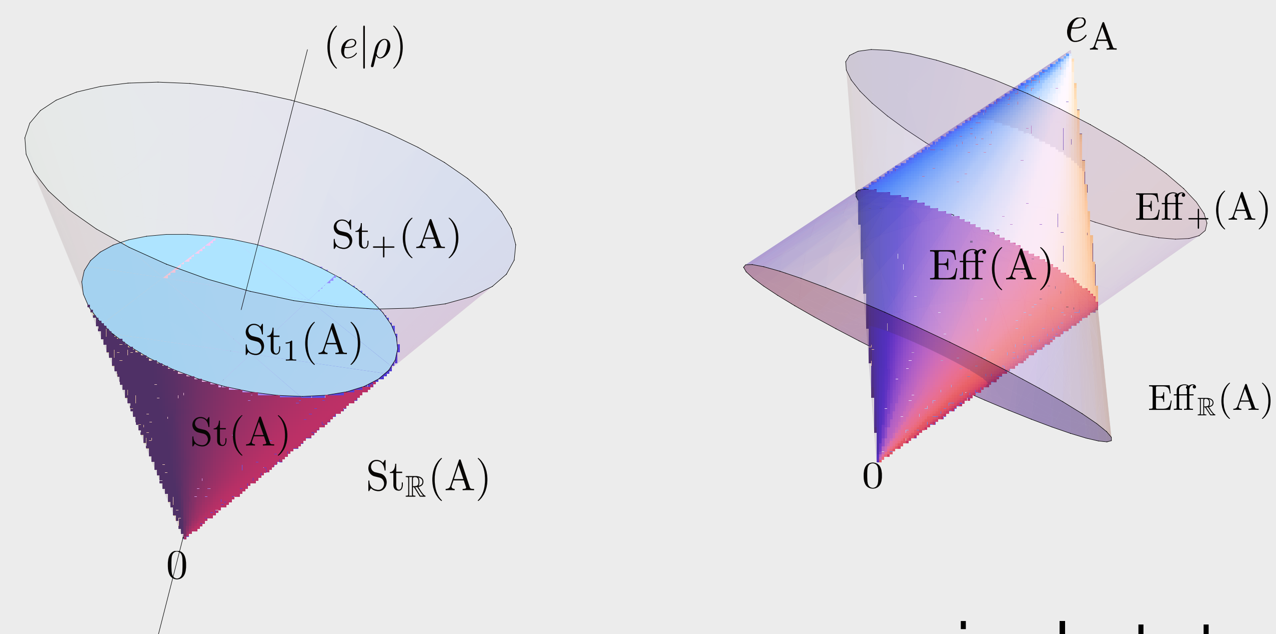




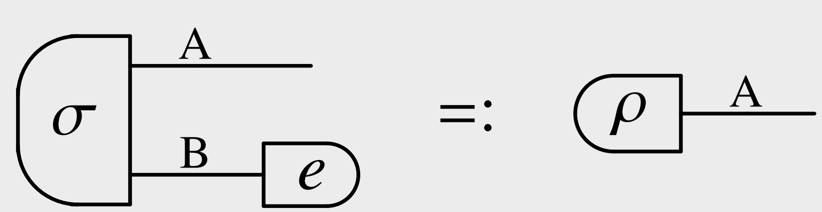
$$p(i, j|\mathscr{X}, \mathscr{Y}) := (a_j|\rho_i)$$

$$p(i|\mathscr{X}, \mathscr{Y}) = p(i|\mathscr{X}, \mathscr{Y}') = p(i|\mathscr{X})$$

Iff conditions: a) the deterministic effect is unique; b) states are "normalizable"



marginal state



Principles for Quantum Theory

P1. Causality

P2. Local discriminability

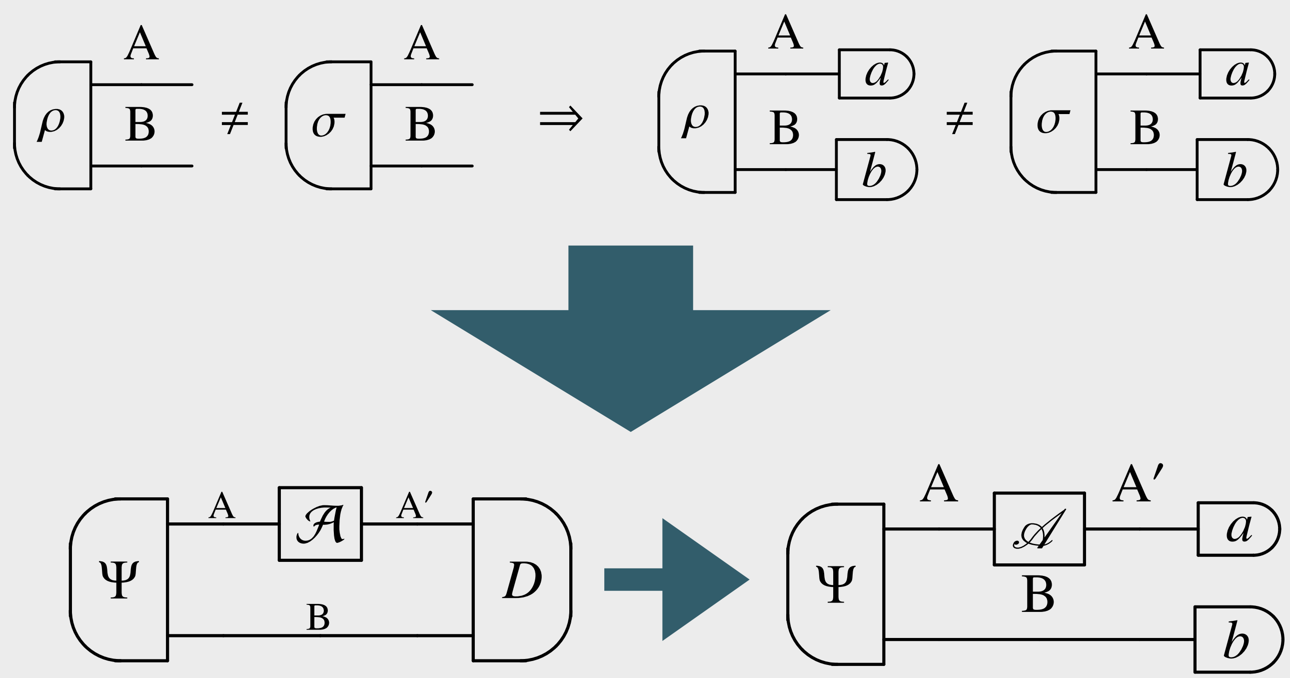
P3. Purification

P4. Atomicity of composition

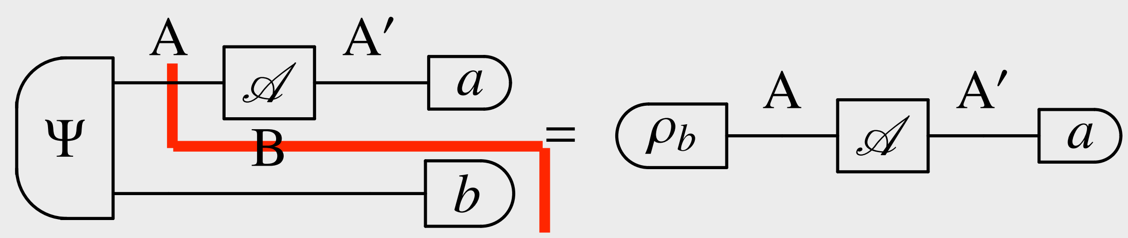
P5. Perfect distinguishability

P6. Lossless Compressibility

It is possible to discriminate any pair of states of composite systems using only local measurements.



Local characterization of transformations



Principles for Quantum Theory

P1. Causality

P2. Local discriminability

P3. Purification

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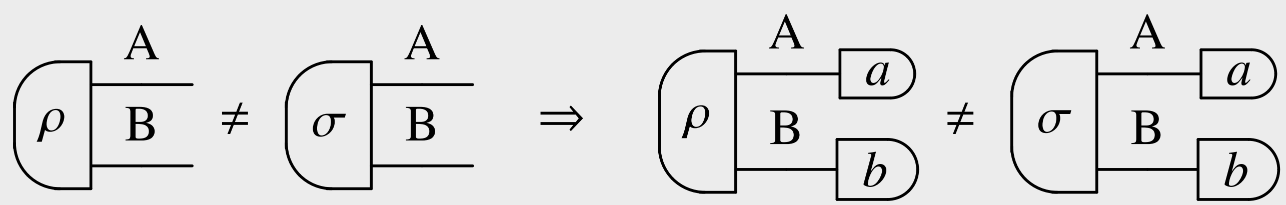
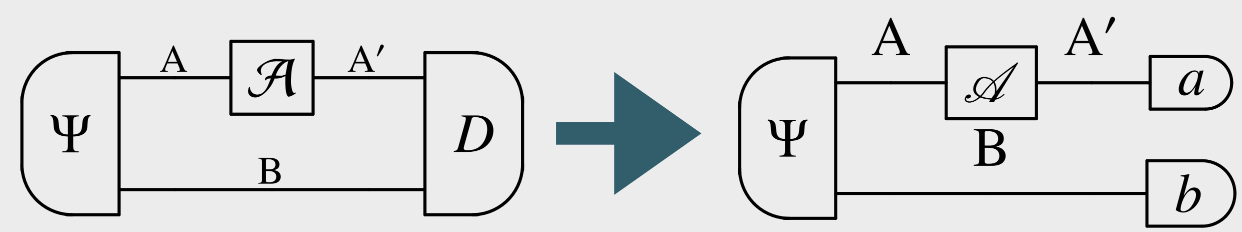
It is possible to discriminate any pair of states of composite systems using only local measurements.

Origin of the complex tensor product





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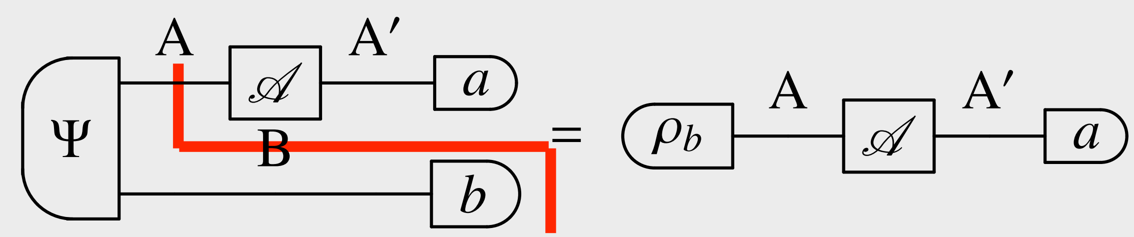
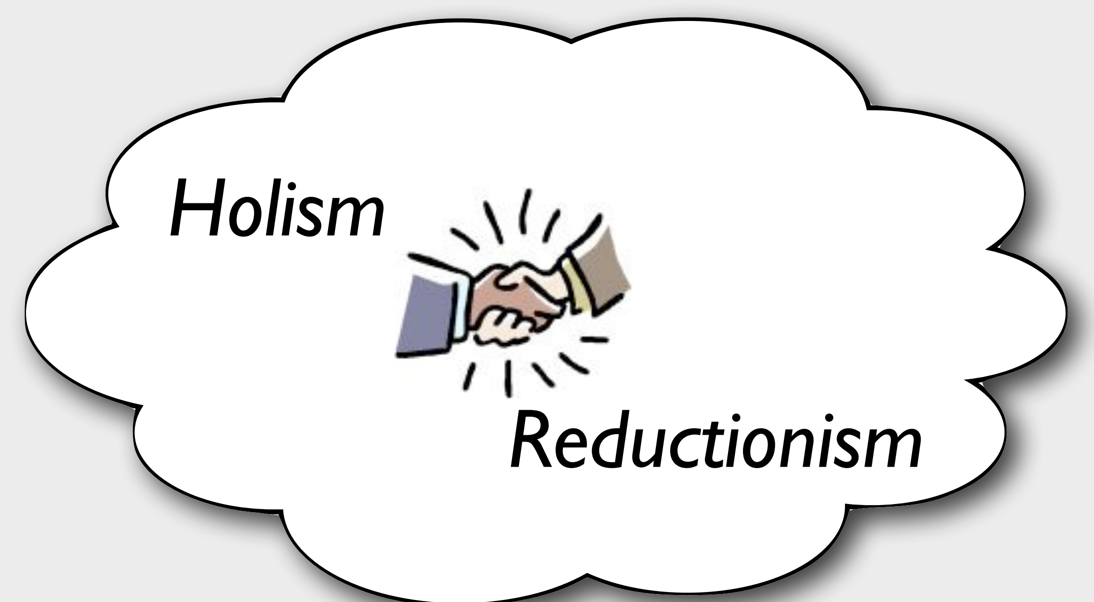
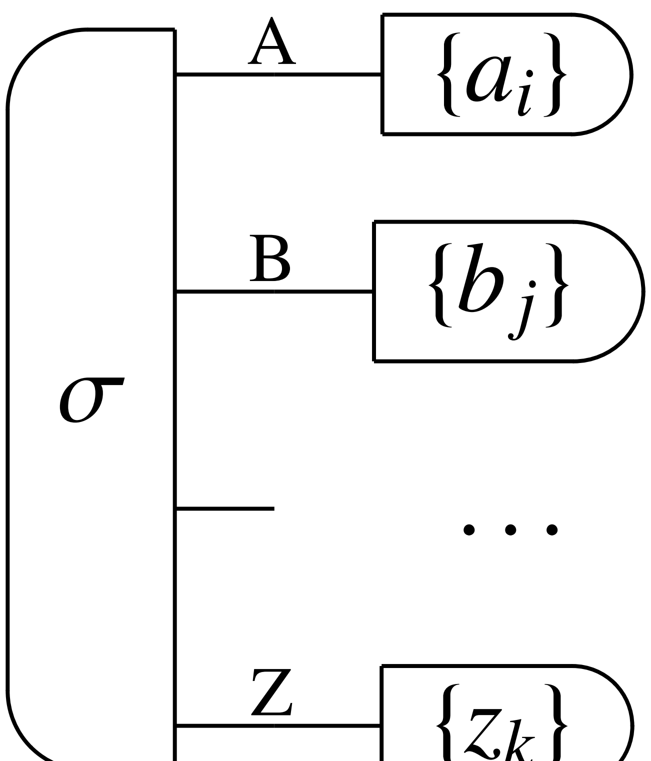


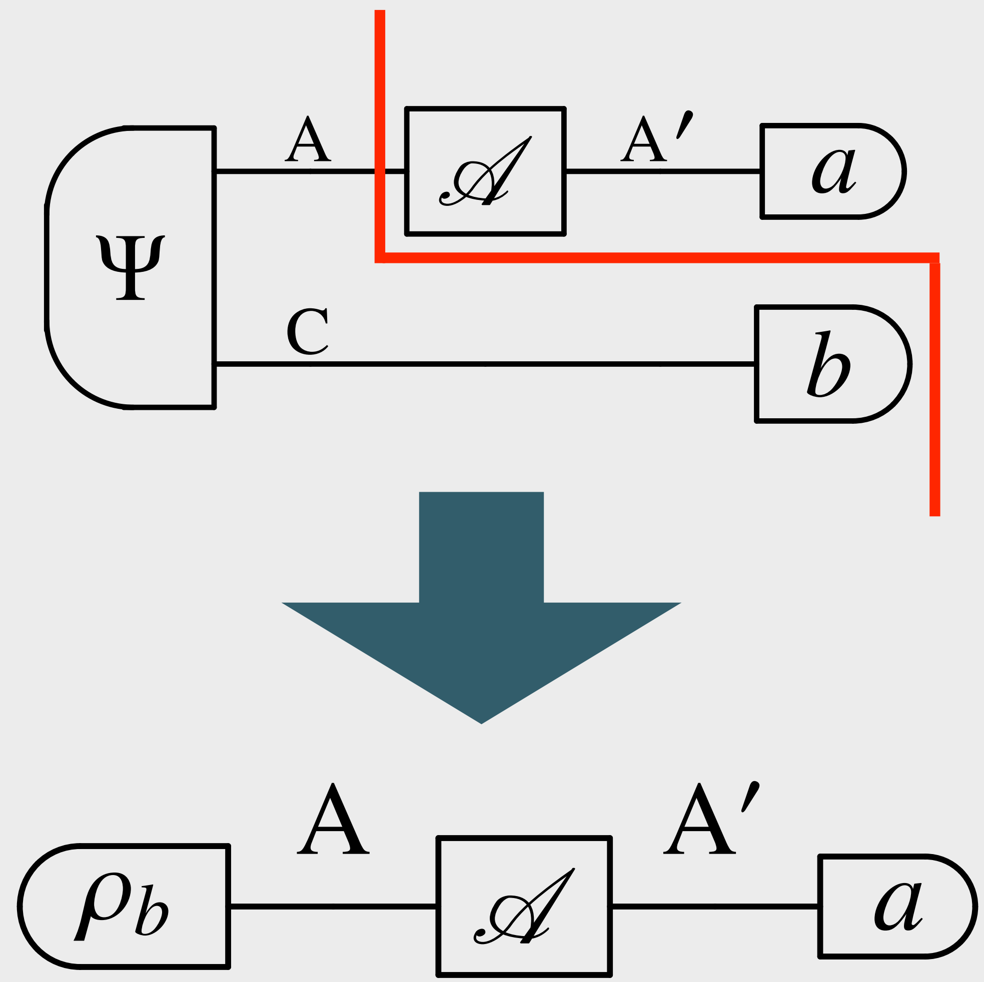


Principles for Quantum Theory

Local effects are separating for joint states



Tomography



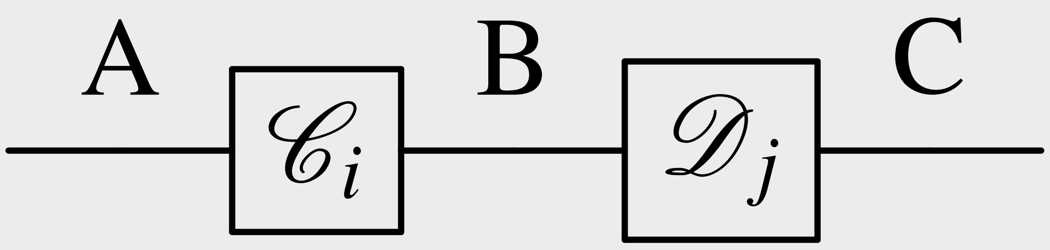
Counter-examples: Real QT, Fermionic QT

Principles for Quantum Theory

P1. Causality
P2. Local discriminability
P3. Purification
P4. Atomicity of composition
P5. Perfect distinguishability
P6. Lossless Compressibility

The composition of two atomic transformations is atomic

Complete information can be accessed on a step-by-step basis



Principles for Quantum Theory

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P2. Local discriminability
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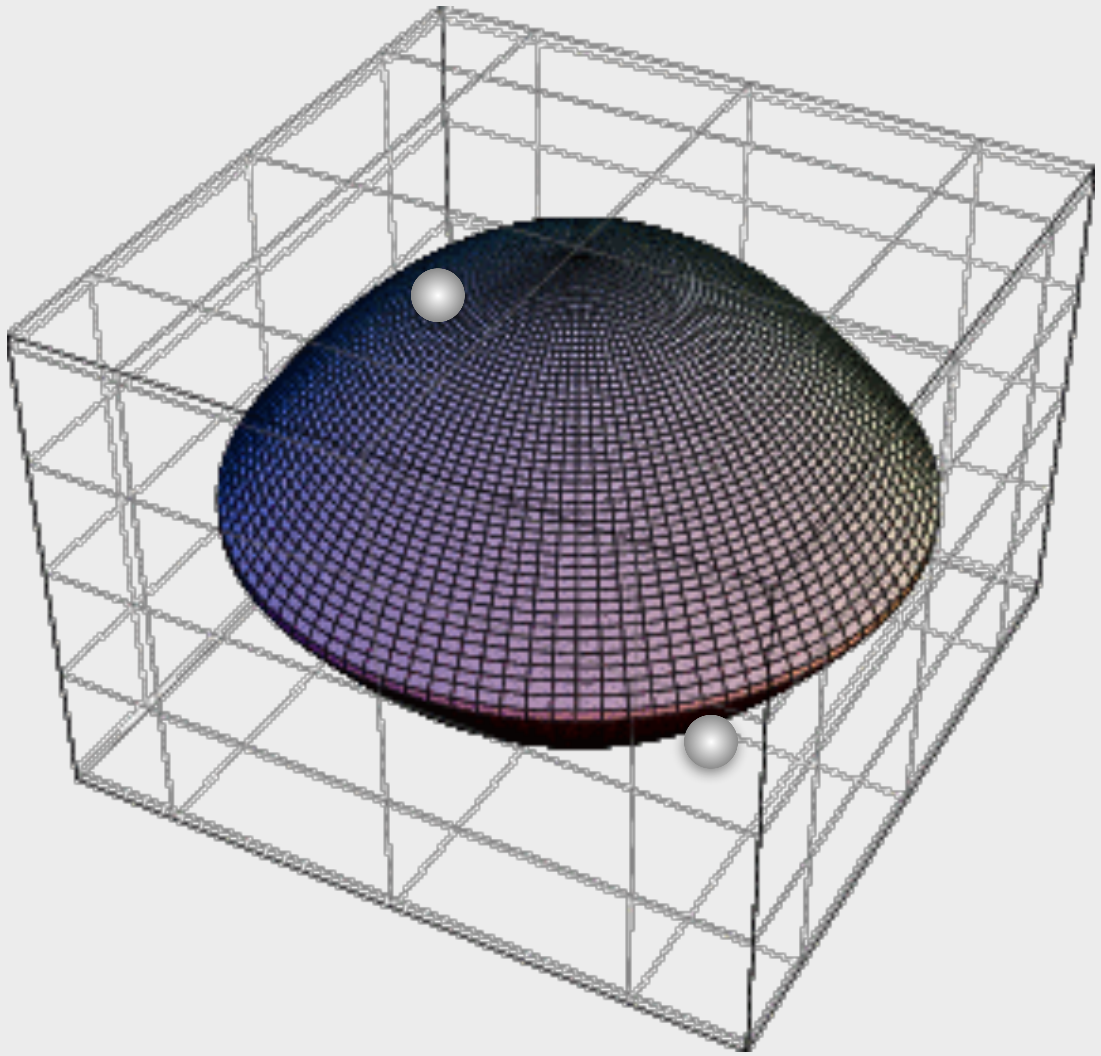
P4. Atomicity of composition
P5. $\boxed{\text{Perfect distinguishability}}$
P6. Lossless Compressibility

Every state that is not completely mixed (i.e. on the boundary of the convex) can be perfectly distinguished from some other state.

Falsifiability of the theory



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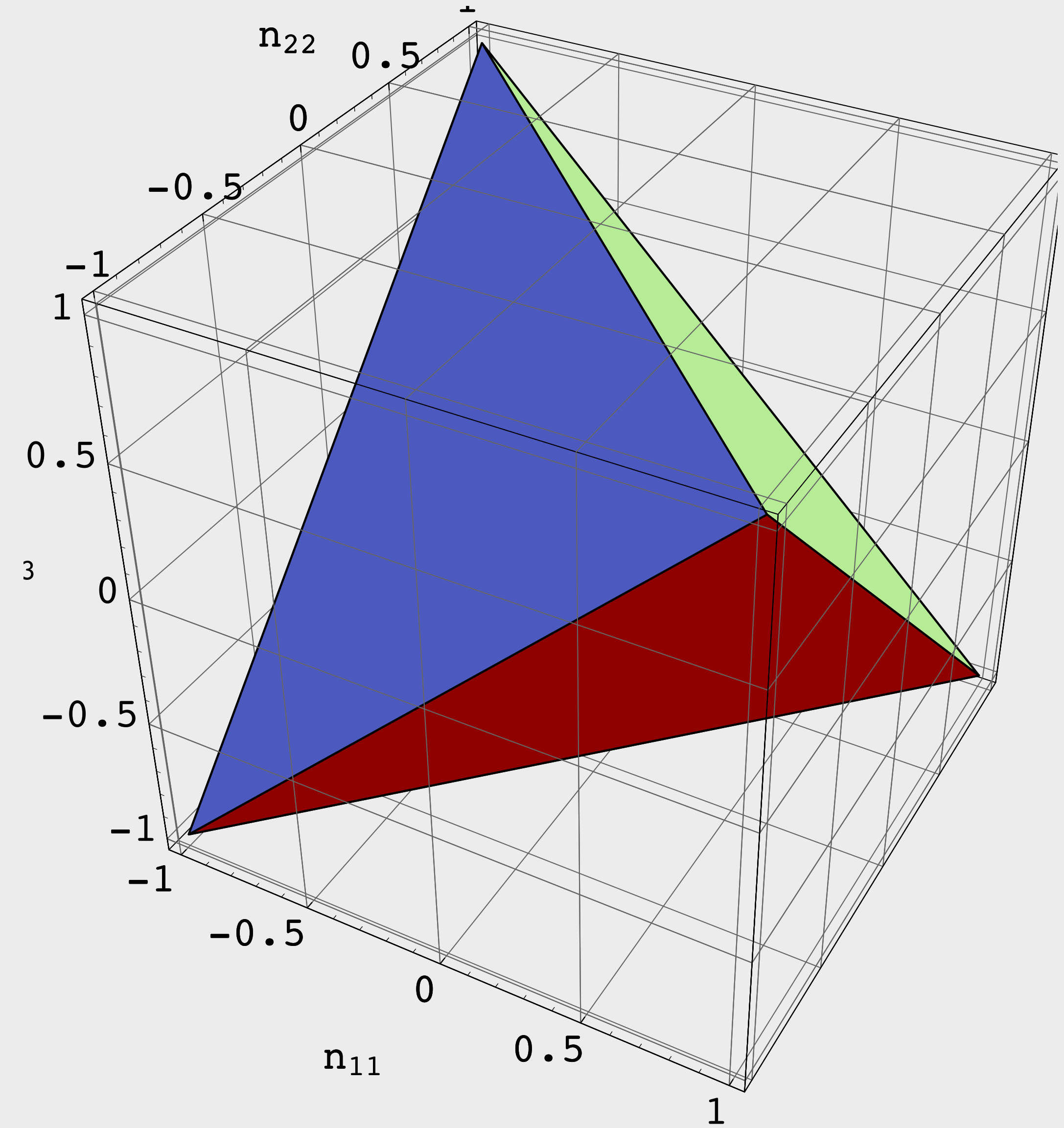
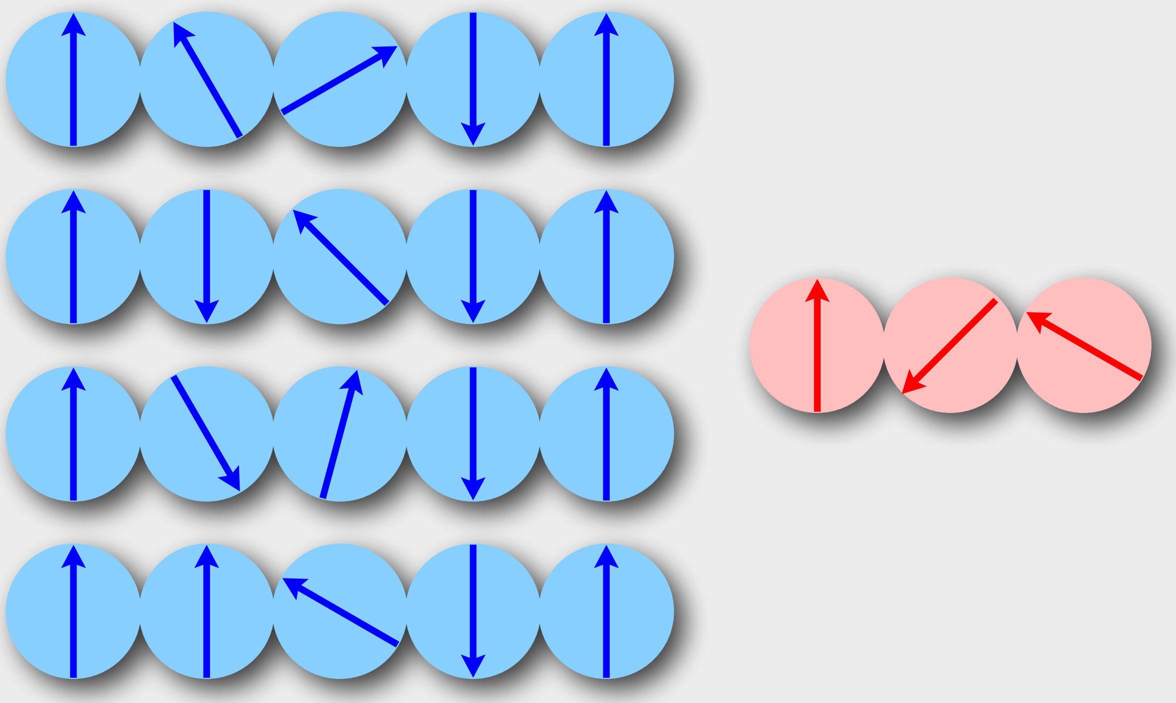
P3. Purification
P4. Atomicity of composition
P5. Perfect distinguishability
P6. Lossless Compressibility

For states that are not completely mixed there exists an ideal compression scheme

Any face of the convex set of states is the convex set of states of some other system





Principles for Quantum Theory

P1. Causality

P2. Local discriminability

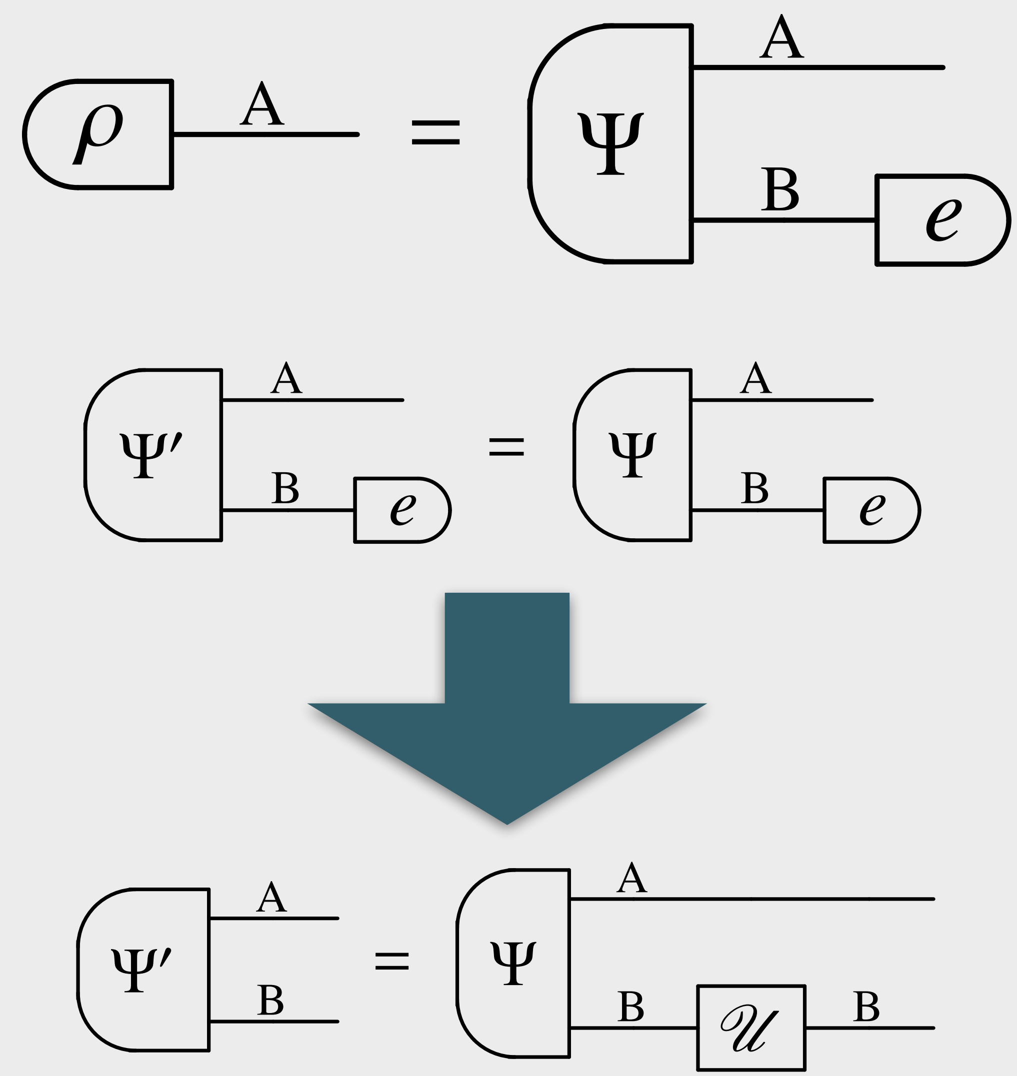
P3. Purification

P4. Atomicity of composition

P5. Perfect distinguishability

P6. Lossless Compressibility

Every state has a purification. For fixed purifying system, every two purifications of the same state are connected by a reversible transformation on the purifying system



Principles for Quantum Theory

P1. Causality

P2. Local discriminability

P3. $\boxed{\text{Purification}}$

P4. Atomicity of composition

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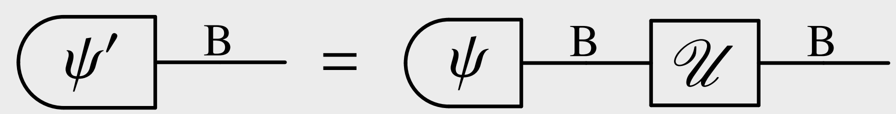
P6. Lossless Compressibility

Every state has a purification. For fixed purifying system, every two purifications of the same state are connected by a reversible transformation on the purifying system

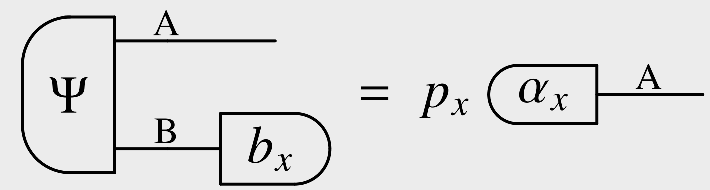
Consequences

1. ***Existence of entangled states:***
the purification of a mixed state is an entangled state;
the marginal of a pure entangled state is a mixed state;

2. Every two normalized pure states of the same system are connected by a reversible transformation

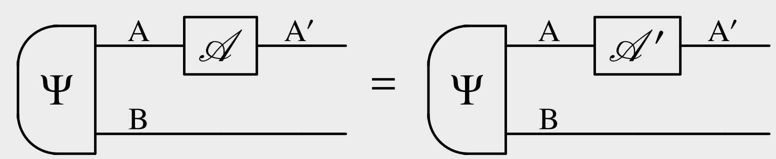


3. ***Steering:*** Let $\mathbf{\Psi}$ purification of ρ. The for every ensemble decomposition $\rho=\sum_x p_x \alpha_x$ there exists a measurement $\{b_x\}$, such that



$\forall x \in$ X

4. ***Process tomography (pure faithful state):***



$$\mathscr{A}\rho = \mathscr{A}'\rho \quad \forall \rho$$

5. ***No information without disturbance***

Principles for Quantum Theory

P1. Causality

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P3. Purification

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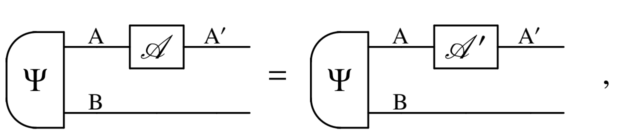
P6. Lossless Compressibility

Every state has a purification. For fixed purifying system, every two purifications of the same state are connected by a reversible transformation on the purifying system

Purification establishes an interesting correspondence between transformations and states. This is easy to see: let us take a set of states $\{\alpha_x \mid x \in \mathsf{X}\}$ that span the whole state space of system A and a set of positive probabilities $\{p_x\}_{x\in\mathsf{X}}$. Then, take a purification of the mixed state $\rho = \sum_x p_x \alpha_x$—say $\Psi \in \mathsf{PurSt}(\mathrm{AB})$. Now, if two transformations $\mathscr{A}$ and $\mathscr{A}'$ satisfy

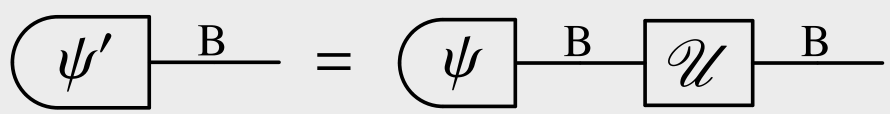


it is clear that $\mathscr{A}$ must be equal to $\mathscr{A}'$, namely the correspondence $\mathscr{A} \mapsto (\mathscr{A} \otimes \mathscr{I}_\mathrm{B})\Phi$ is injective.

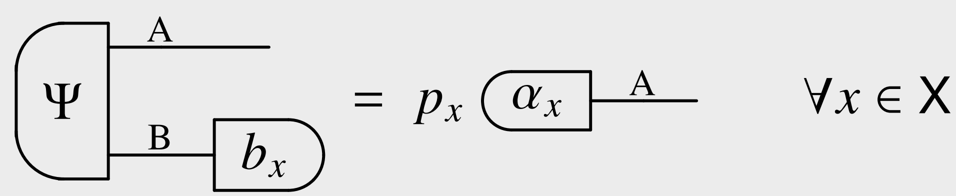
Consequences

1. ***Existence of entangled states:***
the purification of a mixed state is an entangled state;
the marginal of a pure entangled state is a mixed state;

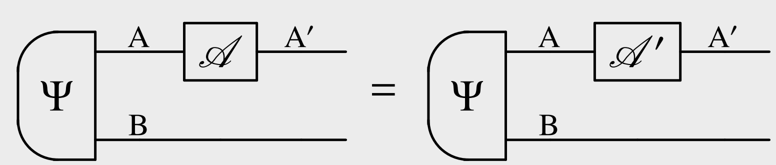
2. *Every two normalized pure states of the same system are connected by a reversible transformation*



3. ***Steering:*** Let $\boldsymbol{\Psi}$ purification of ρ. The for every ensemble decomposition $\rho=\sum_x p_x \alpha_x$ there exists a measurement $\{b_x\}$, such that



4. ***Process tomography (pure faithful state):***



$$\mathscr{A}\rho = \mathscr{A}'\rho \quad \forall \rho$$

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Principles for Quantum Theory

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different. If we take a pure state $\Psi \in \mathsf{PurSt}(\mathrm{AB})$ that can be used for process tomography, then the no-disturbance condition implies $\sum_x (\mathscr{A}_x \otimes \mathscr{I}_\mathrm{B})\Psi = \Psi$. But Ψ is pure: hence, each unnormalized state $(\mathscr{A}_x \otimes \mathscr{I}_\mathrm{B})\Psi$ must be proportional to Ψ. Precisely, there must be a set of probabilities $\{p_x\}$ such that $(\mathscr{A}_x \otimes \mathscr{I}_\mathrm{B})\Psi = p_x\Psi$. Since the map $\mathscr{A} \mapsto (\mathscr{A} \otimes \mathscr{I}_\mathrm{B})\Psi$ is injective (see Sect. 8.6), we conclude that $\mathscr{A}_x = p_x \mathscr{I}_\mathrm{A}$. In other

Consequences

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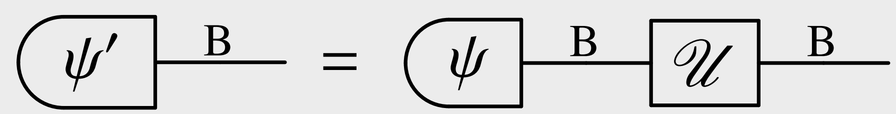

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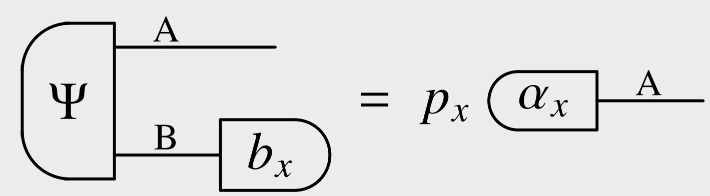
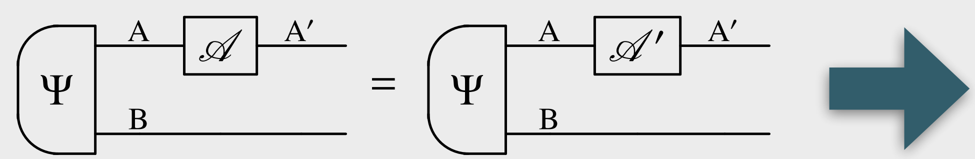


$\forall x \in \mathrm{X}$

4. ***Process tomography (pure faithful state):***



$\mathscr{A}\rho = \mathscr{A}'\rho$
$\forall\rho$

5. ***No information without disturbance***

Principles for Quantum Theory

P1. Causality

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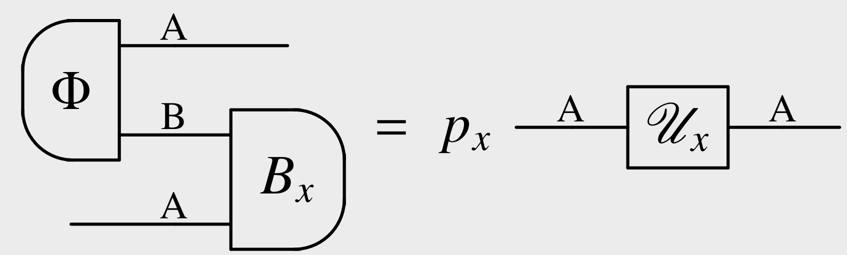
Every state has a purification. For fixed purifying system, every two purifications of the same state are connected by a reversible transformation on the purifying system

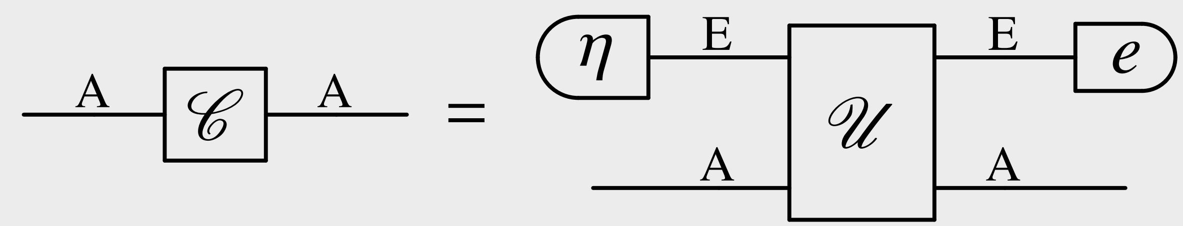
Consequences

6. ***Teleportation***

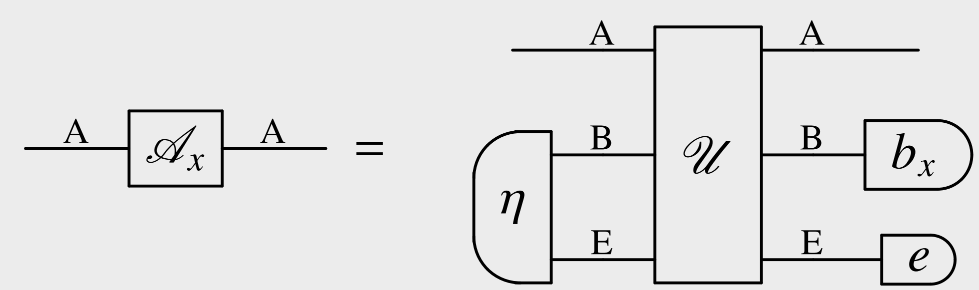


$\forall x \in$ X

7. ***Reversible dilation of "channels"***



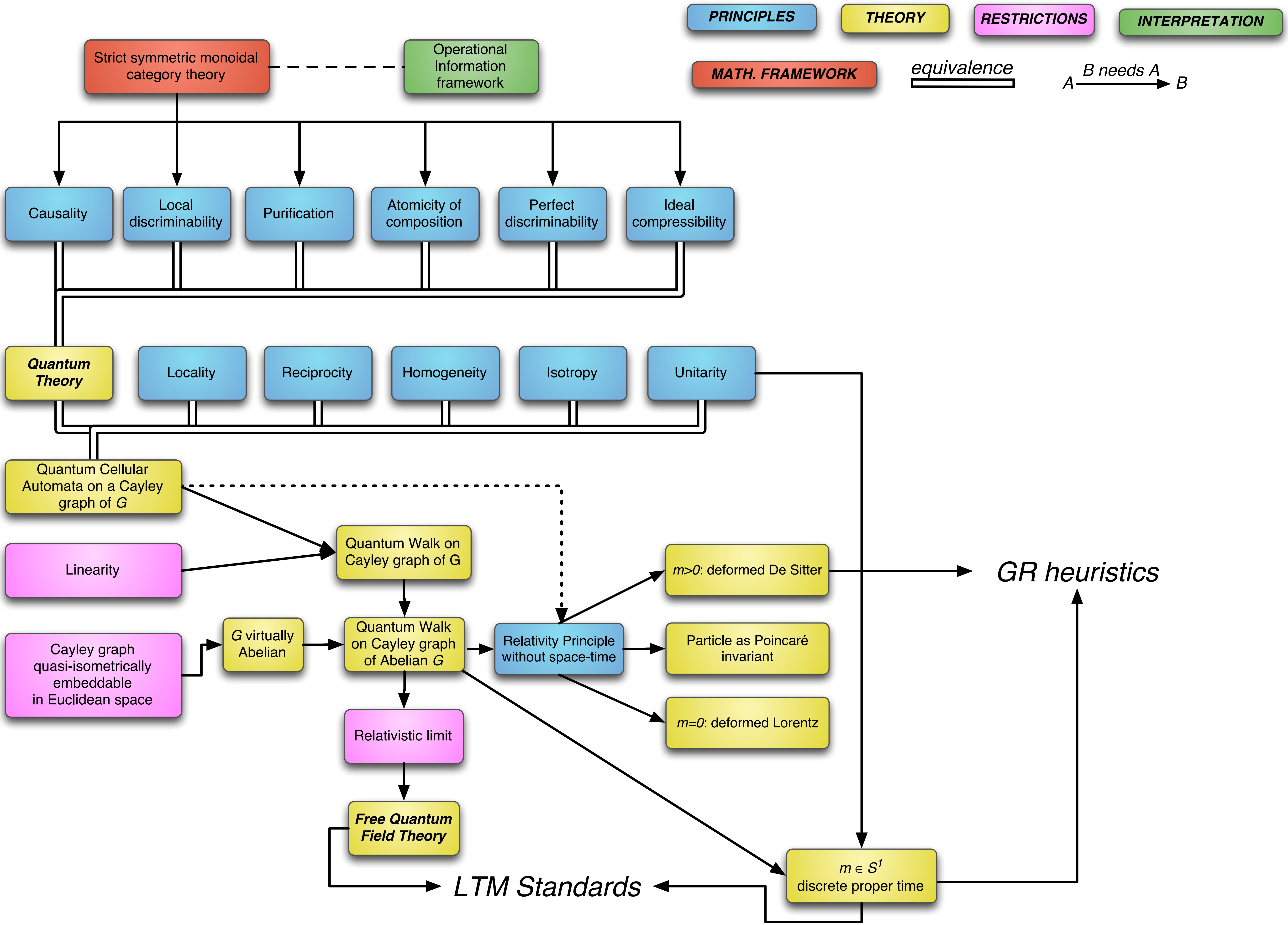
8. ***Reversible dilation of "instruments"***



$\forall x \in$ X

9. ***State-transformation cone isomorphism***

10. ***Rev. transform. for a system make a compact Lie group***



PRINCIPLES
THEORY
RESTRICTIONS
INTERPRETATION
MATH. FRAMEWORK
equivalence
B needs A
A
B
Strict symmetric monoidal category theory
Operational Information framework
Causality
Local discriminability
Purification
Atomicity of composition
Perfect discriminability
Ideal compressibility
Quantum Theory
Locality
Reciprocity
Homogeneity
Isotropy
Unitarity
Quantum Cellular Automata on a Cayley graph of G
Linearity
Quantum Walk on Cayley graph of G
Cayley graph quasi-isometrically embeddable in Euclidean space
G virtually Abelian
Quantum Walk on Cayley graph of Abelian G
Relativity Principle without space-time
m>0: deformed De Sitter
Particle as Poincaré invariant
m=0: deformed Lorentz
GR heuristics
Relativistic limit
Free Quantum Field Theory
m ∈ S¹ discrete proper time
LTM Standards

This is more or less what I wanted to say

Thank you for your attention