Quantum Theory and Quantum Field theory derived from information-theoretic principles

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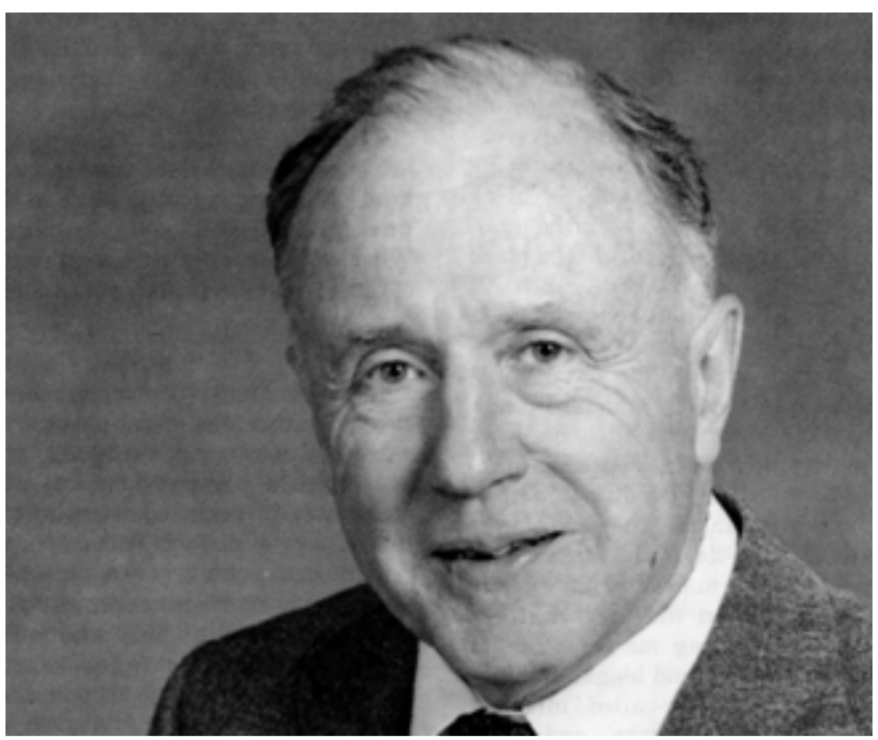
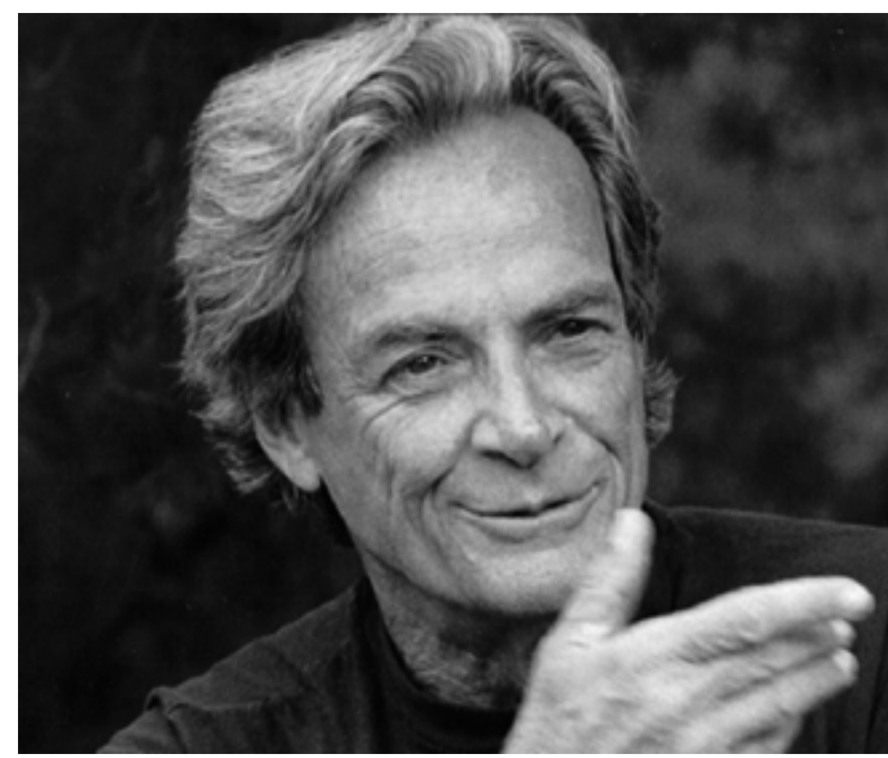
The information-theoretical paradigm:

the physical law as an *algorithm*





The New Axiomatization Program

To derive the whole Physics axiomatically

from "principles" stated in form of purely mathematical axioms without physical primitives,
but having a thorough physical interpretation.

Solution: informationalism

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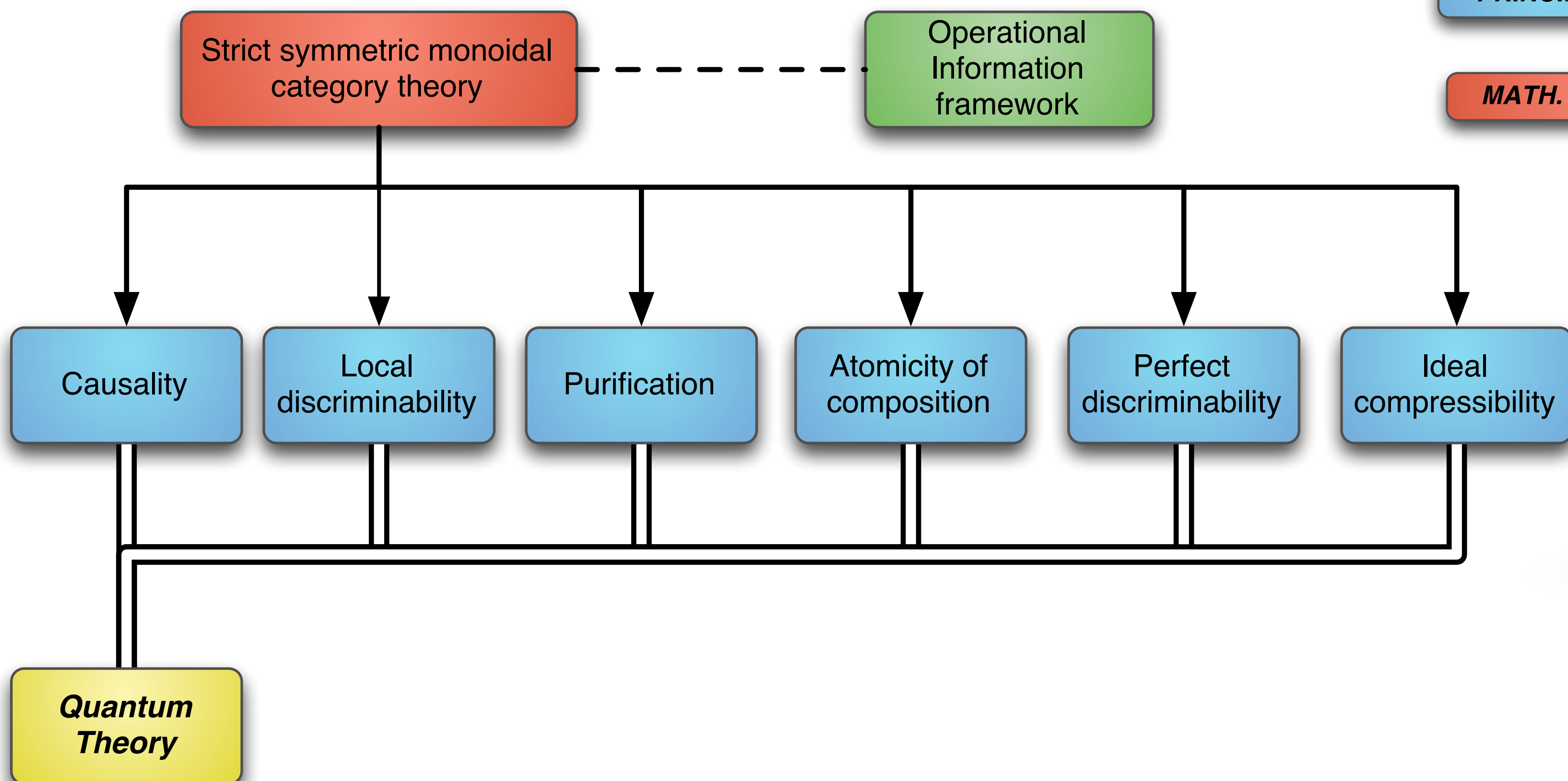
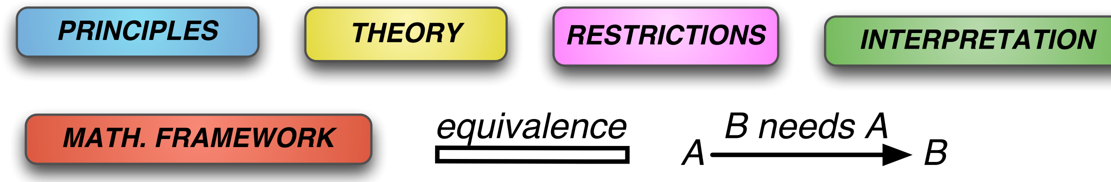
physical primitives: mass, force, rods, clocks,…





Principles for Quantum Theory

Selected for a Viewpoint in *Physics*

PHYSICAL REVIEW A **84**, 012311 (2011)

Informational derivation of quantum theory

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We derive quantum theory from purely informational principles. Five elementary axioms—causality, perfect distinguishability, ideal compression, local distinguishability, and pure conditioning—define a broad class of theories of information processing that can be regarded as standard. One postulate—purification—singles out quantum theory within this class.

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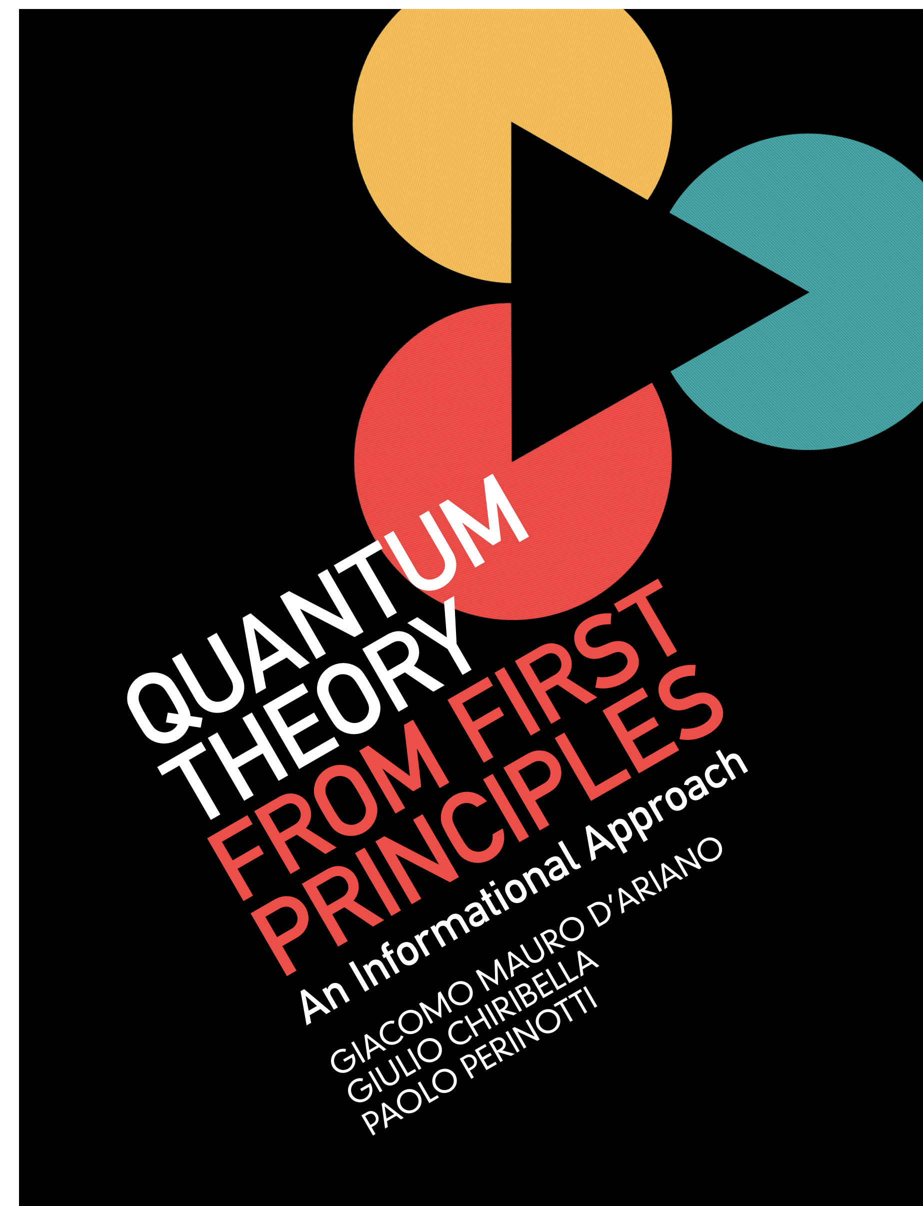
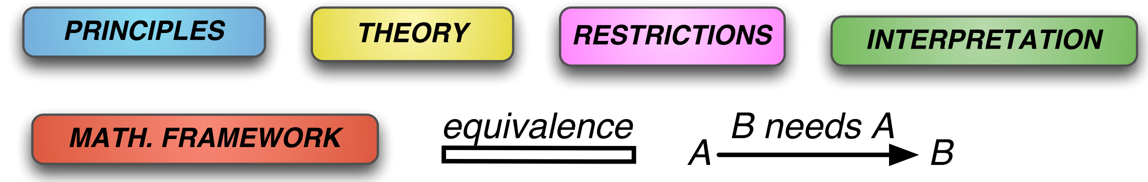
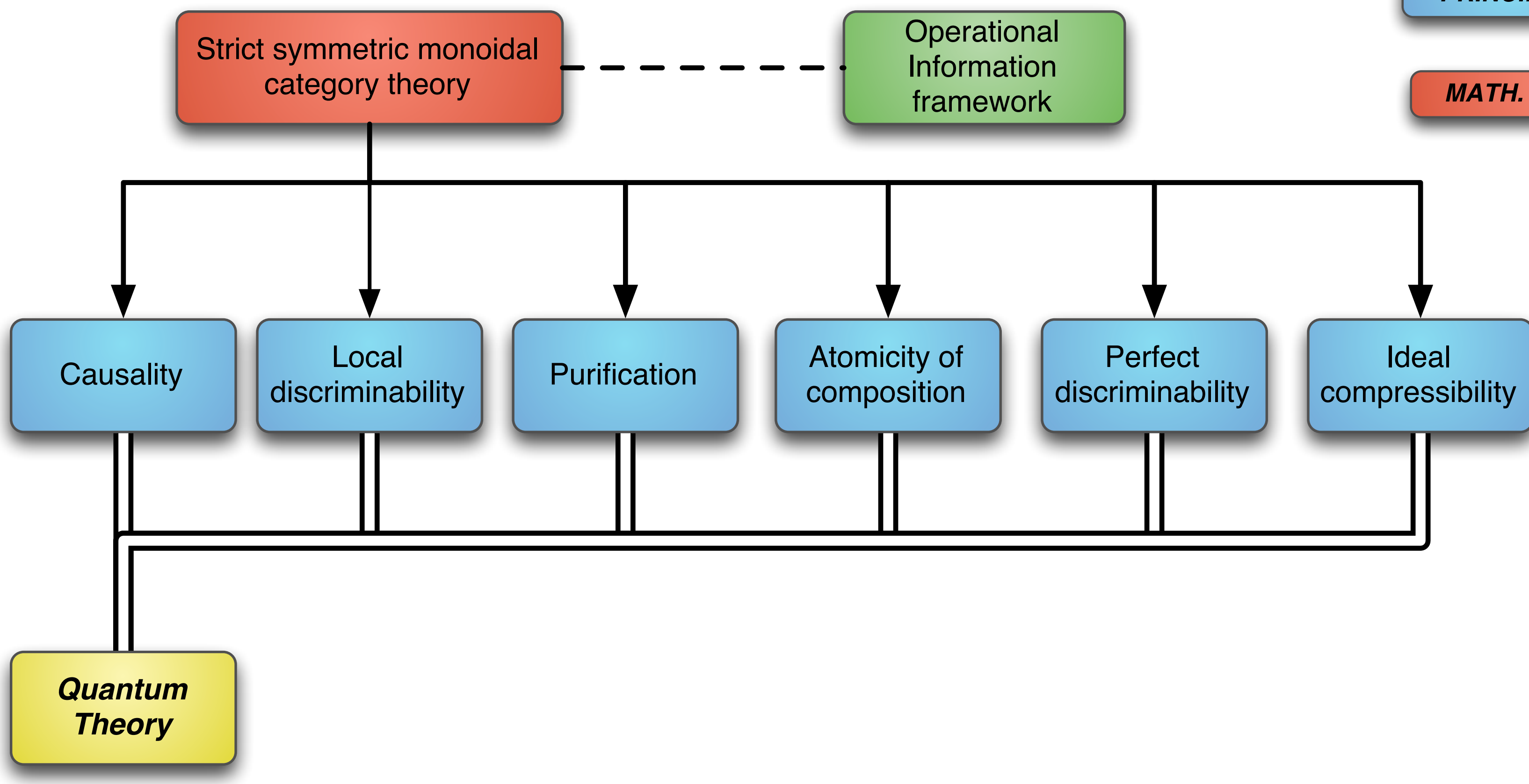






Principles for Mechanics

JOHN TEMPLETON FOUNDATION

SUPPORTING SCIENCE~INVESTING IN THE BIG QUESTIONS

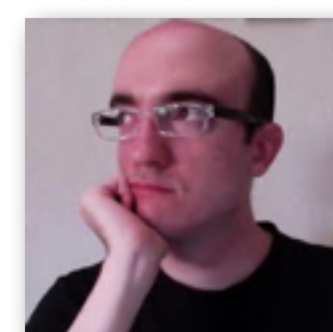
Project: *A Quantum-Digital Universe*, Grant ID: 43796

Paolo Perinotti

Alessandro Bisio

Alessandro Tosini

Marco Erba



Nicola Mosco

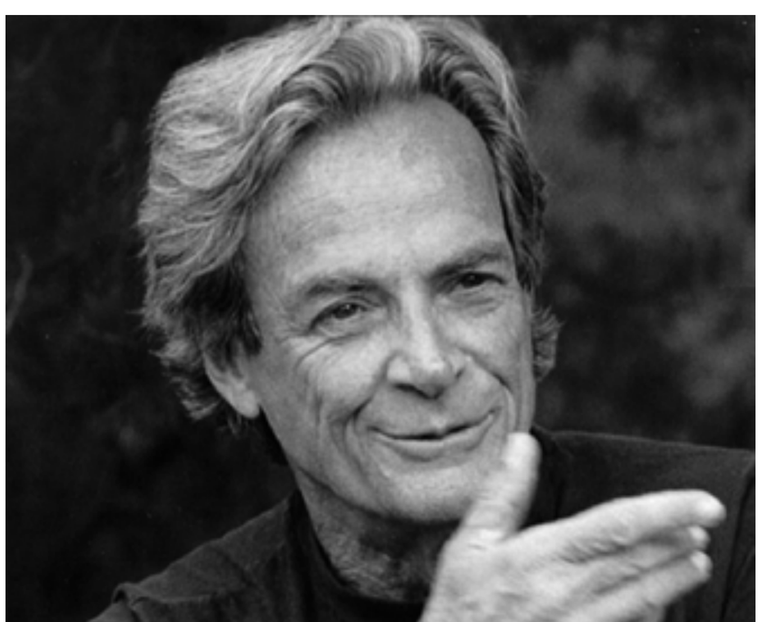
- *Mechanics (QFT) derived in terms of countably many quantum systems in interaction*

add principles

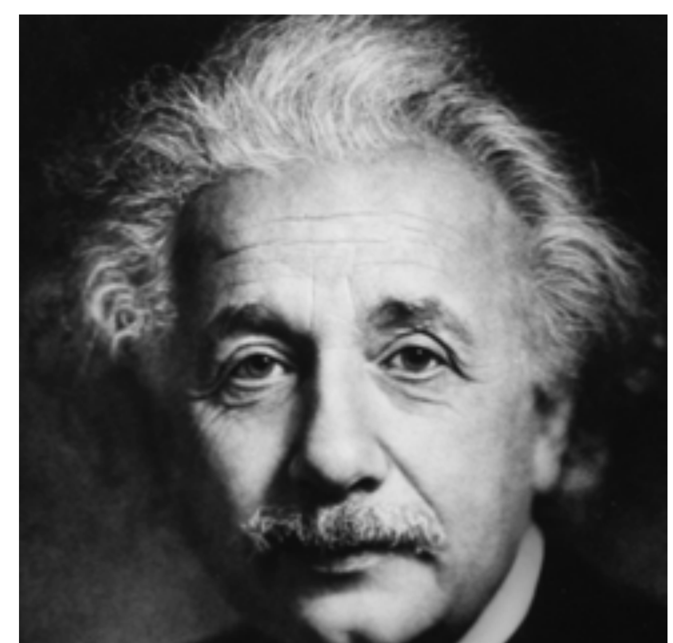
Min algorithmic complexity principle

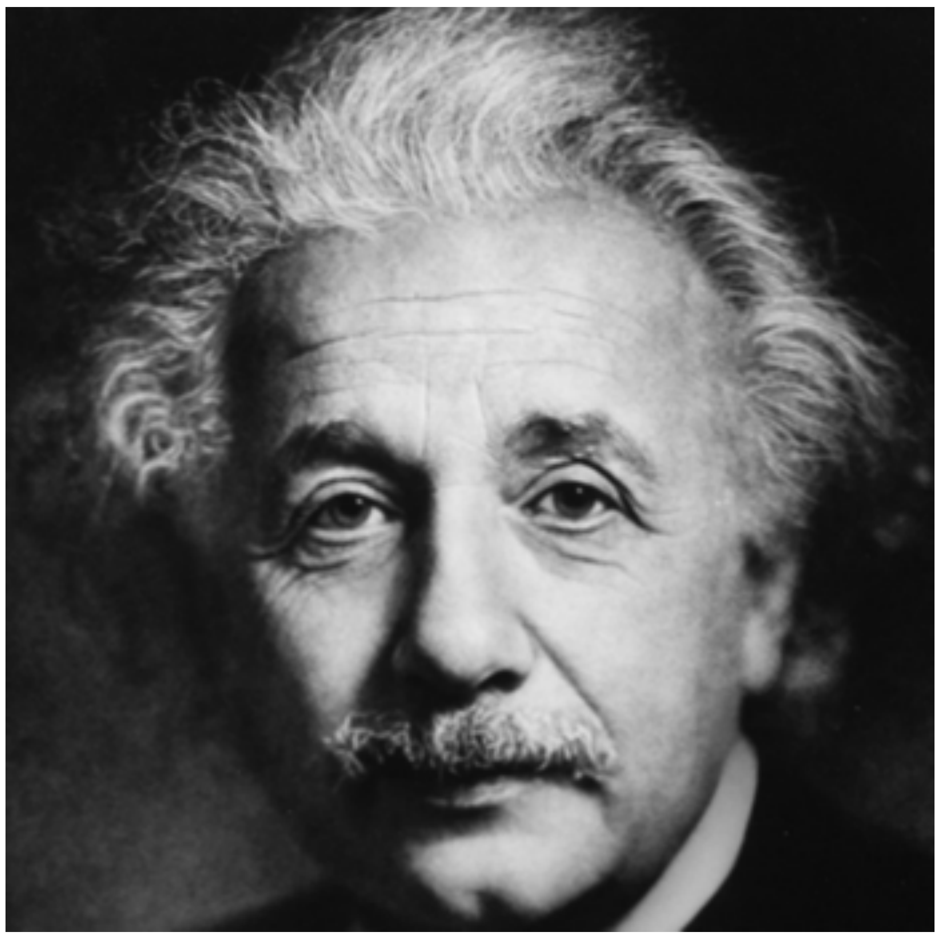
Algorithm → discreteness!

discrete



continuum



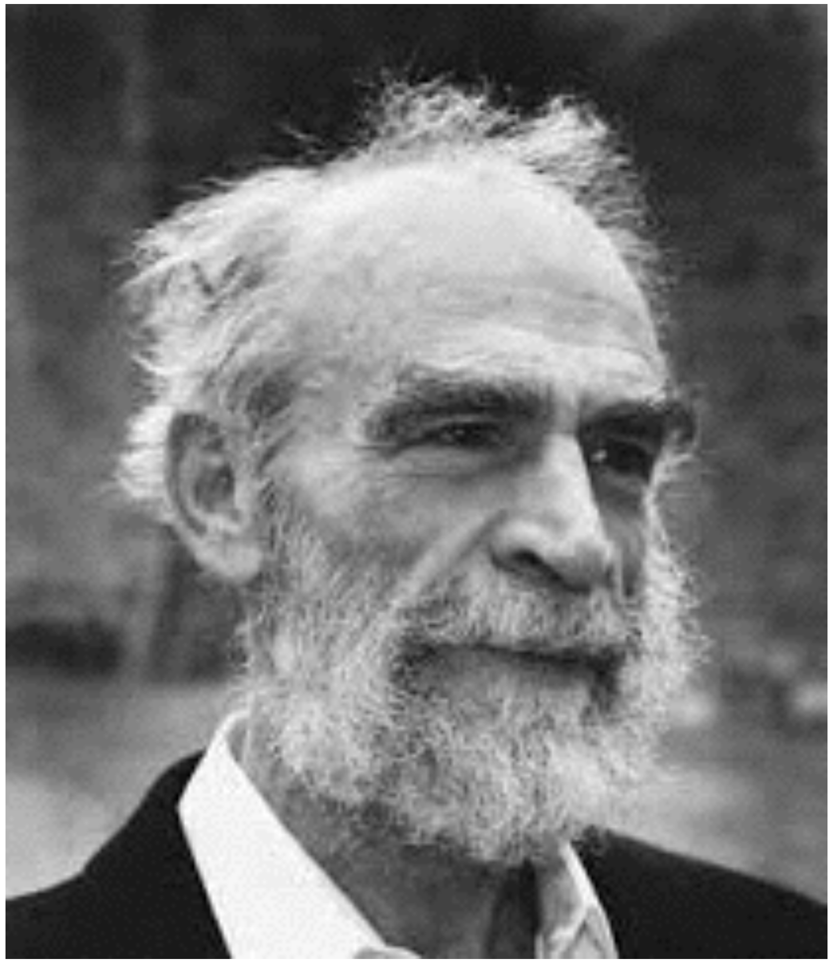


"But you have correctly grasped <u>the drawback that the continuum brings</u>. If the molecular view of matter is the correct (appropriate) one, i.e., if a part of the universe is to be represented by a finite number of moving points, then the continuum of the present theory contains too great a manifold of possibilities. I also believe that this too great is responsible for the fact that our present means of description <u>miscarry with the quantum theory</u>. The problem seems to me how one can formulate statements about a discontinuum without calling upon a continuum (space-time) as an aid; the latter should be banned from the theory as a supplementary construction not justified by the essence of the problem, which corresponds to nothing "real". <u>But we still lack the mathematical structure unfortunately.</u> How much have I already plagued myself in this way!"

John Stachel in *From Quarks to Quasars: Philosophical Problems of Modern Physics,* University of Pittsburg Press, pag. 379

A new mathematics: *geometric group theory*

The *geometrization* of group theory



Mikhail Gromov

Geometric group theory: a primer

$$G = \langle \underbrace{h_1, h_2, \ldots}_{\textit{generators}} | \underbrace{r_1, r_2, \ldots}_{\textit{relators}} \rangle$$

Dehn problems: undecidable!

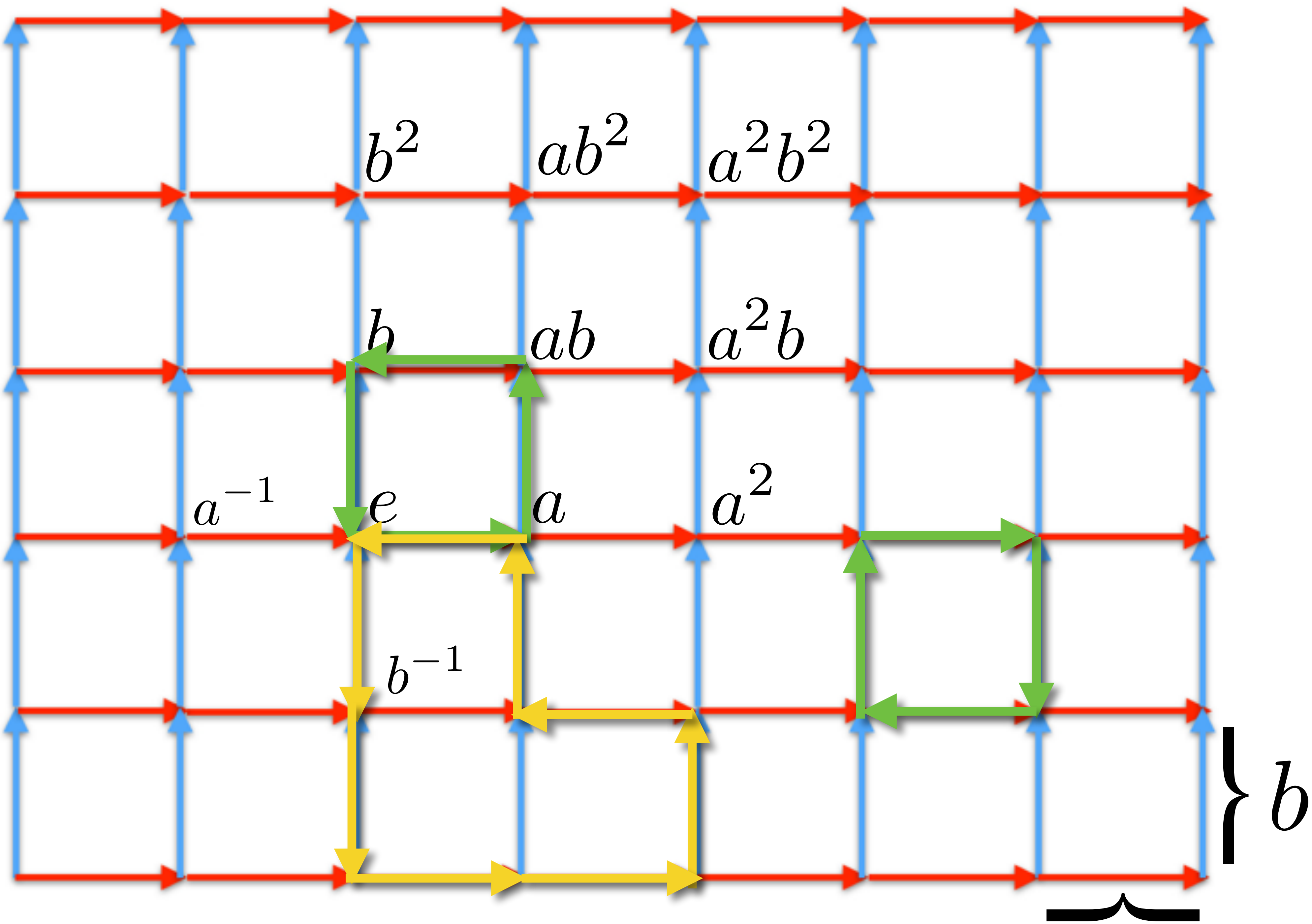
Example: $Z^2 = \{(n,m) | n,m \in Z\}$

$$Z^2 = \langle a, b | aba^{-1}b^{-1} \rangle \qquad e = (0,0),\ a = (1,0),\ b = (0,1),$$

$$ab = ba \Longrightarrow aba^{-1}b^{-1} = e$$

$$\Longrightarrow abaab^{-1}abbba^{-1}baabb^{-1}aa^{-1} = a^5b^4$$

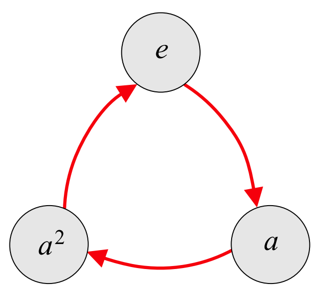
Geometric Group Theory: Cayley graph of a group



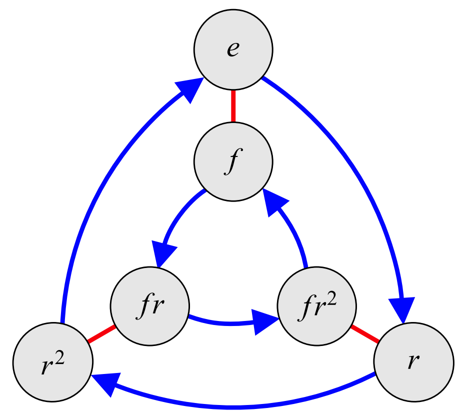
$$a = (1, 0)$$
$$b = (0, 1)$$

$$Z^2 = \langle a, b | aba^{-1}b^{-1} \rangle$$

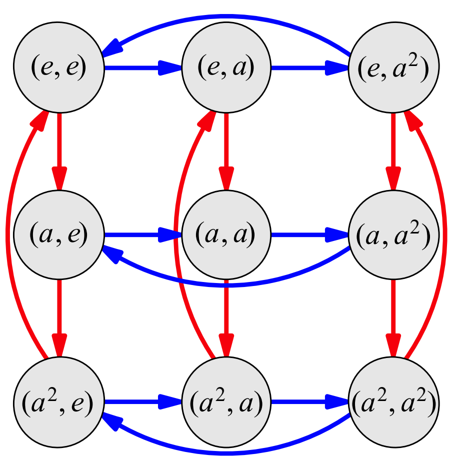
Geometric Group Theory: Cayley graph of a group



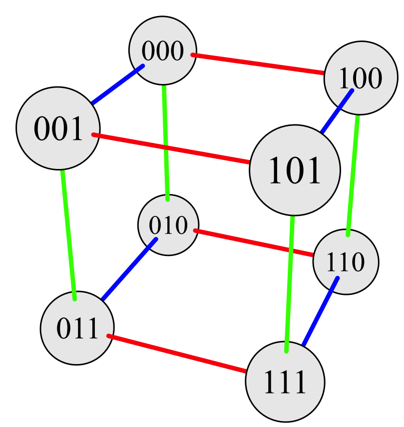
Cyclic group C_3 (or $\mathbb{Z}_3$)



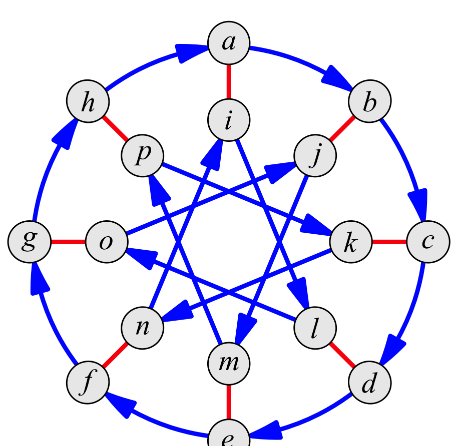
Symmetric group S_3



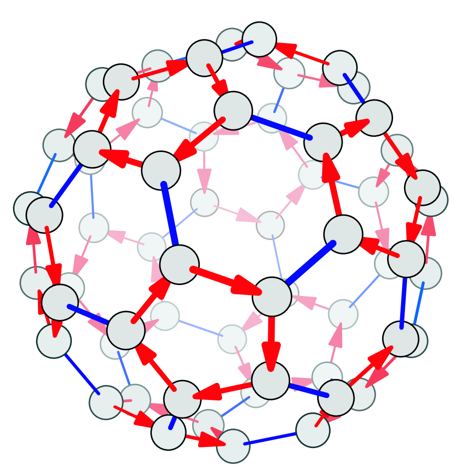
Direct product group $C_3 \times C_3$



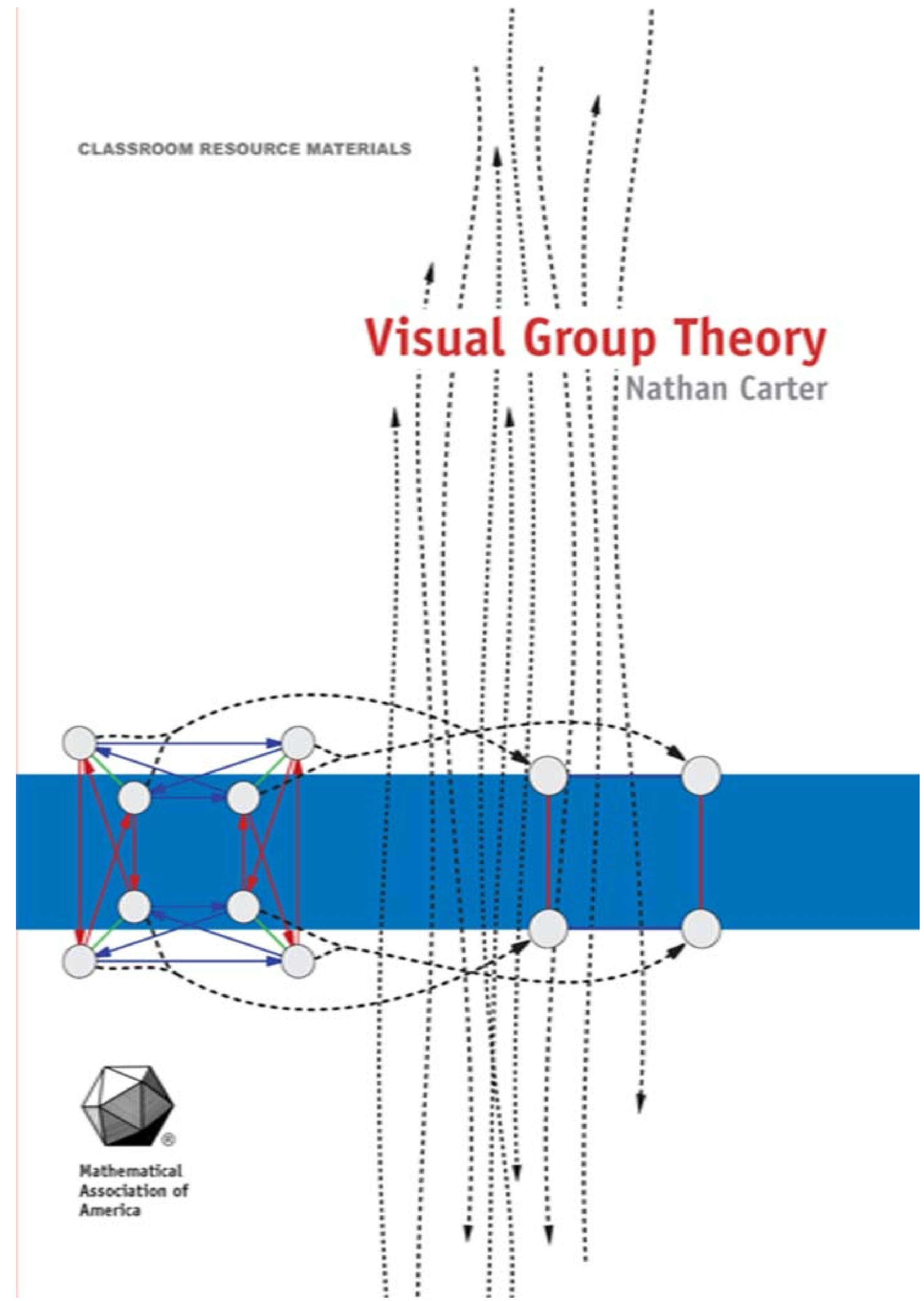
Direct product group $C_2 \times C_2 \times C_2$

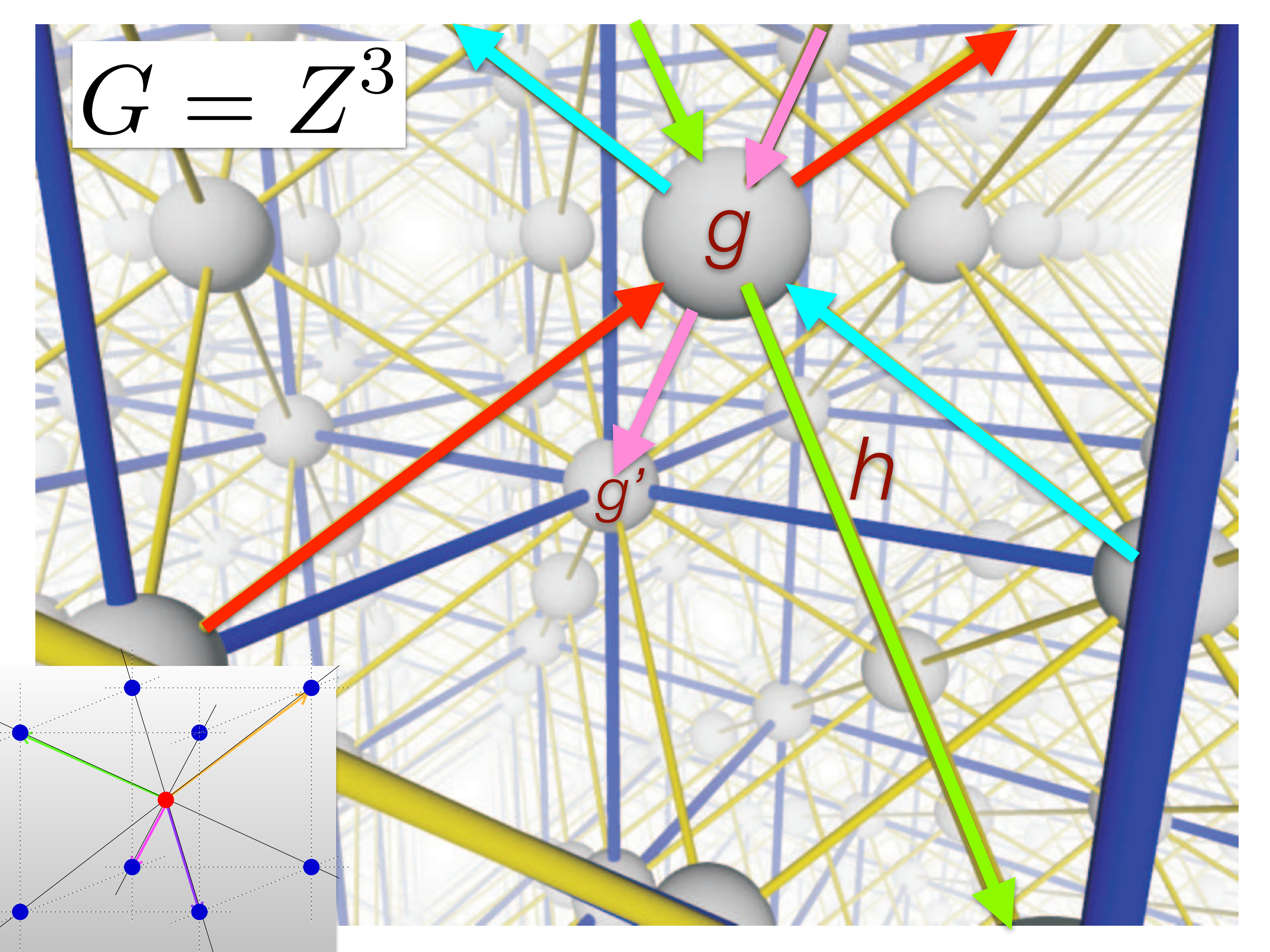


Quasihedral group with 16 elements



Alternating group A_5

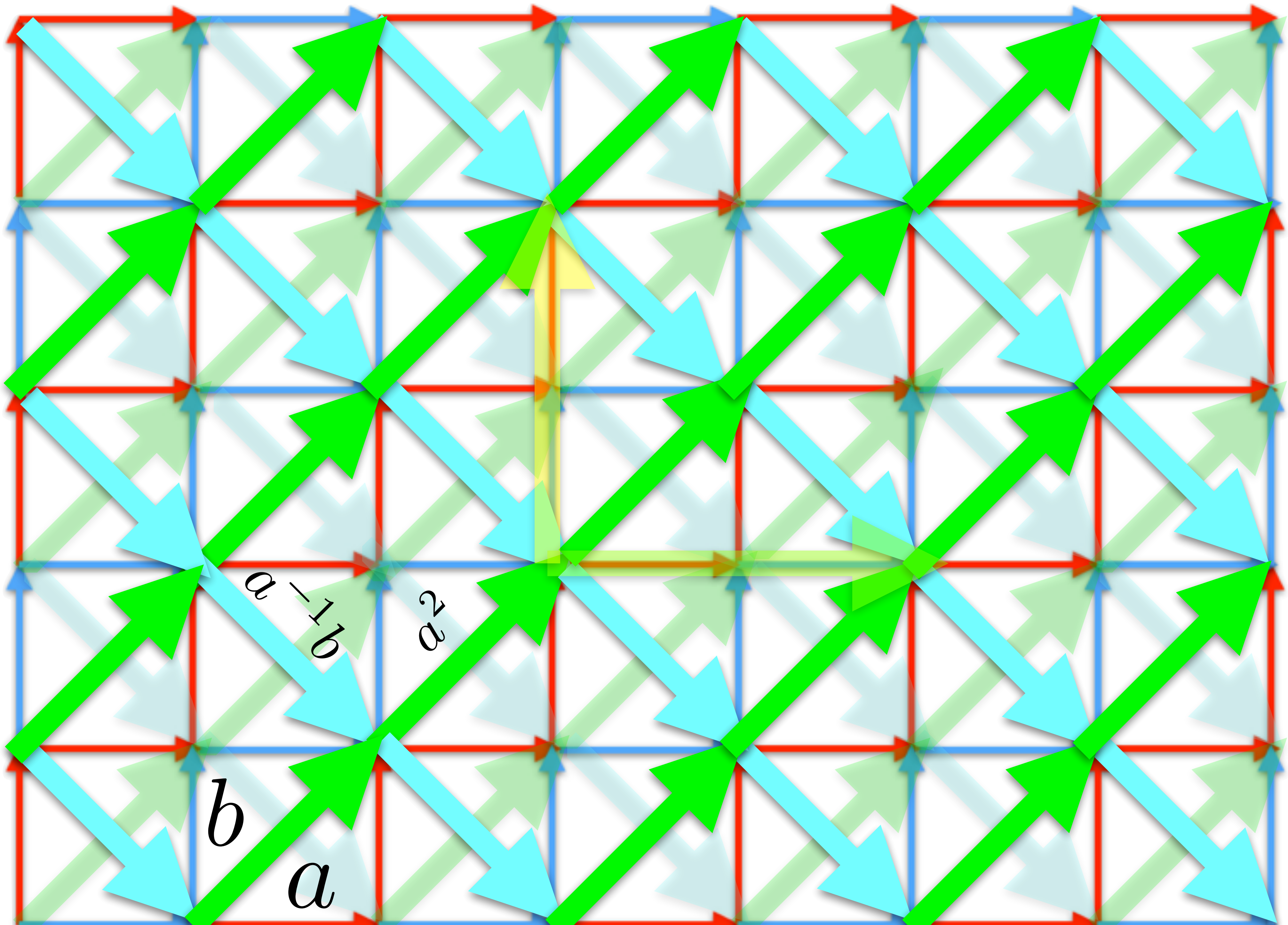



$G = Z^3$
g
g'
h

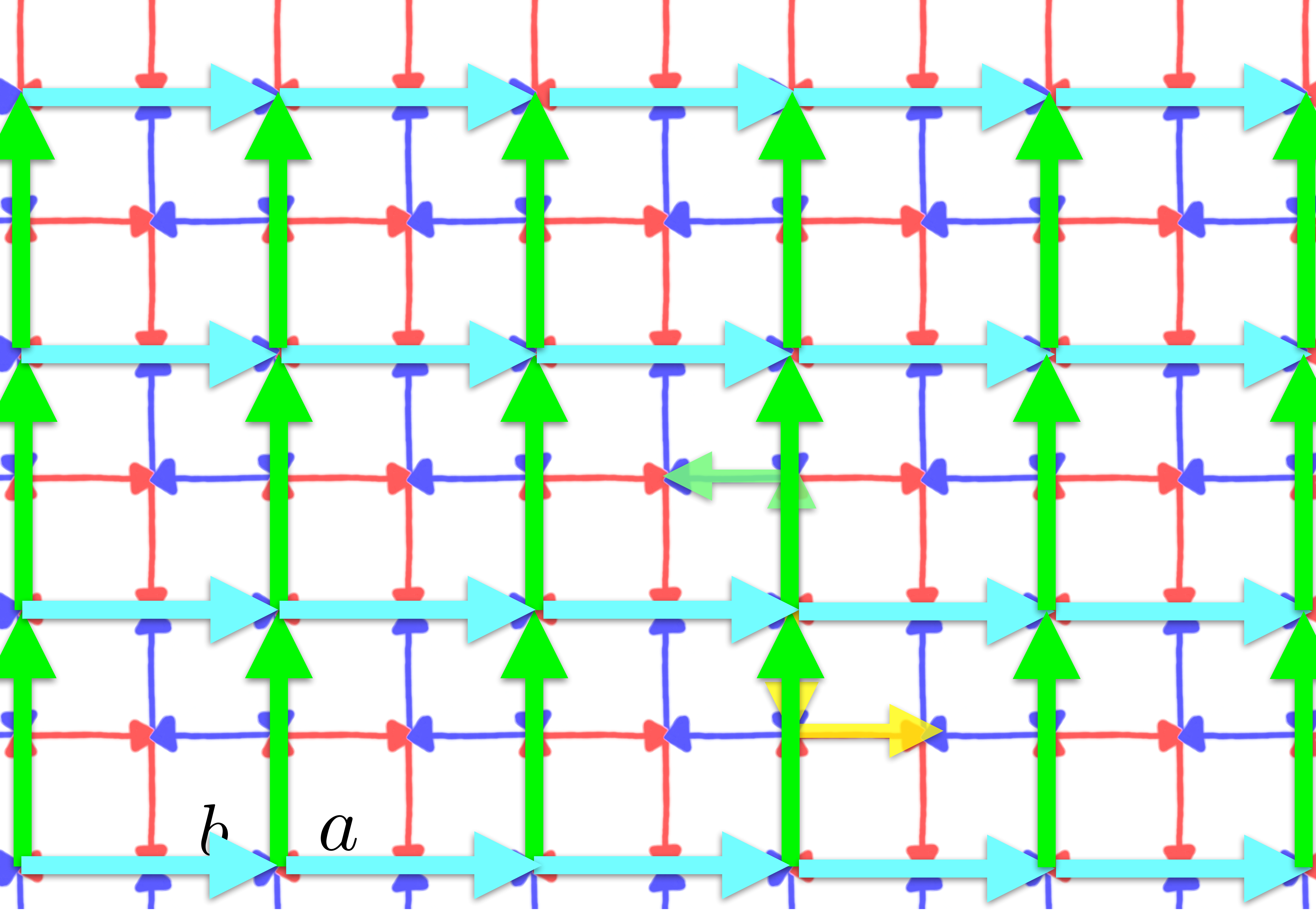
Virtually Abelian groups

$Z^2 \subset G$

Index =2

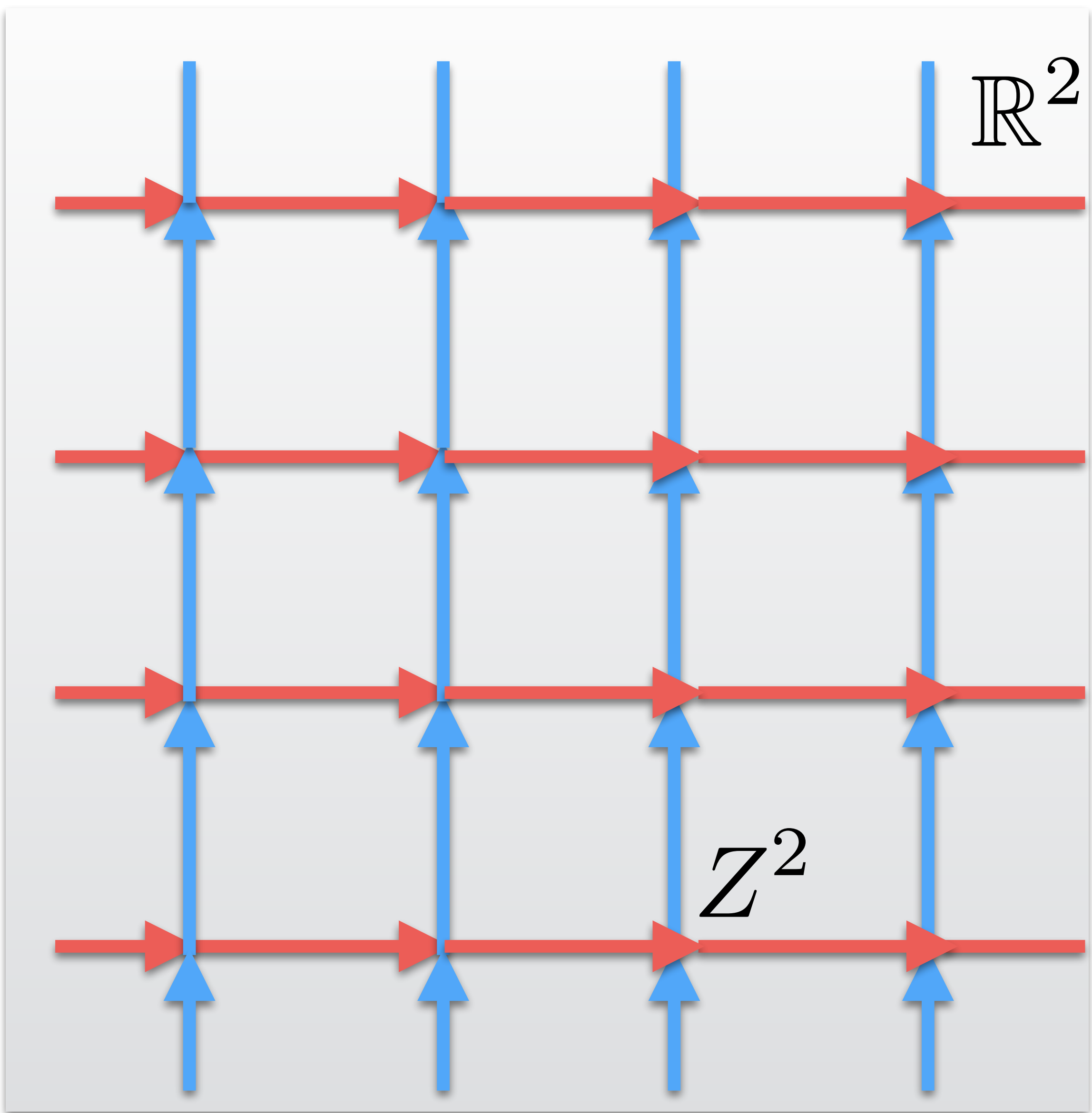


$G = \langle a, b | a^2 b^{-2} \rangle$



$Z^2 \subset G$ Index =4 $G = \langle a, b | a^4, b^4, (ab)^2 \rangle$

Geometric Group Theory



Quasi isometric embedding

Suppose that *f* is a (not necessarily continuous) function from one metric space M_1 to a second metric space M_2. Then it is called a *quasi-isometry* from M_1 to M_2 if there exist constants $A \geq 1$, $B \geq 0$, $C \geq 0$ such that the following two properties both hold:

1) for every two points and in M_1, the distance between their images is (up to the additive constant B) *within a factor* of A of their original distance: $\forall\, \boldsymbol{x,u} \in M_1$

$$\frac{1}{A} d_1(x,y) - B \leqslant d_2(f(x), f(y)) \leqslant A d_1(x,y) + B$$

2) every point of M_2 is within a constant distance C from the image point:

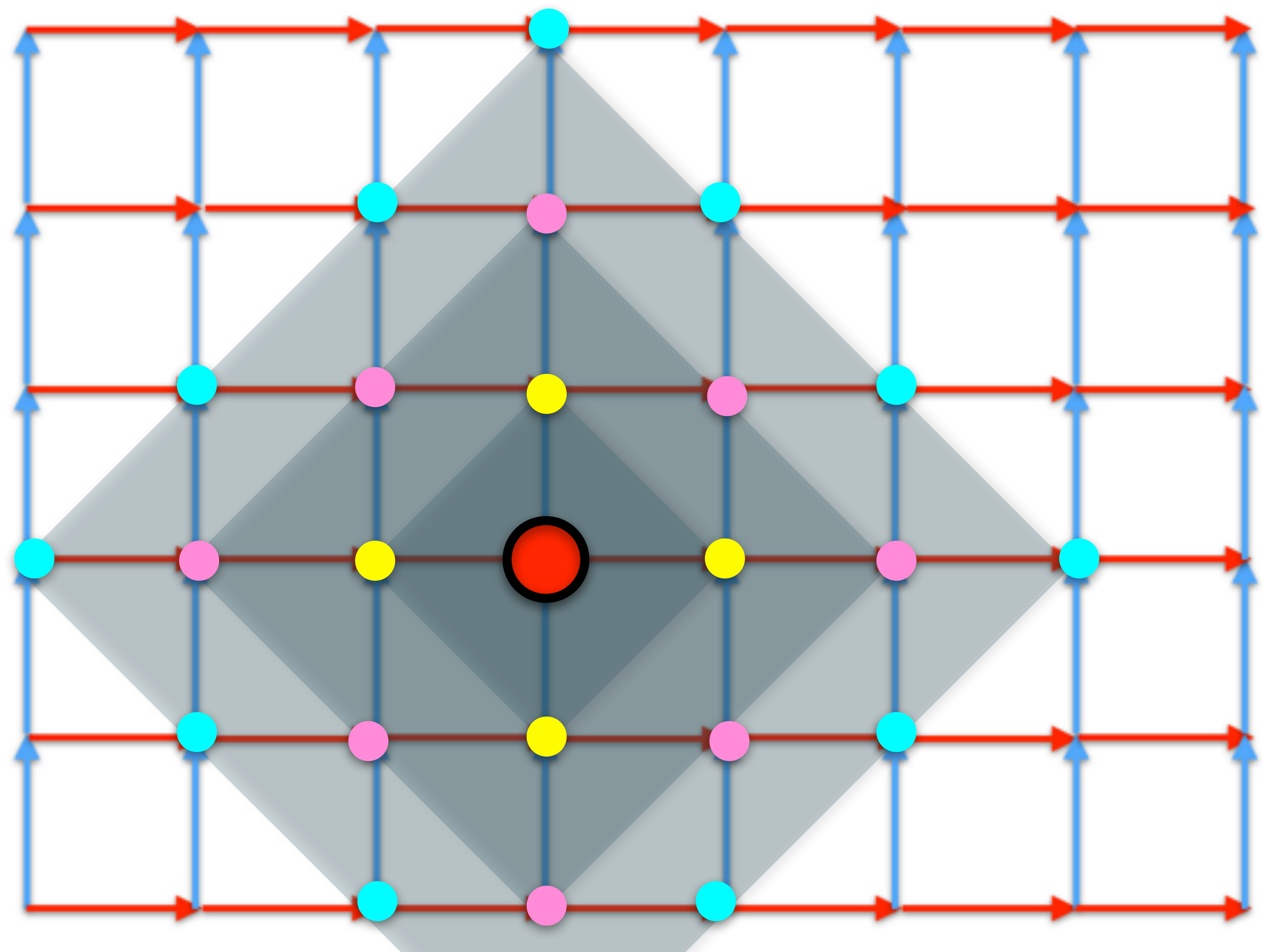
$$\forall z \in M_2 : \exists x \in M_1 : d_2(z, f(x)) \leqslant C$$

Quantum walk on Cayley graph

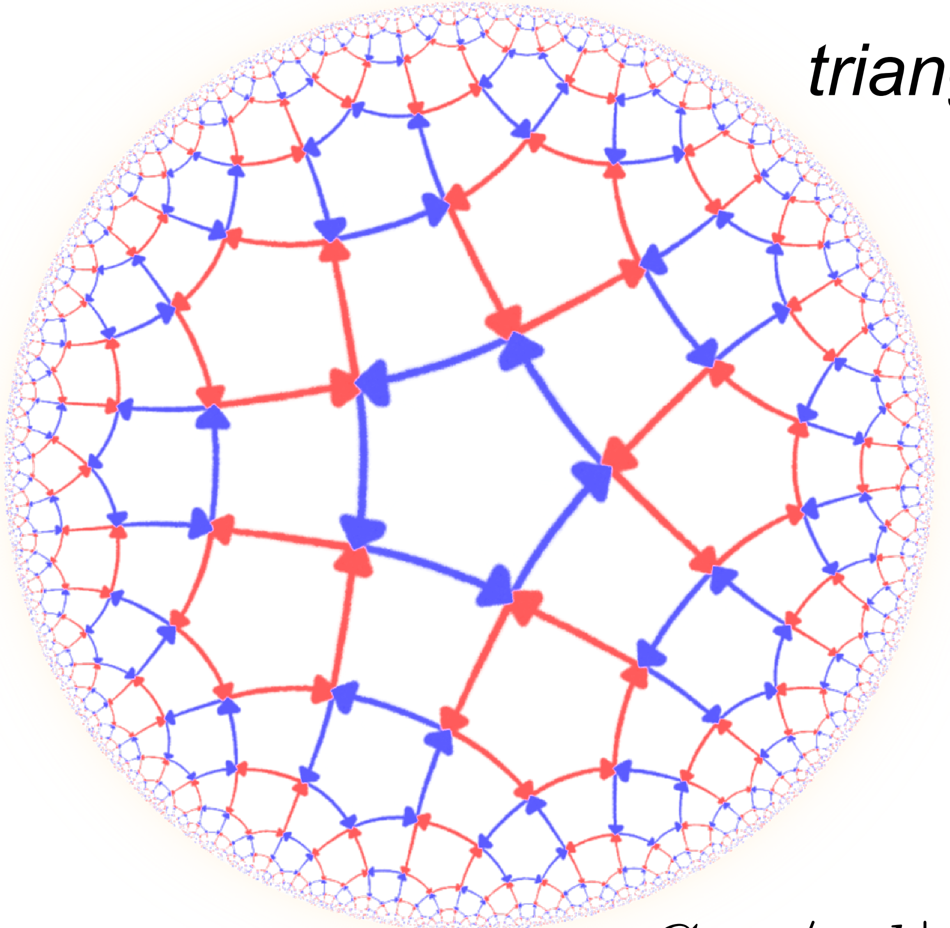
Theorem: A group is quasi-isometrically embeddable in R^d iff it is virtually Abelian

Theorem: A group has polynomial growth iff it is virtually nihilpotent

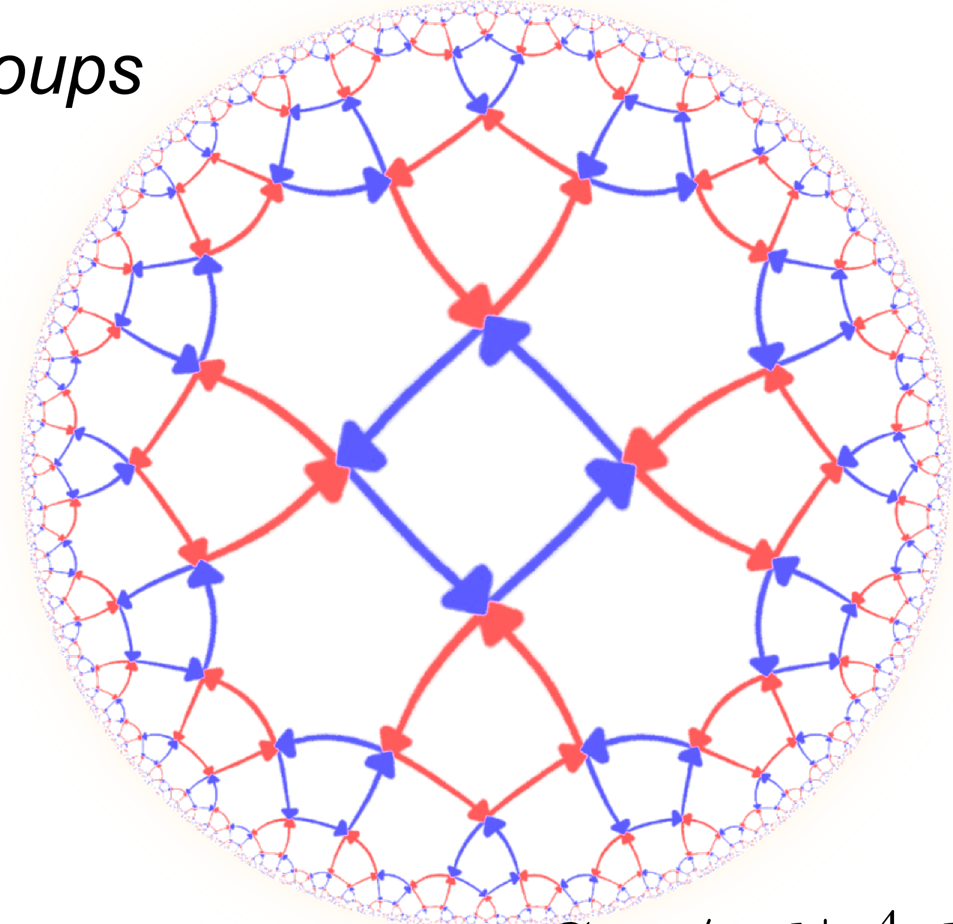
points $\sim r^d$



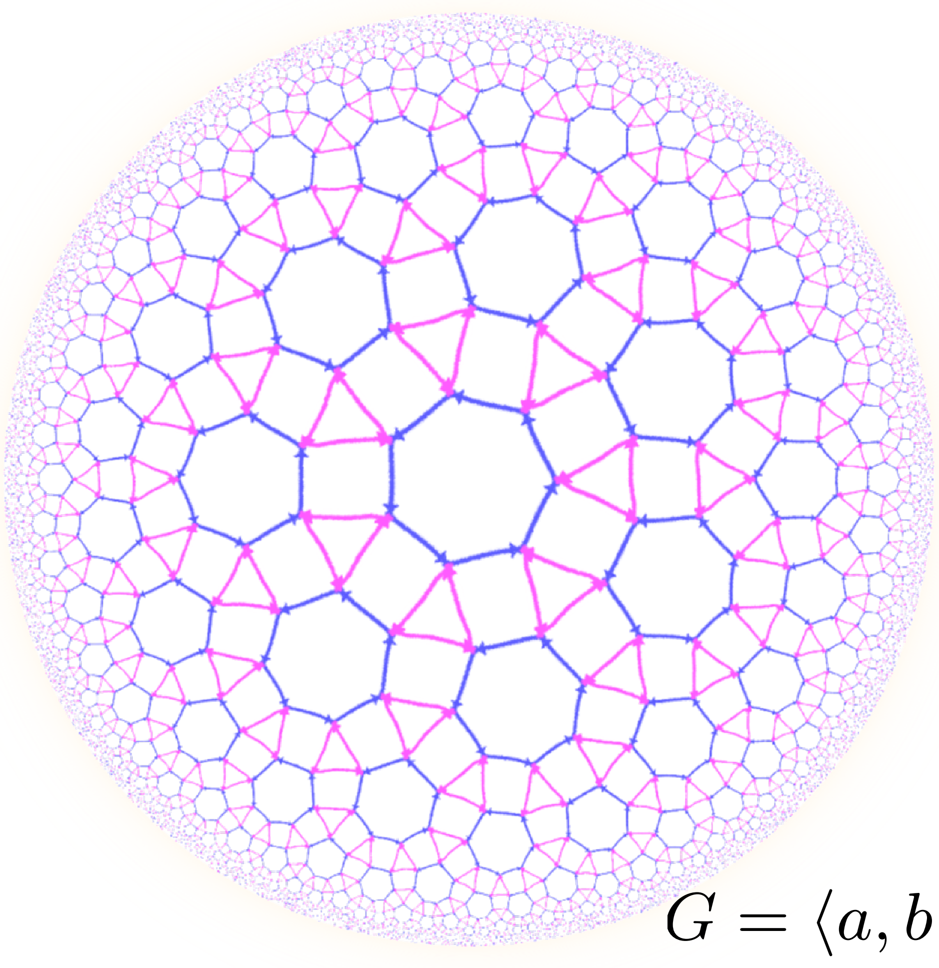
triangle orbifold groups



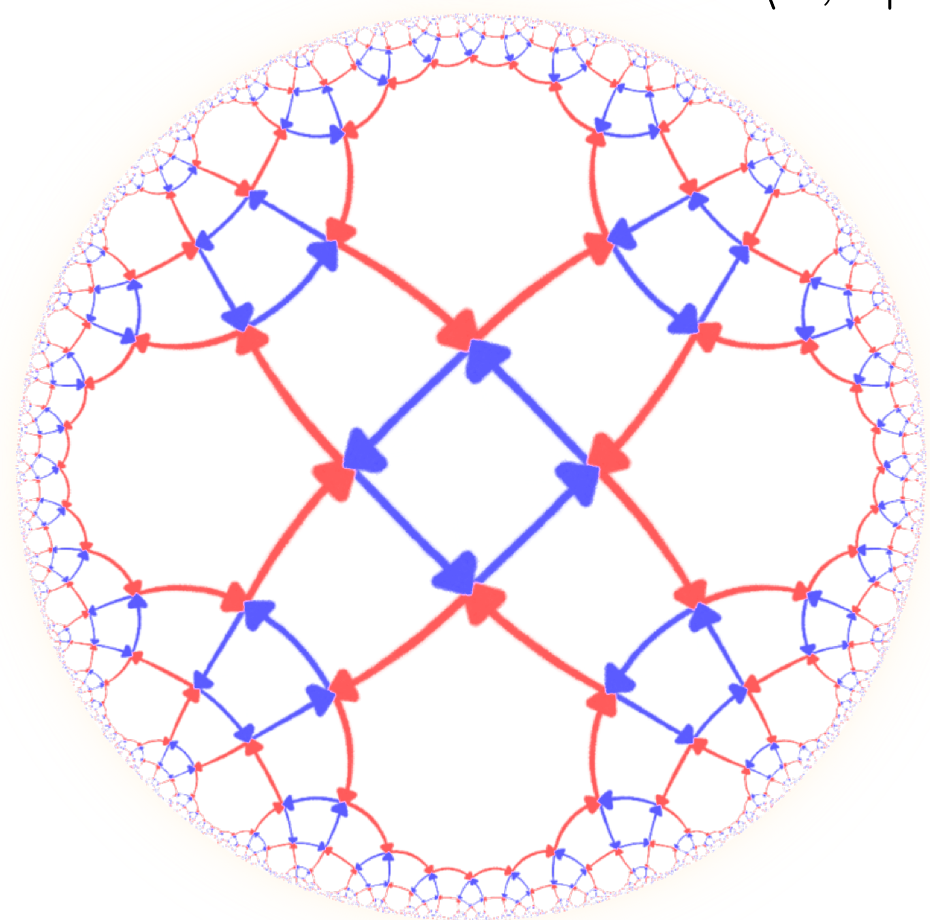
$G = \langle a, b | a^5, b^5, (ab)^2 \rangle$



$G = \langle a, b | a^4, b^4, (ab)^3 \rangle$

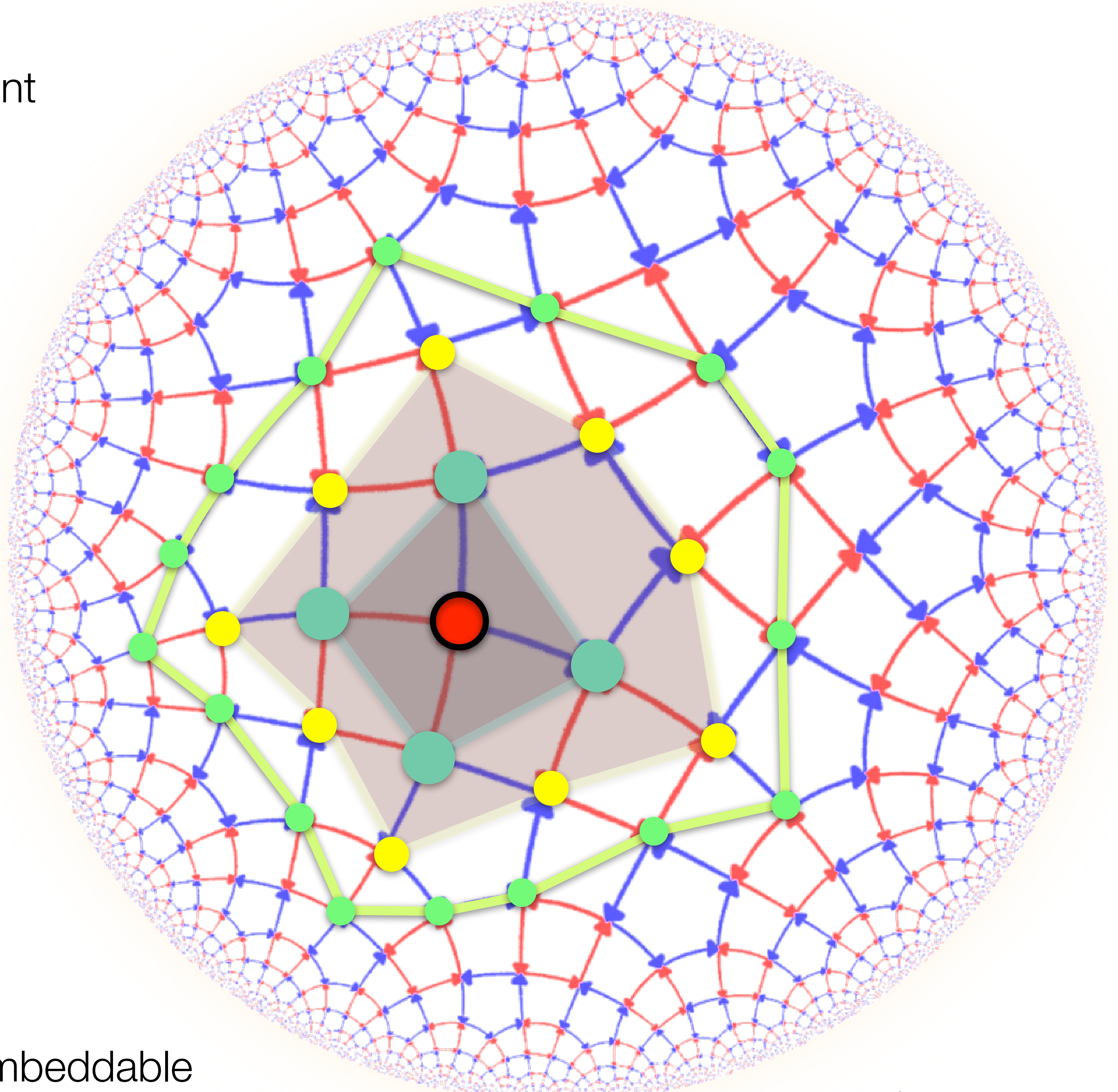


$G = \langle a, b | a^7, b^3, (ab)^2 \rangle$



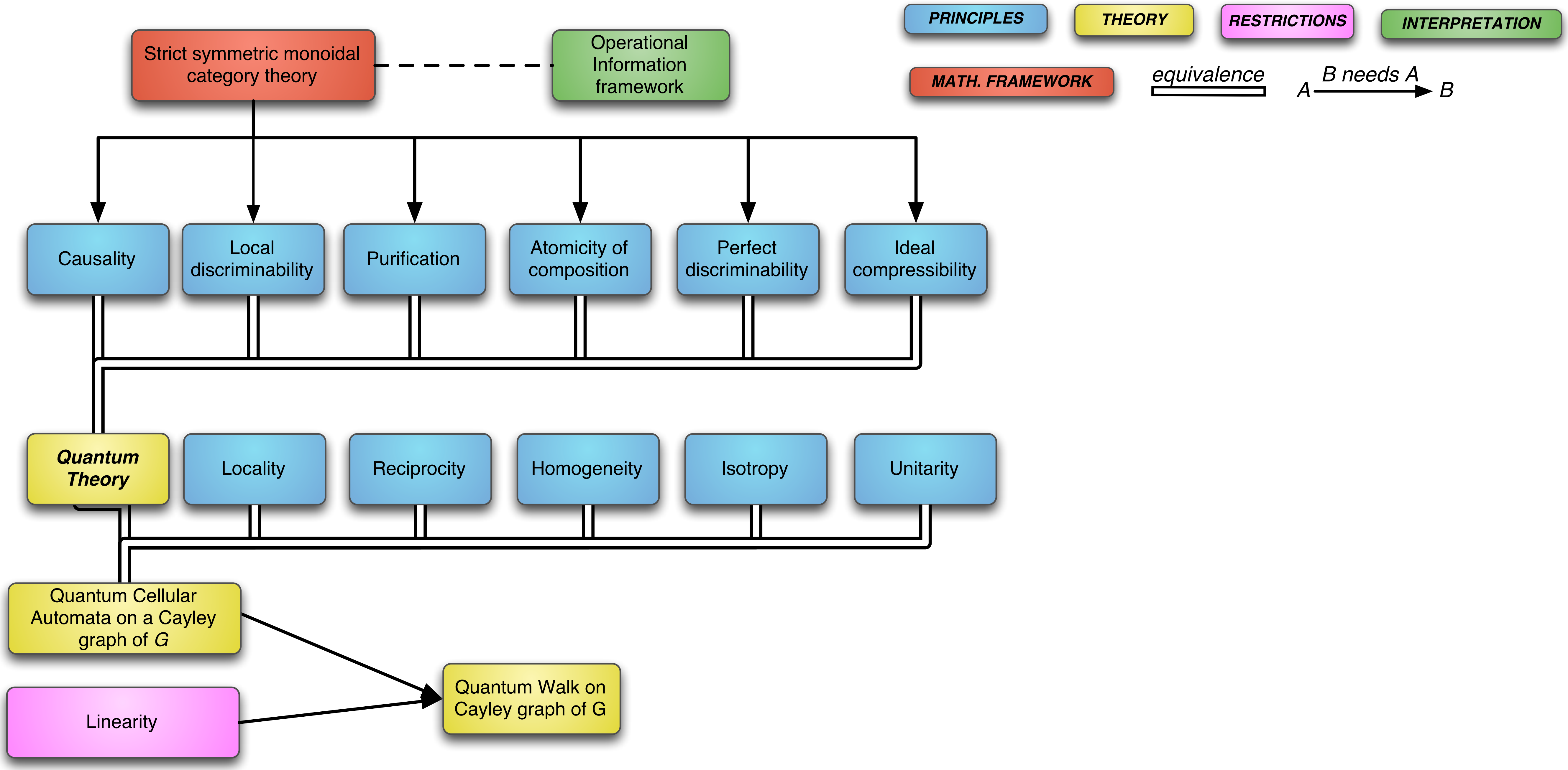
$G = \langle a, b | a^4, b^{10}, (ab)^2 \rangle$

G non virtually nihilpotent
→ exponential growth



quasi-isometrically embeddable
in hyperbolic space

$$G = \langle a, b | a^5, b^4, (ab)^2 \rangle$$



Linearity in the field corresponds to work with transition matrices between blocks of direct-sum of Hilbert spaces instead of unitary interactions on tensor product of Hilbert spaces.

$$\mathscr{A}\psi_g = U\psi_g U^\dagger = \sum_{g'} A_{g,g'}\psi_{g'}$$

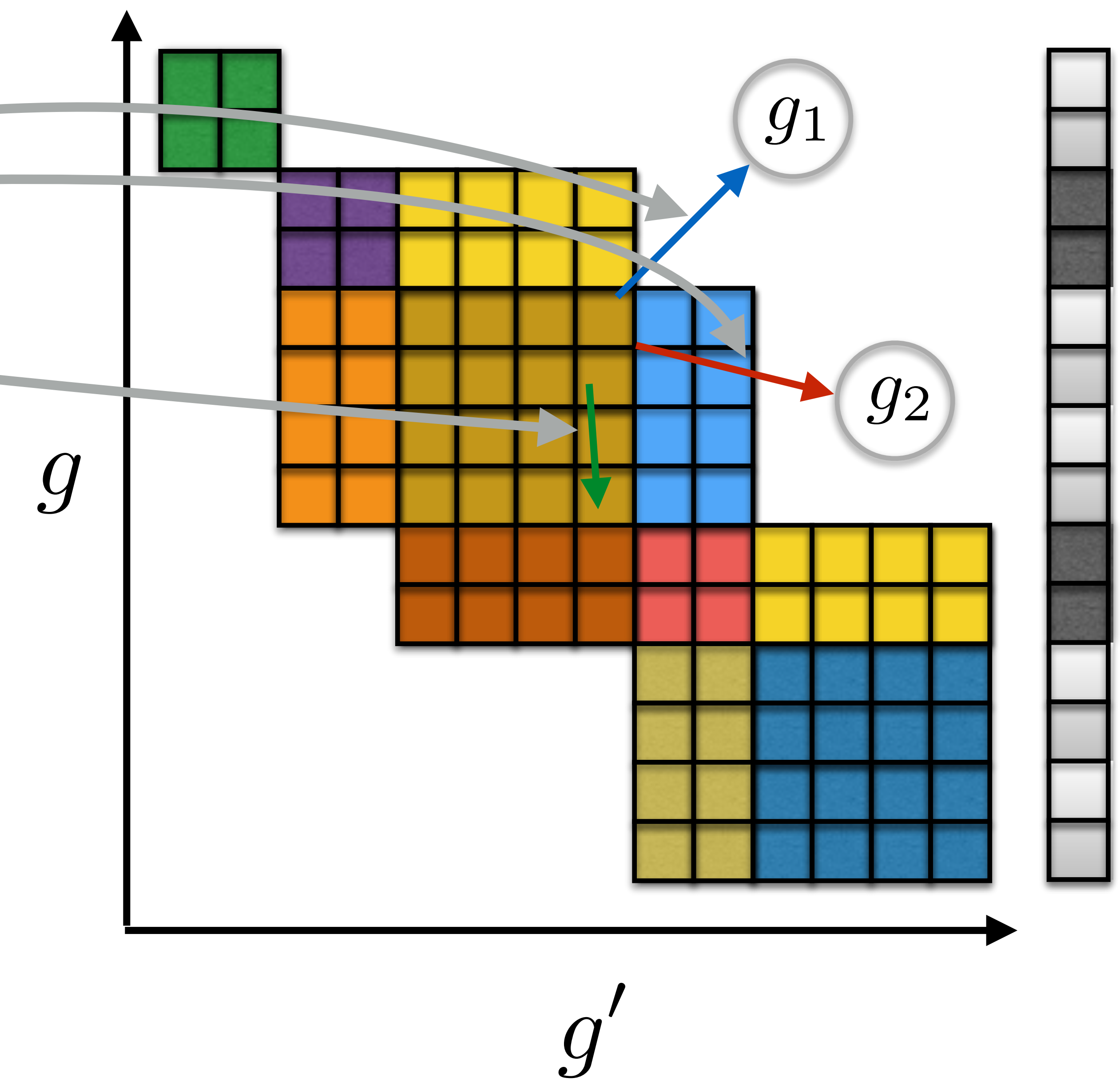
Quantum walk on Cayley graph

w.l.g. Hilbert space $\mathcal{H} = \bigoplus_{g\in G}\mathbb{C}^{s_g}$ $\quad |G| \leqslant \aleph,\ s_g \in \mathbb{N}$

Evolution

$$\psi_g(t+1) = \sum_{g'\in S_g} A_{gg'}\psi_{g'}(t)$$

$$\sum_{g'} A_{gg'}A^\dagger_{g''g'} = \sum_{g'} A^\dagger_{gg'}A_{g''g'} = \delta_{gg''}I_{s_g}$$



Build a <u>directed colored graph</u> with an arrow from g to g' *wherever they* are connected by $A_{gg'} \neq 0$

D'Ariano, Perinotti,
PRA **90** 062106 (2014)

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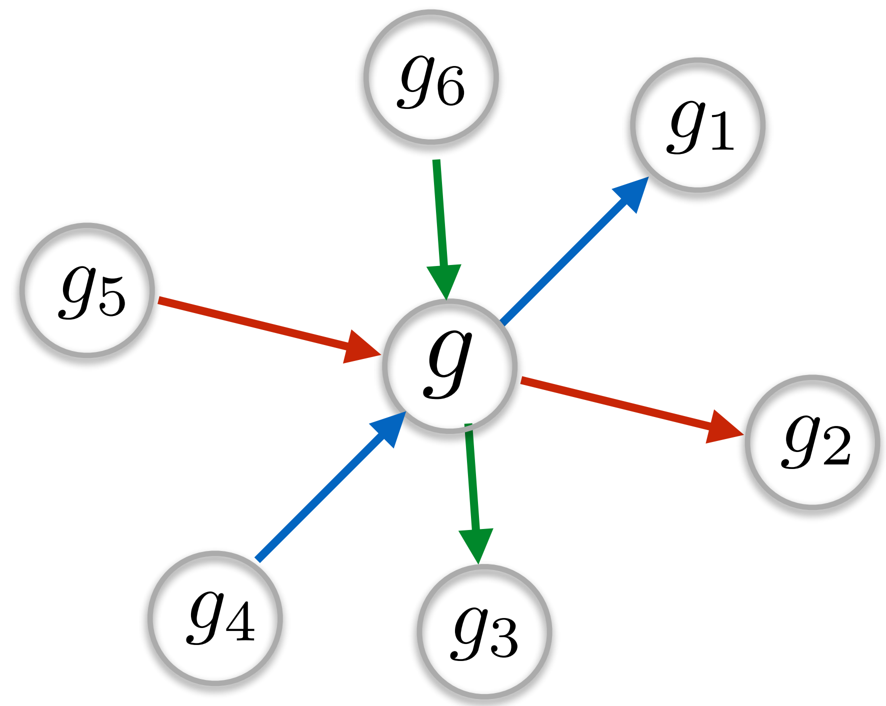

1) *Locality*: S_g uniformly bounded

2) *Reciprocity*: $A_{gg'} \neq 0 \implies A_{g'g} \neq 0$

3) *Homogeneity*: all $g \in G$ are "equivalent"

$S_g = S,\ s_g = s\ \ldots$ label $A_{gg'} =: A_h,\ h \in: S$

define the "action" on the set of vertices *G*: $gh := g'$ whenever $A_{gg'} = A_h$

D'Ariano, Perinotti,
PRA **90** 062106 (2014)

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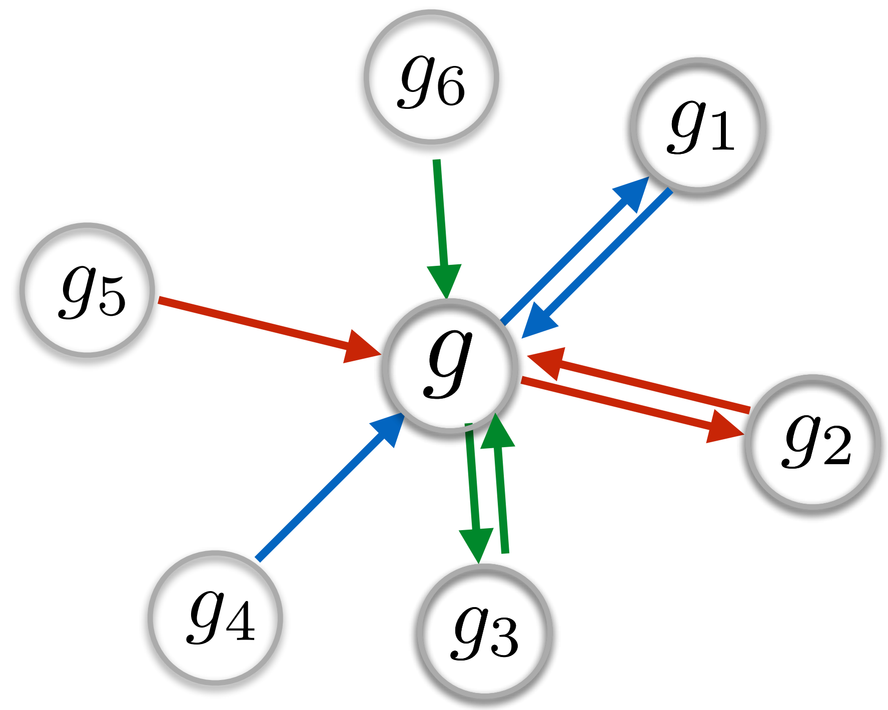
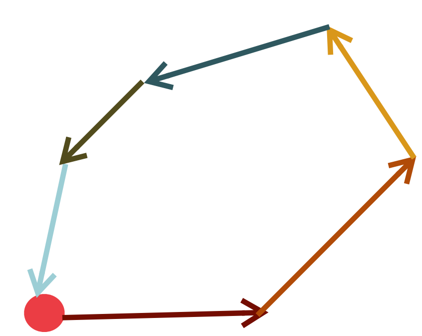


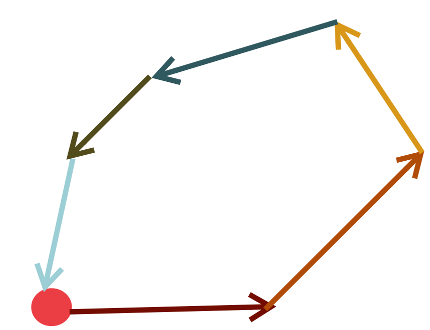
1) *Locality*: S_g uniformly bounded

2) *Reciprocity*: $A_{gg'} \neq 0 \implies A_{g'g} \neq 0$

3) *Homogeneity*: all $g \in G$ are equivalent

A sequence $A_{h_N}A_{h_{N-1}}\ldots A_{h_1}$ connects g to itself, namely $gh_1h_2\ldots h_N = g$, then it must also connect any other g' to itself, i.e. $g'h_1h_2\ldots h_N = g'$.





From 2): two-loop $ghh^{-1} = g$ defines uniquely h^{-1} for h and viceversa

$$A_{gg'} =: A_h,\ A_{g'g} =: A_{h^{-1}},\ h \in S \equiv S_+ \cup S_-,\ S_- := S_+^{-1}$$

D'Ariano, Perinotti, PRA **90** 062106 (2014)

Quantum walk on Cayley graph

w.l.g. Hilbert space $\mathcal{H} = \oplus_{g\in G}\mathbb{C}^{s_g} \quad |G| \leqslant \aleph, \; s_g \in \mathbb{N}$

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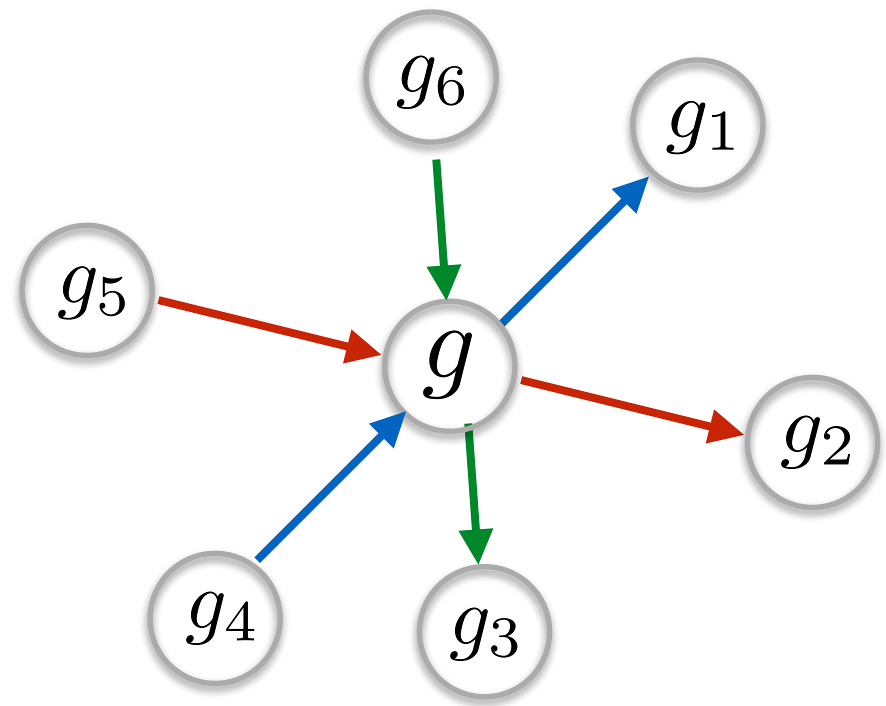

1) *Locality*: S_g uniformly bounded

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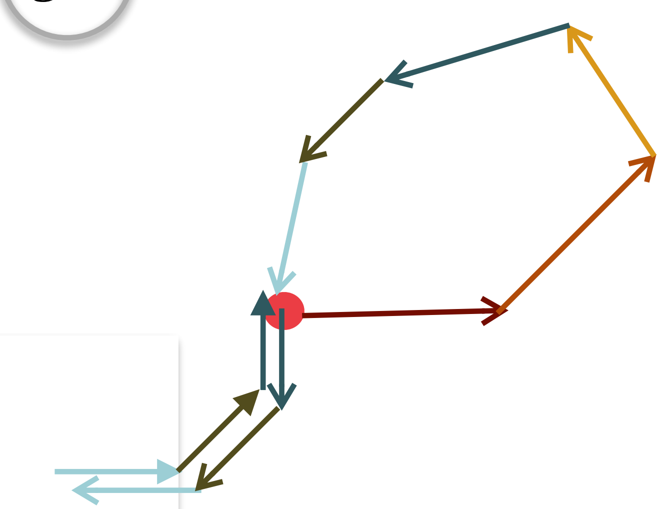

Build the free group *F* of words made with letters:

$$h \in S := S_+ \cup S_-$$

with action on vertices in *G*: $gh := g'$ whenever $A_{gg'} = A_h$

Consider the subgroup *H* of closed paths ➡ *H* normal subgroup of *F*

D'Ariano, Perinotti, PRA **90** 062106 (2014)

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$\Gamma(G,S_+)$ *colored directed graph* with vertices $g \in G$ and edges (g,g') with $g' = gh$

Either the graph is connected, or it consists of disconnected copies.
W.l.g. assume it as connected.

Being H normal, one concludes that:

$G = F/H = \langle S|R\rangle$ is a group with Cayley graph $\Gamma(G,S_+)$ (the identity any element $e \in G$).

D'Ariano, Perinotti,
PRA **90** 062106 (2014)

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The following operator over the Hilbert space $\ell^2(G)\otimes\mathbb{C}^s$ is unitary

$$A = \sum_{h\in S} T_h \otimes A_h$$

where T is the right regular representation of G on $\ell^2(G)$ acting as

$$T_g|g'\rangle = |g'g^{-1}\rangle$$

1) *Locality*: S_g uniformly bounded

2) *Reciprocity*: $A_{gg'} \neq 0 \implies A_{g'g} \neq 0$

3) *Homogeneity*: all $g \in G$ are equivalent

iff for Quantum Walk on Cayley graph

4) *Isotropy:*

There exist:

- a group L of permutations of S_+, transitive over S_+ that leaves the Cayley graph invariant
- a unitary s-dimensional (projective) representation $\{L_l\}$ of L such that:

$$\boxed{A = \sum_{h\in S} T_h \otimes A_h = \sum_{h\in S} T_{lh}\otimes L_l A_h L_l^\dagger}$$

Quantum walk on Cayley graph

The quantum walk on the Cayley graph is completely specified as

$$Q = (G, S_+, s, \{A_h\}_{h \in S})$$

D'Ariano, Erba, Perinotti,Tosini
arXiv::1603.07666,1511.03992

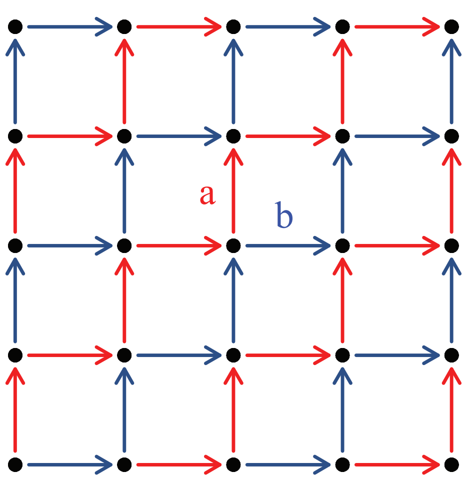
Induced representation theorem

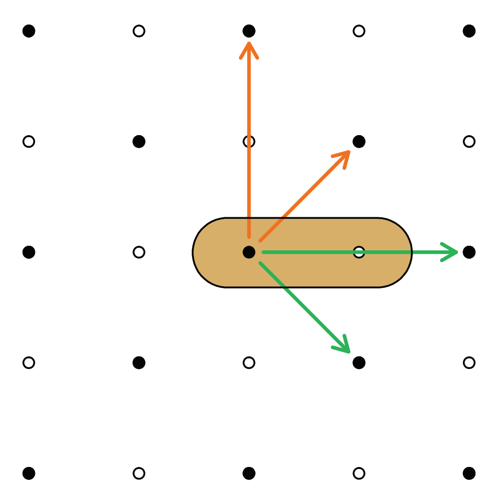
$Q = (G, S_+, s, \{A_h\}_{h\in S})$ with virtually Abelian G

equivalent to

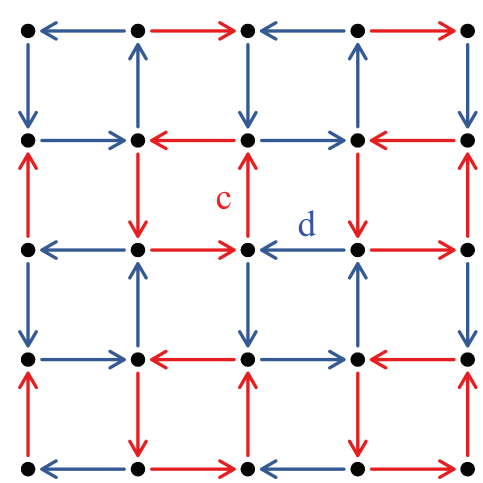
$Q' = (H, S_+, si_H, \{A_h\}_{h\in S}), H \subset G,\ i_H$: index of H

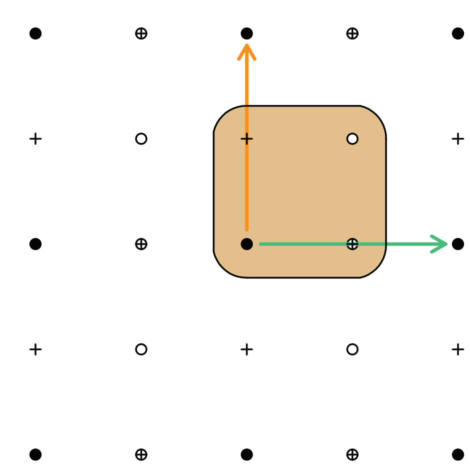
$\langle\ a, b \mid a^2 b^{-2}\ \rangle$



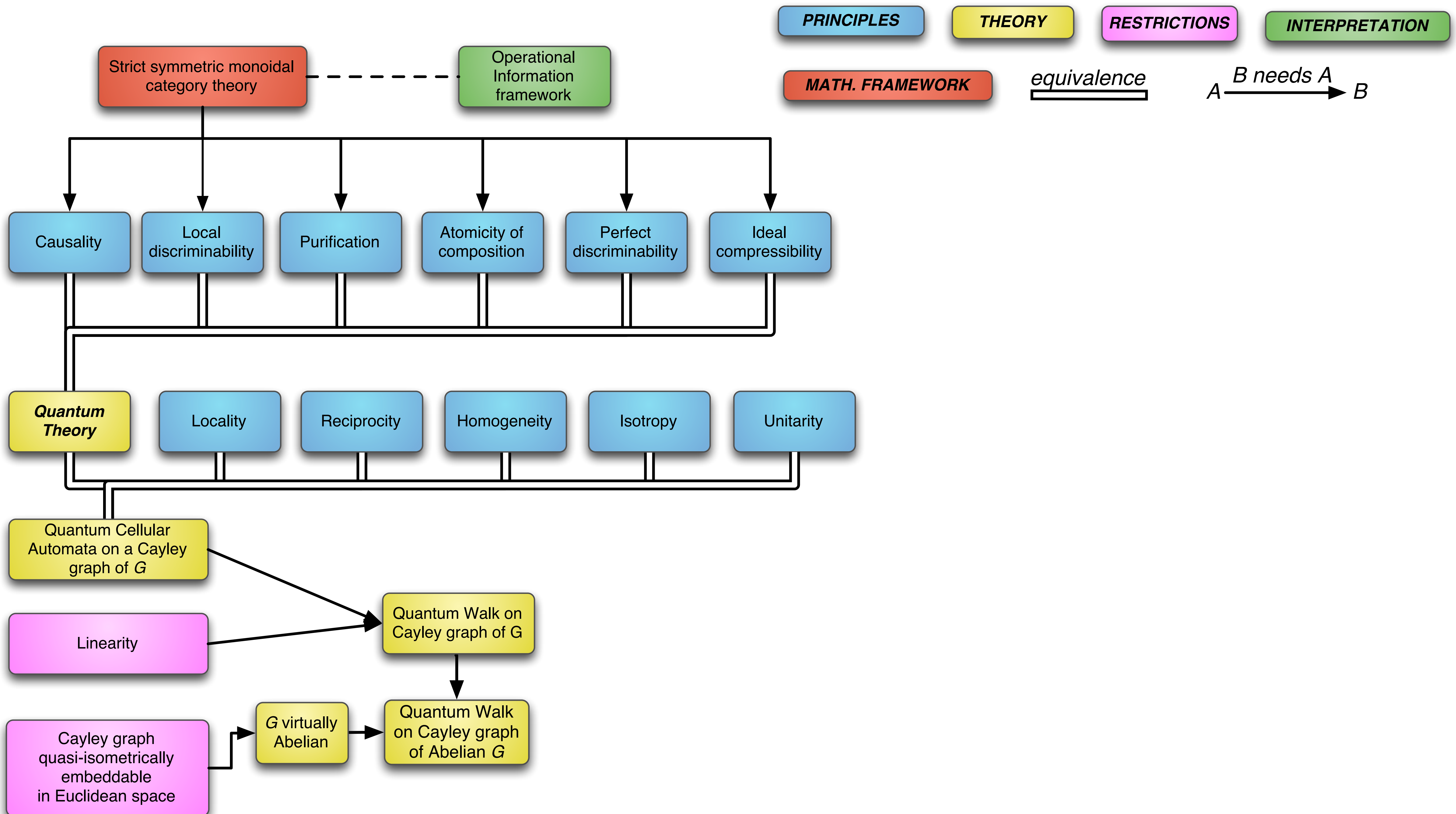


$\langle\ c, d \mid c^4, d^4, (cd)^2\ \rangle$





(isotropy is not transferred between G and H)



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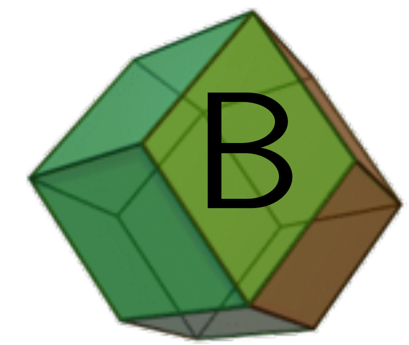
D'Ariano, Perinotti, PRA **90** 062106 (2014)

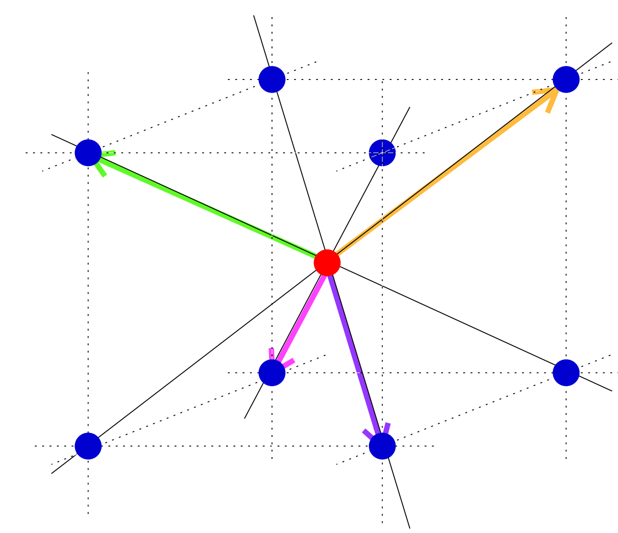
The Weyl QW

☞ Minimal dimension for nontrivial unitary *A:* s=2



- Unitarity ⇒ for d=3 the only possible *G* is the BCC!!

- Isotropy ⇒ Fermionic ψ (d=3)



Unitary operator: $$A = \int_B^{\oplus} d\mathbf{k}\, A_{\mathbf{k}}$$

Two QWs connected by P

$$\boxed{\begin{aligned} A_{\mathbf{k}}^{\pm} = &- i\sigma_x(s_x c_y c_z \pm c_x s_y s_z) \\ &\mp i\sigma_y(c_x s_y c_z \mp s_x c_y s_z) \\ &- i\sigma_z(c_x c_y s_z \pm s_x s_y c_z) \\ &+ I(c_x c_y c_z \mp s_x s_y s_z) \end{aligned}}$$

$$s_\alpha = \sin\frac{k_\alpha}{\sqrt{3}}$$

$$c_\alpha = \cos\frac{k_\alpha}{\sqrt{3}}$$

The Weyl QW

D'Ariano, Perinotti,
PRA **90** 062106 (2014)

$$i\partial_t\psi(t) \simeq \tfrac{i}{2}[\psi(t+1)-\psi(t-1)] = \tfrac{i}{2}(A-A^\dagger)\psi(t)$$

$$\begin{aligned}\tfrac{i}{2}(A^{\pm}_{\mathbf{k}}-A^{\pm\dagger}_{\mathbf{k}}) = &+\sigma_x(s_xc_yc_z \pm c_xs_ys_z)\\ &\pm\sigma_y(c_xs_yc_z \mp s_xc_ys_z)\\ &+\sigma_z(c_xc_ys_z \pm s_xs_yc_z)\end{aligned}$$

"Hamiltonian"

$k \ll 1$ $\Rightarrow$ $\boxed{i\partial_t\psi = \frac{1}{\sqrt{3}}\boldsymbol{\sigma}^{\pm}\cdot\mathbf{k}\,\psi}$ ☜ Weyl equation! $\quad \boldsymbol{\sigma}^{\pm} := (\sigma_x, \pm\sigma_y, \sigma_z)$

Two QCAs connected by P

$$\boxed{\begin{aligned}A^{\pm}_{\mathbf{k}} = &-i\sigma_x(s_xc_yc_z \pm c_xs_ys_z)\\ &\mp i\sigma_y(c_xs_yc_z \mp s_xc_ys_z)\\ &-i\sigma_z(c_xc_ys_z \pm s_xs_yc_z)\\ &+I(c_xc_yc_z \mp s_xs_ys_z)\end{aligned}}$$

$$s_\alpha = \sin\frac{k_\alpha}{\sqrt{3}}$$

$$c_\alpha = \cos\frac{k_\alpha}{\sqrt{3}}$$

D'Ariano, Perinotti, PRA **90** 062106 (2014)

Dirac QW $\oplus$

Local coupling: $A_{\mathbf{k}}$ coupled with its inverse with off-diagonal identity block matrix

$$\boxed{\begin{aligned} E^{\pm}_{\mathbf{k}} &= \begin{pmatrix} nA^{\pm}_{\mathbf{k}} & imI \\ imI & nA^{\pm\dagger}_{\mathbf{k}} \end{pmatrix} \\ n^2+m^2=1 &\qquad n,m\in\mathbb{R} \end{aligned}}$$

$$\omega^E_{\pm}(\mathbf{k}) = \cos^{-1}[n(c_xc_yc_z \mp s_xs_ys_z)]$$

Dirac in relativistic limit $\quad k \ll 1$

m: mass, $m^2 \leq 1$

n^{-1}: refraction index

± 1 m **!**

Bisio, D'Ariano, Perinotti, Ann. Phys. **368** 177 (2016)

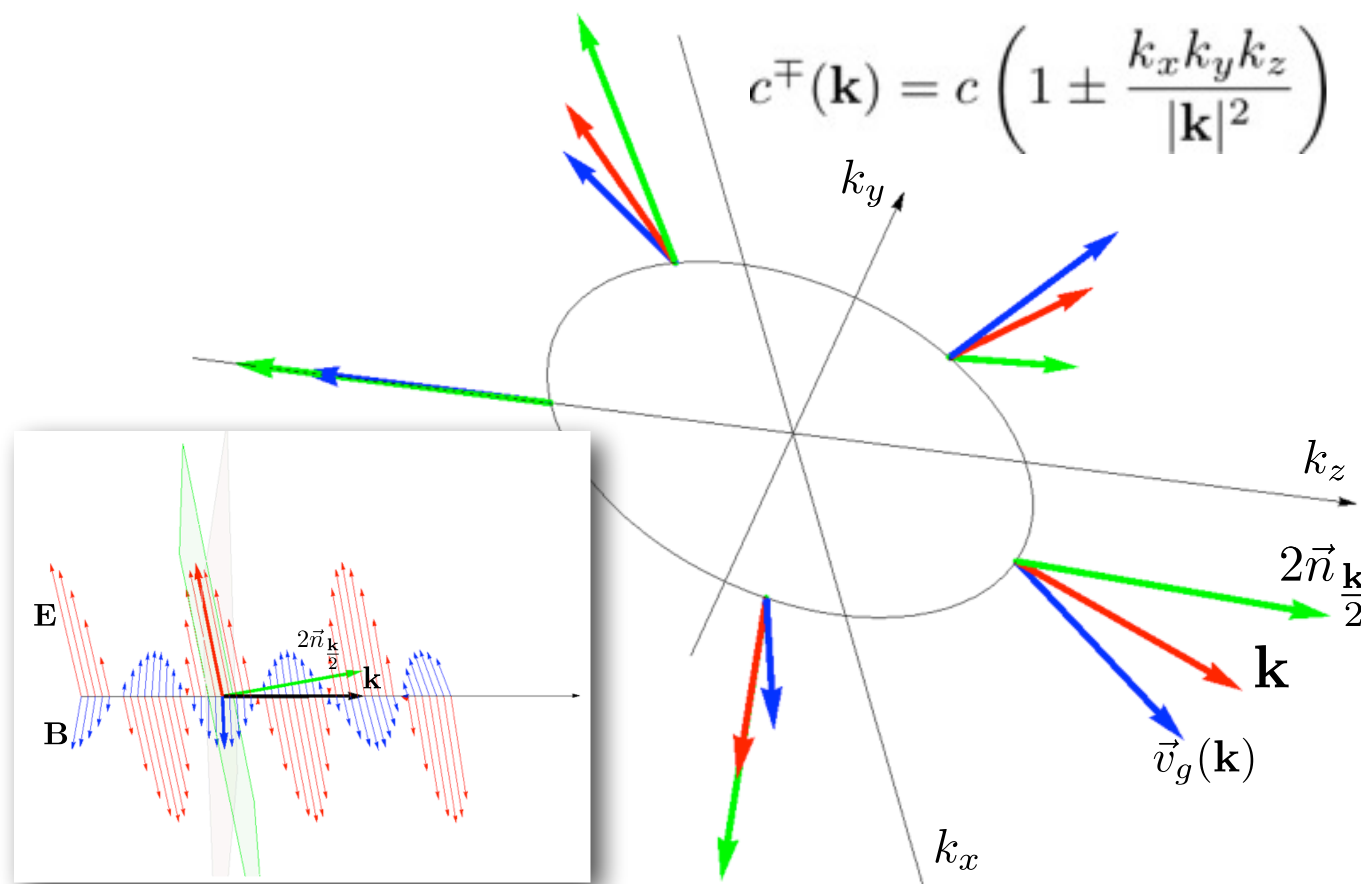
Maxwell QW $\otimes$

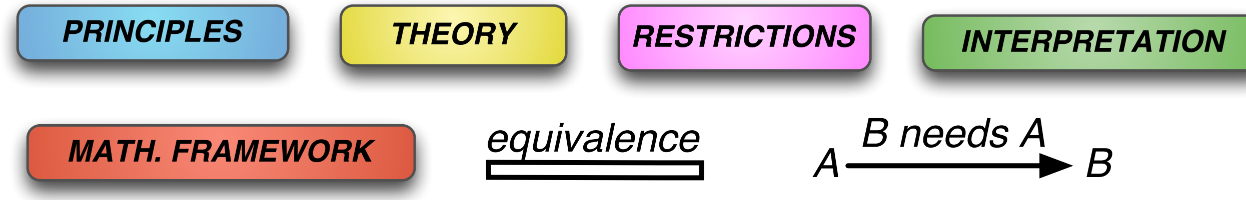
$$\boxed{\begin{aligned} M^{\pm}_{\mathbf{k}} &= A^{\pm}_{\mathbf{k}} \otimes A^{\pm *}_{\mathbf{k}} \\ F^{\mu}(\mathbf{k}) &= \int \frac{\mathrm{d}\,\mathbf{q}}{2\pi} f(\mathbf{q})\tilde{\psi}(\tfrac{\mathbf{k}}{2}-\mathbf{q})\sigma^{\mu}\varphi(\tfrac{\mathbf{k}}{2}+\mathbf{q}) \end{aligned}}$$

Maxwell in relativistic limit $\quad k \ll 1$

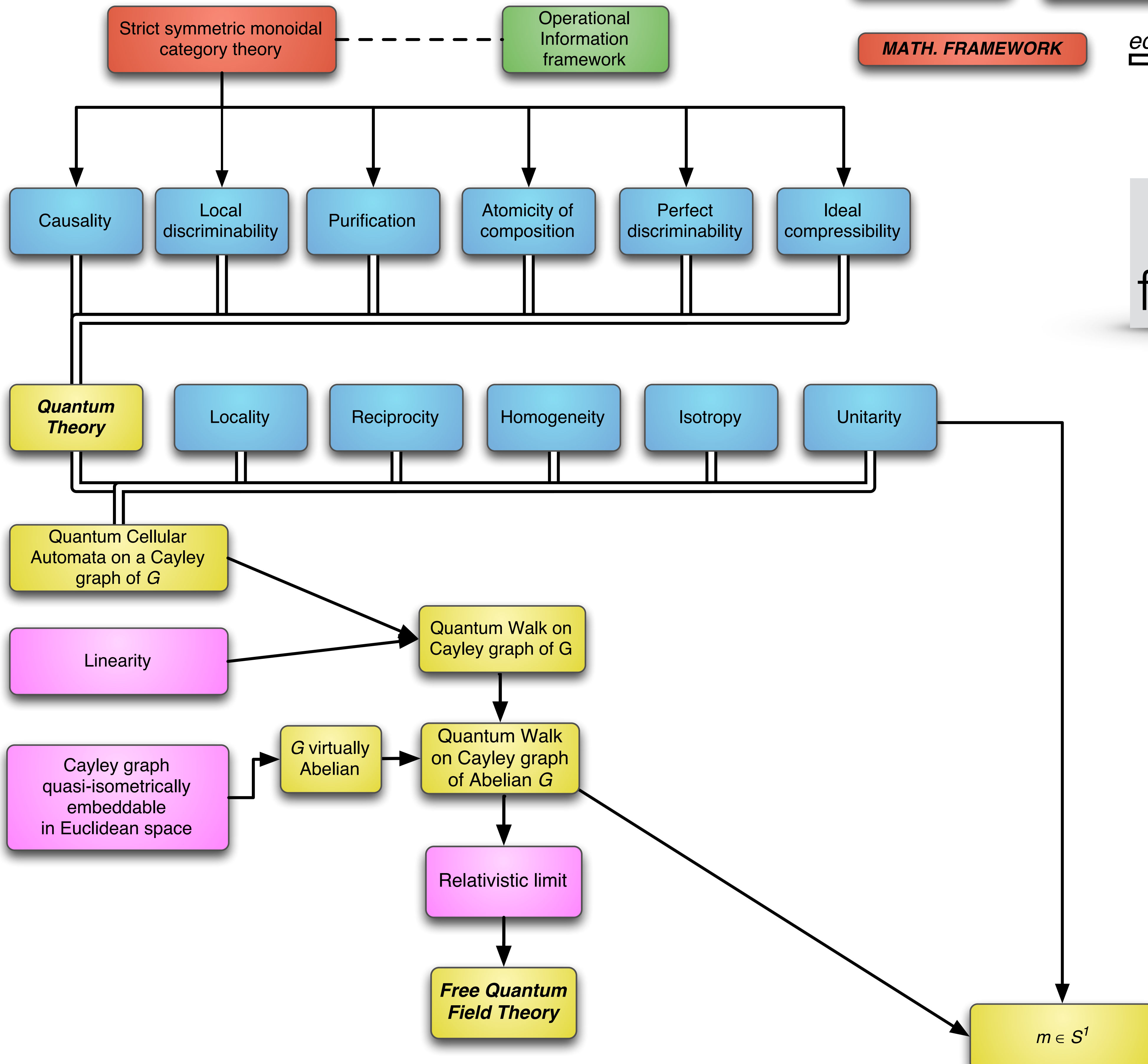
Boson: made with pairs of entangl Fermions

(De Broglie neutrino-theory of photon)





Principles for Mechanics

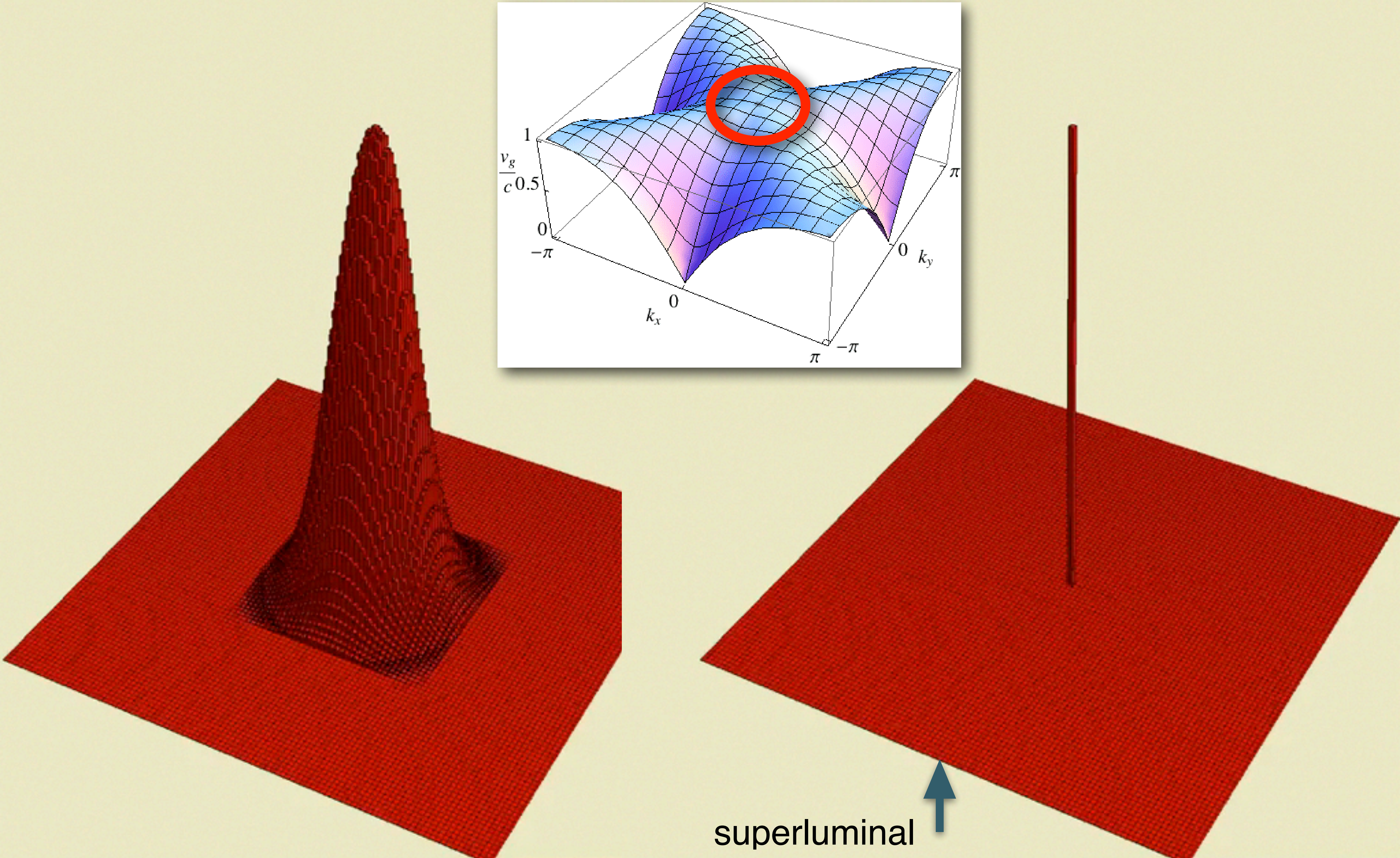


D'Ariano, Perinotti,
PRA **90** 062106 (2014)

Bisio, D'Ariano, Perinotti,
Ann. Phys. **368** 177 (2016)

Free Quantum Field Theory

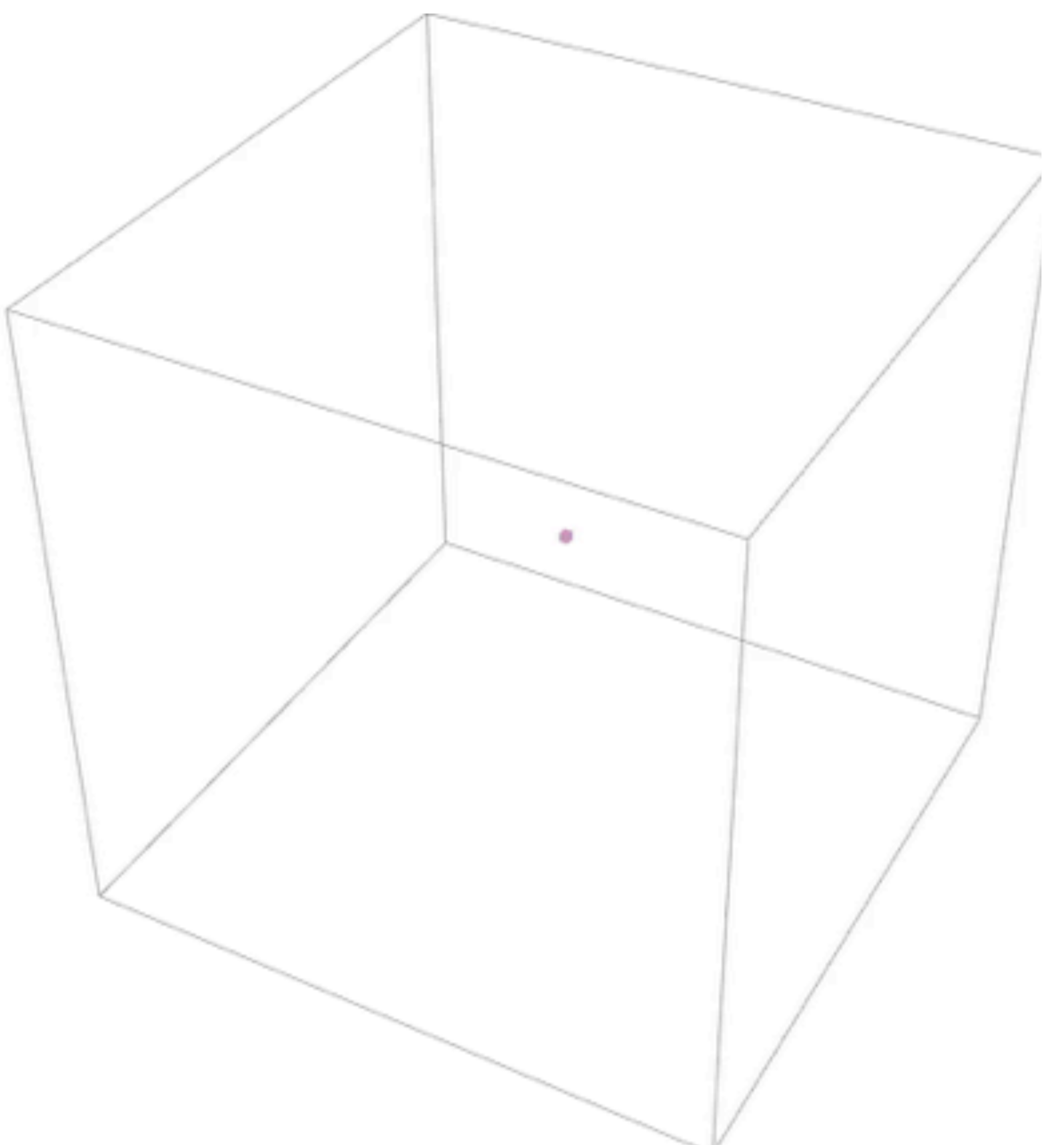
Got it!

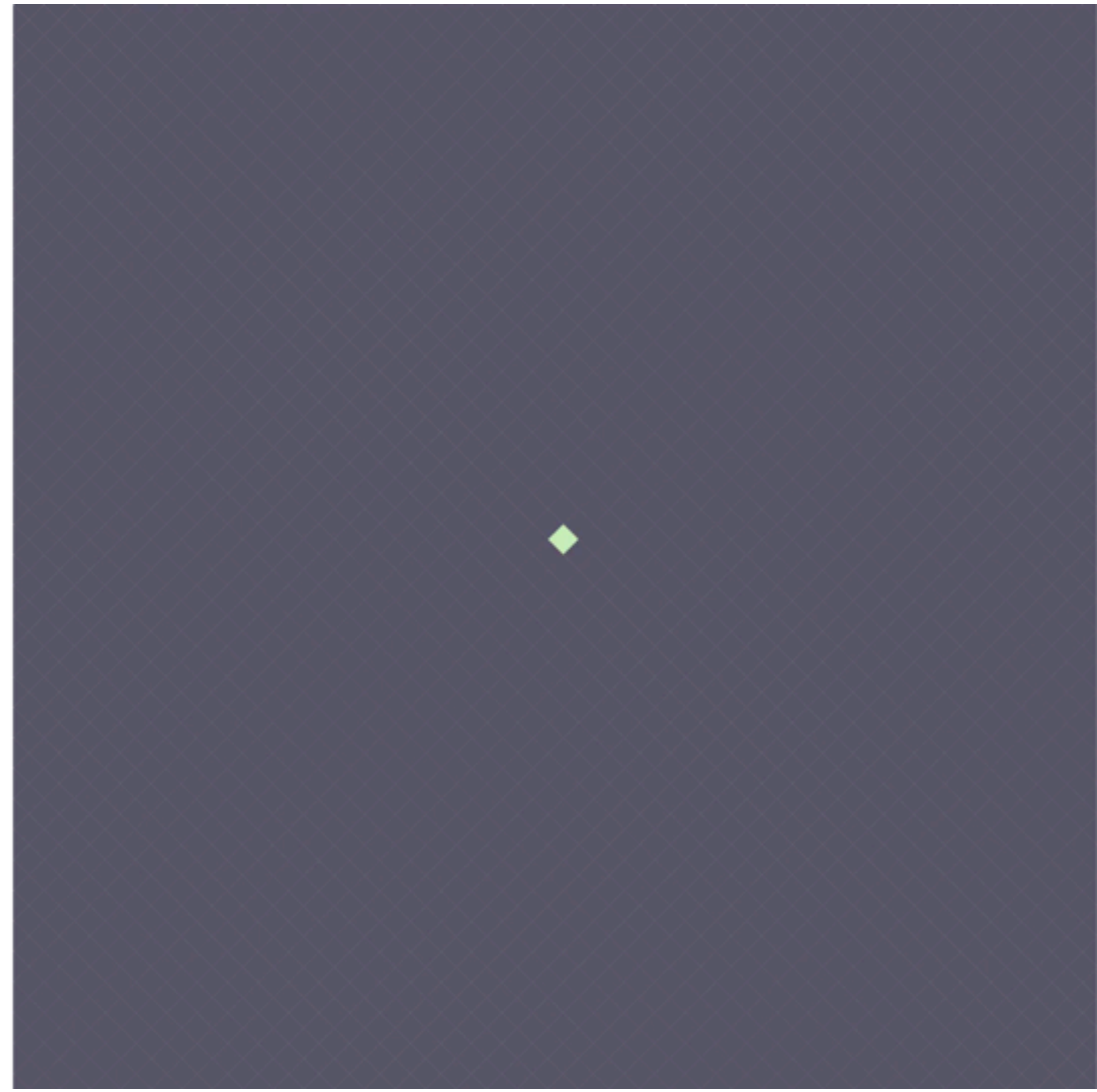


2d Dirac

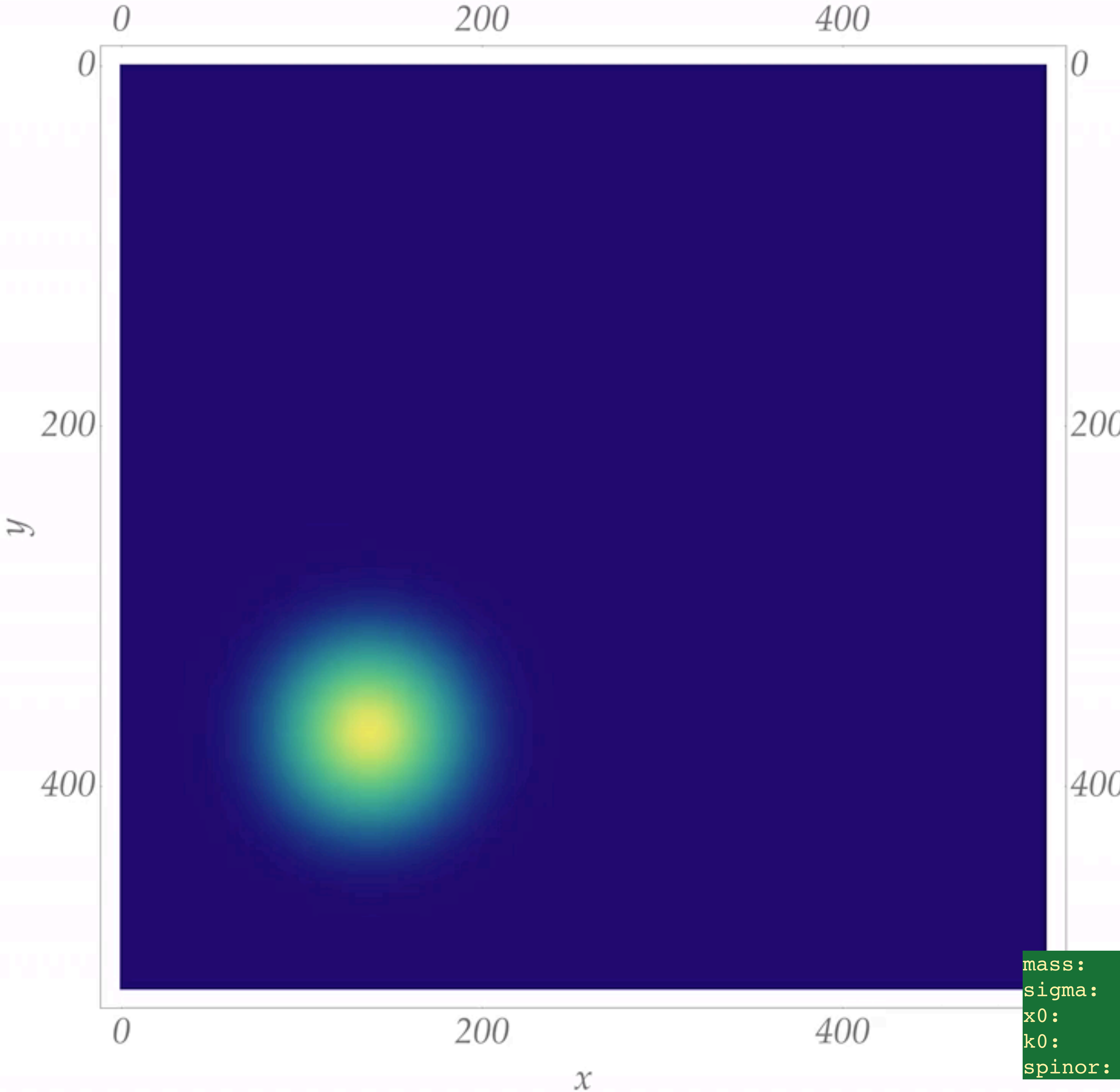
- Evolution of a *narrow-band particle-state*
- Evolution of a *localized state*

D'Ariano, Mosco, Perinotti, Tosini, arXiv:1603.06442





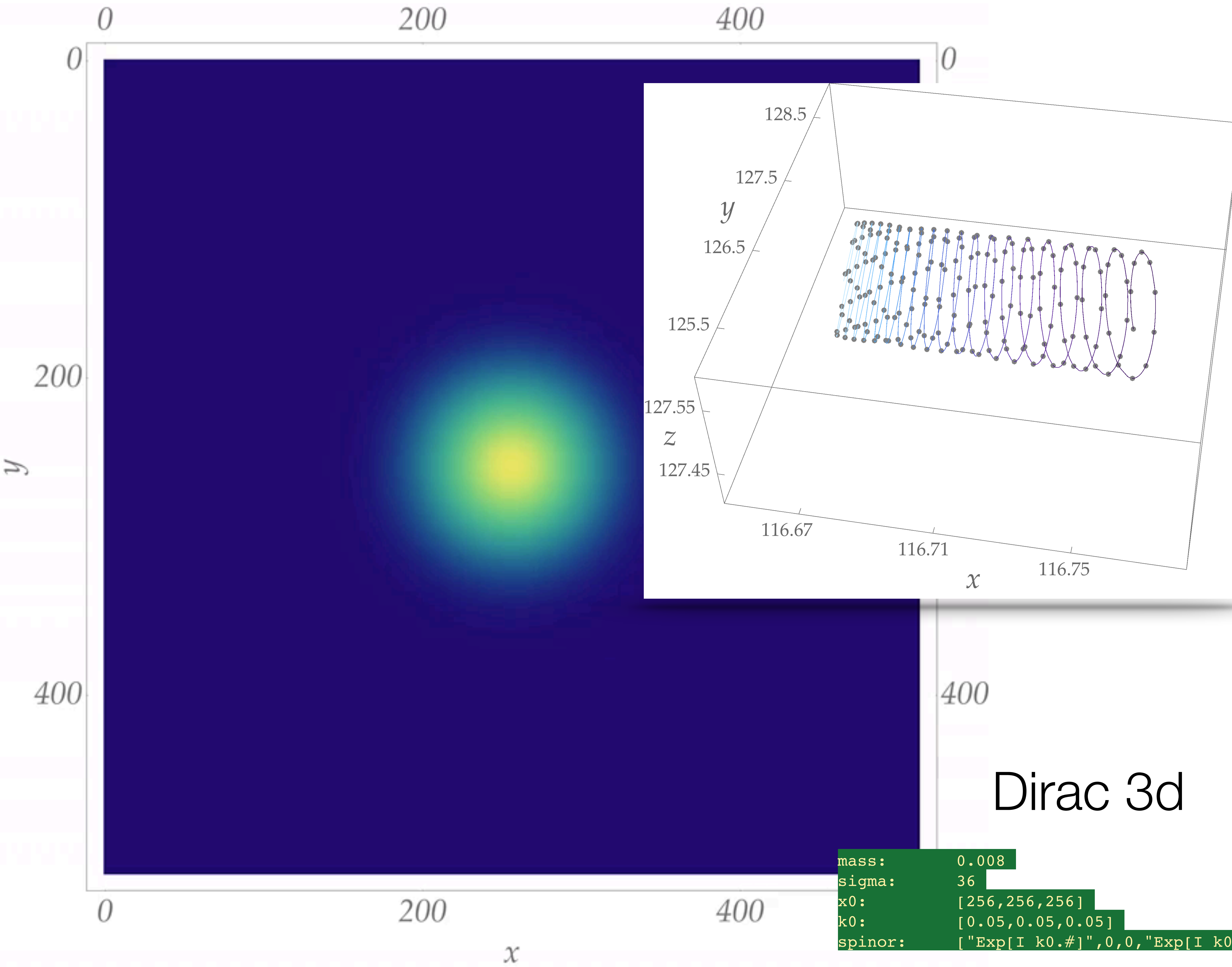
Weyl 3d



D'Ariano, Mosco, Perinotti, Tosini, arXiv:1603.06442

Dirac 3d

```
mass:        0.002
sigma:       32
x0:          [140,140,140]
k0:          [0.05,0.05,0.05]
spinor:      ["Exp[I k0.#]",0,0,0]
```



D'Ariano, Mosco, Perinotti, Tosini, arXiv:1603.06442

Dirac 3d

```
mass:        0.008
sigma:       36
x0:          [256,256,256]
k0:          [0.05,0.05,0.05]
spinor:      ["Exp[I k0.#]",0,0,"Exp[I k0.#]"]
```

Dirac emerging from the QCA

D'Ariano, Perinotti, PRA **90** 062106 (2014)

fidelity with Dirac for a narrowband packets in the relativistic limit $k \simeq m \ll 1$

$$F = |\langle \exp\left[-iN\boldsymbol{\Delta}(\mathbf{k})\right]\rangle|$$

$$\begin{aligned}\boldsymbol{\Delta}(\mathbf{k}) &:= (m^2 + \tfrac{k^2}{3})^{\frac{1}{2}} - \omega^E(\mathbf{k}) \\ &= \frac{\sqrt{3}k_x k_y k_z}{(m^2 + \frac{k^2}{3})^{\frac{1}{2}}} - \frac{3(k_x k_y k_z)^2}{(m^2 + \frac{k^2}{3})^{\frac{3}{2}}} + \tfrac{1}{24}(m^2 + \tfrac{k^2}{3})^{\frac{3}{2}} + \mathcal{O}(k^4 + N^{-1}k^2)\end{aligned}$$

relativistic proton: $N \simeq m^{-3} = 2.2 * 10^{57} \;\Rightarrow\; t = 1.2 * 10^{14}\mathrm{s} = 3.7 * 10^6$ y

UHECRs: $k = 10^{-8} \gg m \;\Rightarrow N \simeq k^{-2} = 10^{16} \;\Rightarrow\; 5 * 10^{-28}$ s

G. M. D'Ariano, N. Mosco, P. Perinotti, A. Tosini,
PLA **378** 3165 (2014); EPL **109** 40012 (2015)

Analytical solution of Dirac (d=1) and Weyl (d=1,2,3)

The analytical solution of the Dirac automaton can also be expressed in terms of Jacobi polynomials $P_k^{(\zeta,\rho)}$ performing the sum over f in Eq. (16) which finally gives

$$\psi(x,t) = \sum_{y} \sum_{a,b\in\{0,1\}} \gamma_{a,b} P_k^{(1,-t)}\left(1 + 2\left(\frac{m}{n}\right)^2\right) A_{ab}\psi(y,0),$$

$$k = \mu_+ - \frac{a \oplus b + 1}{2},$$

$$\gamma_{a,b} = -(\mathrm{i}^{a\oplus b}) n^t \left(\frac{m}{n}\right)^{2+a\oplus b} \frac{k!\left(\mu_{(-)^{ab}} + \frac{\overline{a\oplus b}}{2}\right)}{(2)_k}, \qquad (18)$$

where $\gamma_{00} = \gamma_{11} = 0$ ($\gamma_{10} = \gamma_{01} = 0$) for $t + x - y$ odd (even) and $(x)_k = x(x+1)\cdots(x+k-1)$.

Dispersive Schrödinger equation

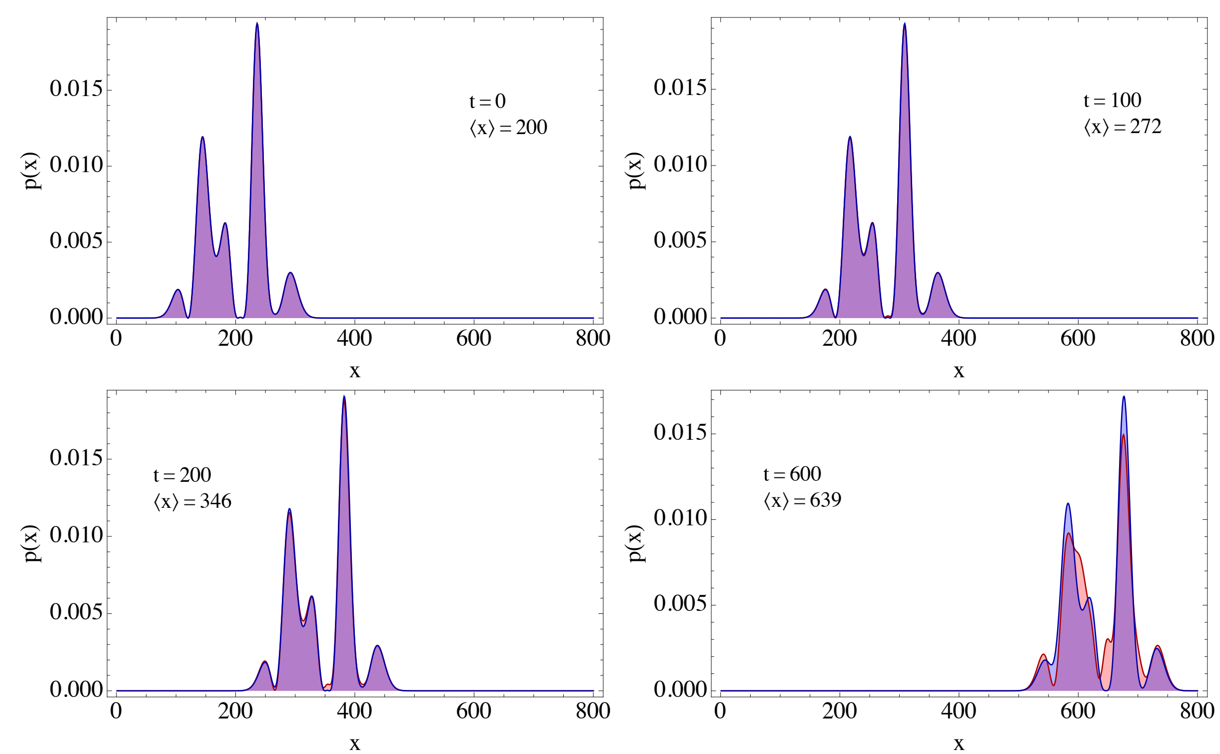
$$i\partial_t e^{-i\mathbf{k}_0\cdot\mathbf{x}+i\omega_0 t}\psi(\mathbf{k},t) = s[\omega(\mathbf{k})-\omega_0]e^{-i\mathbf{k}_0\cdot\mathbf{x}+i\omega_0 t}\psi(\mathbf{k},t)$$

$$i\partial_t\tilde{\psi}(\mathbf{k},t) = s[\omega(\mathbf{k})-\omega_0]\tilde{\psi}(\mathbf{k},t)$$

$$i\partial_t\tilde{\psi}(\mathbf{x},t) = s[\mathbf{v}\cdot\boldsymbol{\nabla}+\frac{1}{2}\mathbf{D}\cdot\boldsymbol{\nabla}\boldsymbol{\nabla}]\tilde{\psi}(\mathbf{x},t)$$

$$\mathbf{v} = (\boldsymbol{\nabla}_{\mathbf{k}}\omega)(\mathbf{k}_0)$$

$$\mathbf{D} = (\boldsymbol{\nabla}_{\mathbf{k}}\boldsymbol{\nabla}_{\mathbf{k}}\omega)(\mathbf{k}_0)$$



D'Ariano, Perinotti, PRA **90** 062106 (2014)

The New Axiomatization Program

Cases of study:

- physical standards
- special relativity from quantum theory without kinematics
- particle notion without mechanics
- proper time

Case of study 1: LTM standards in adimensional theory

$$x = \frac{x_{[m]}}{a_*} \in \mathbb{Z}, \quad t = \frac{t_{[sec]}}{t_*} \in \mathbb{N}, \quad m = \frac{m_{[kg]}}{m_*} \in [0,1]$$

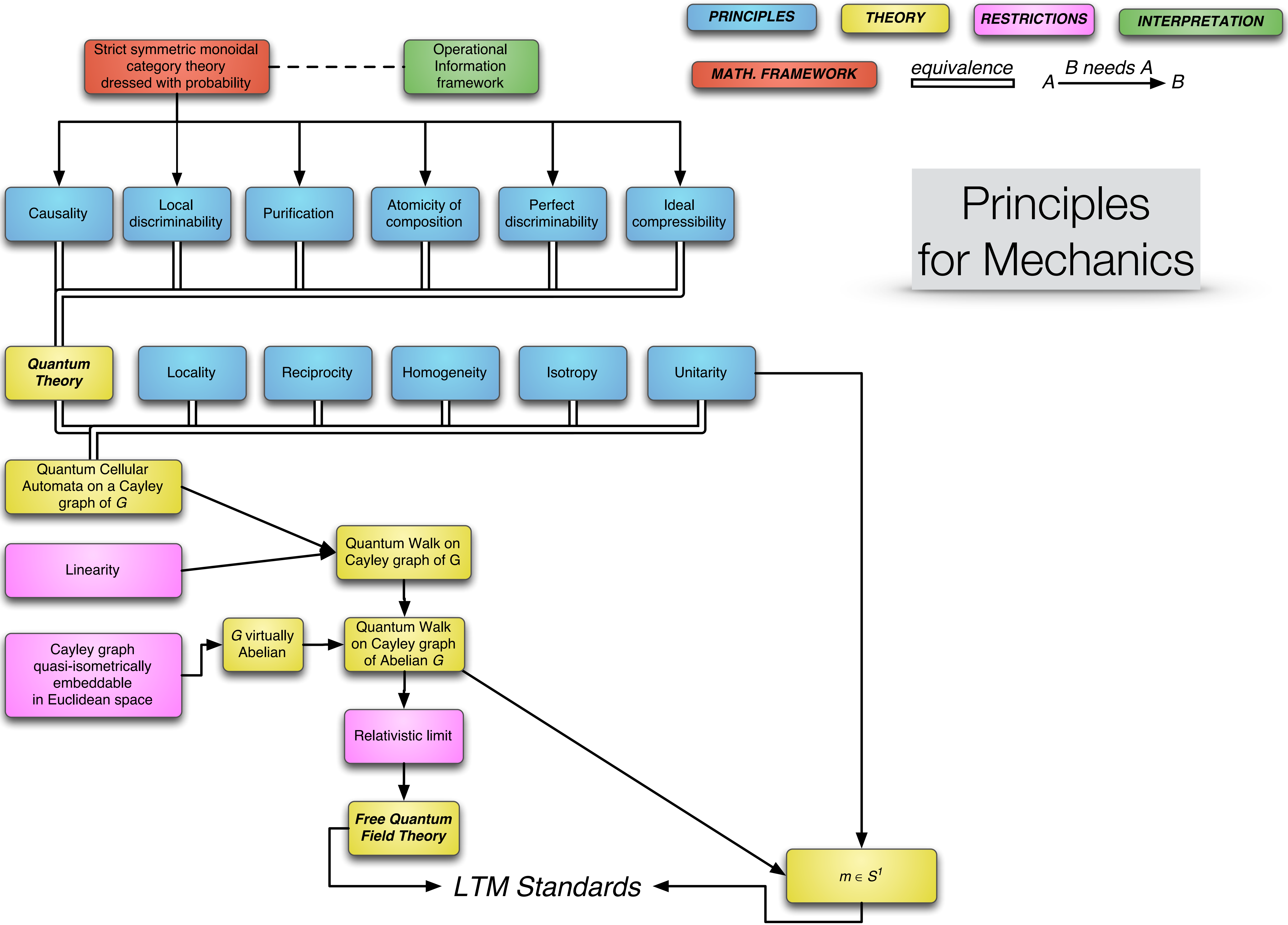
$$m_* \simeq \frac{1}{\sqrt{3}\pi} \frac{\hbar k}{c(k) - c(0)}$$

$$\begin{cases} c \equiv c(0) = \dfrac{a_*}{t_*} \\ \hbar = m_* a_* c \end{cases}$$

Heuristic argument of the mini-black-hole:

m_* Planck mass

from the relativistic limit



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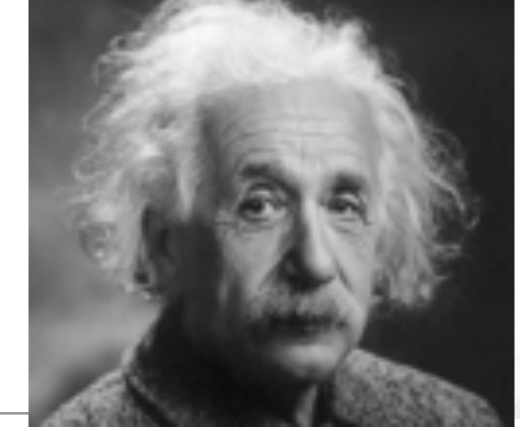
Free Quantum Field Theory
LTM Standards
m ∈ S1

Case of study 2: Special Relativity from quantum theory without kinematics



Relativity Principle: Invariance of the dynamical law with the inertial frame

Inertial frame: a reference frame where the Newton inertia law holds for a mechanically isolated system

Maxwell equations

Einstein Special Relativity

Poincaré group: group of changes of inertial frame that leave the dynamical law invariant.

Case of study 2: Special Relativity from quantum theory without kinematics

Relativity Principle: Invariance of the dynamical law with the inertial frame

Inertial frame: a reference frame where energy and momentum are conserved for a mechanically *isolated* system.

Poincaré group: group of changes of inertial frame that leave the dynamical law invariant.

Case of study 2: Special Relativity from quantum theory without kinematics

Relativity Principle: Invariance of the dynamical law with the inertial frame

Inertial frame: Representation of the dynamical law for given *values* of the constants of motion for an *isolated* system.

Dynamical law: expressed in terms of the values of the constants of motion.

Poincaré group: group of changes of inertial frame that leave the dynamical law invariant.

good for any dynamical system!

Case of study 2: Special Relativity from quantum theory without kinematics

Relativity Principle: Invariance of the dynamical law with the inertial frame

Inertial frame: Representation of the physical law in terms of eigenspaces of the constants of the dynamics $k := (\omega, \mathbf{k})$

Dynamical law: eigenvalue equation

$$A_{\mathbf{k}}\psi(\mathbf{k},\omega) = e^{i\omega}\psi(\mathbf{k},\omega)$$

Poincaré group: group of changes of representations in terms of eigenspaces of the constants of dynamics that leave the eigenvalue equation invariant.

Quantum Walks

Case of study 2: Special Relativity from quantum theory without kinematics

- Mathematical statement:
  invariance of eigenvalue equation under change of representation.

- Physical interpretation:
  invariance of the physical law under change of inertial reference frame.

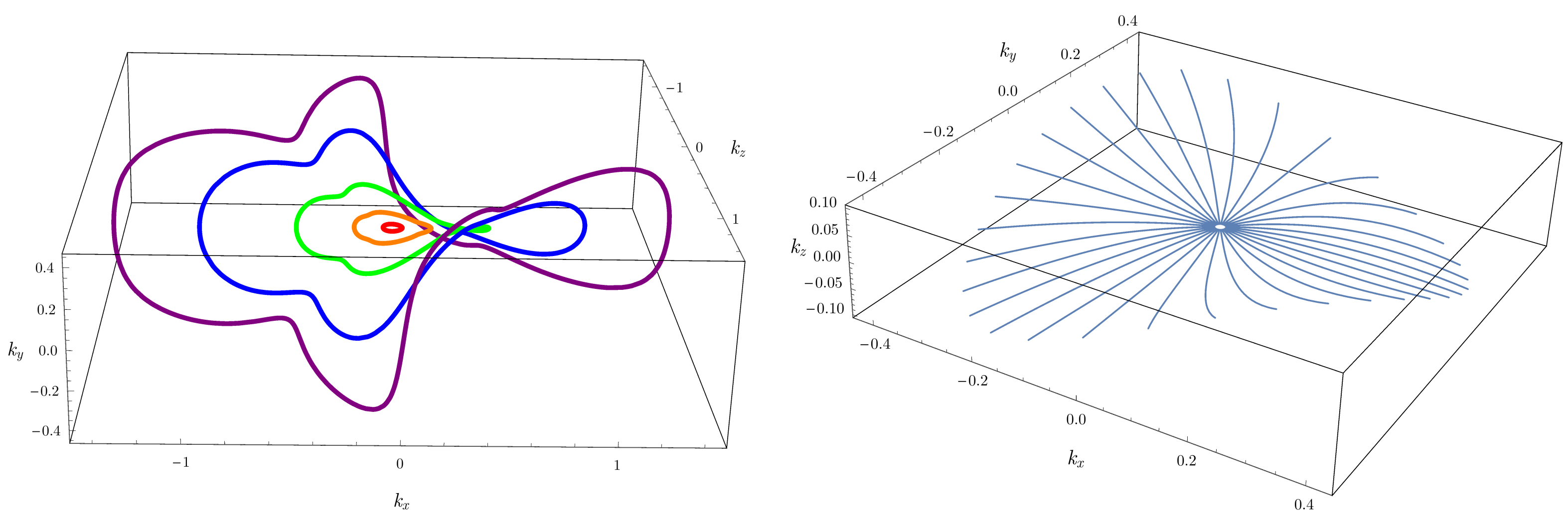


FIG. 2: The distortion effects of the Lorentz group for the discrete Planck-scale theory represented by the quantum walk in Eq. (6). Left figure: the orbit of the wavevectors $\mathbf{k} = (k_x, 0, 0)$, with $k_x \in \{.05, .2, .5, 1, 1.7\}$ under the rotation around the z axis. Right figure: the orbit of wavevectors with $|\mathbf{k}| = 0.01$ for various directions in the (k_x, k_y) plane under the boosts with $\boldsymbol{\beta}$ parallel to $\mathbf{k}$ and $|\boldsymbol{\beta}| \in [0, \tanh 4]$.

m=0 Deformed Poincaré group *SO(1,3)*
m>0 Deformed De Sitter group *SO(1,4)*
Lorentz transformations are perfectly recovered for *k,m≪1*

- For *k~1*:
  - *Double Special Relativity* (Camelia-Smolin).
  - *Relative locality* (in addition to relativity of simultaneity)

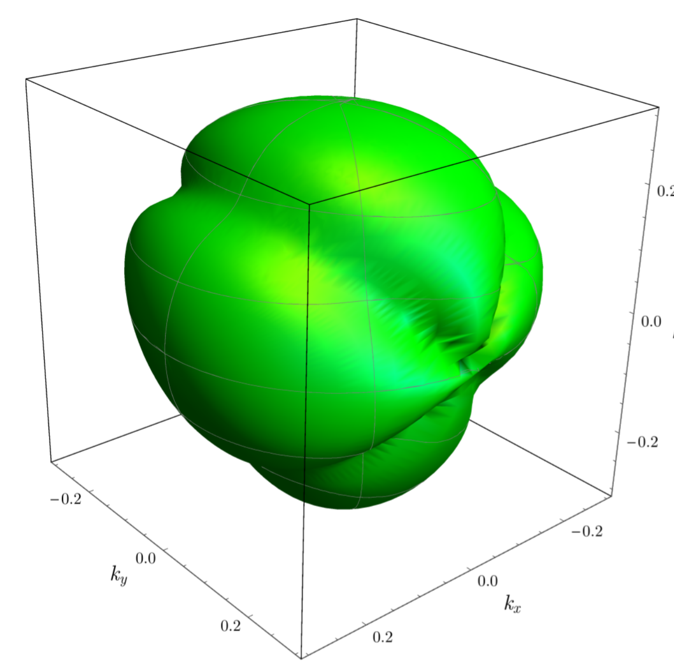
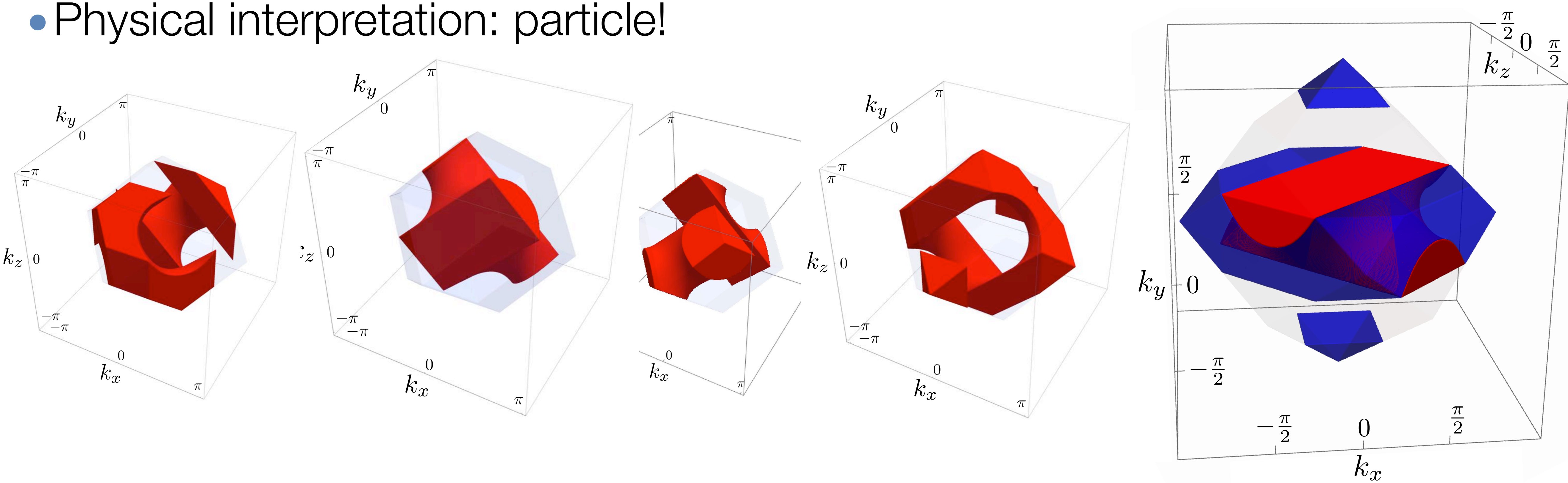


FIG. 3: The green surface represents the orbit of the wavevector $\mathbf{k} = (0.3, 0, 0)$ under the full rotation group $SO(3)$.

Case of study 3: particle notion without mechanics

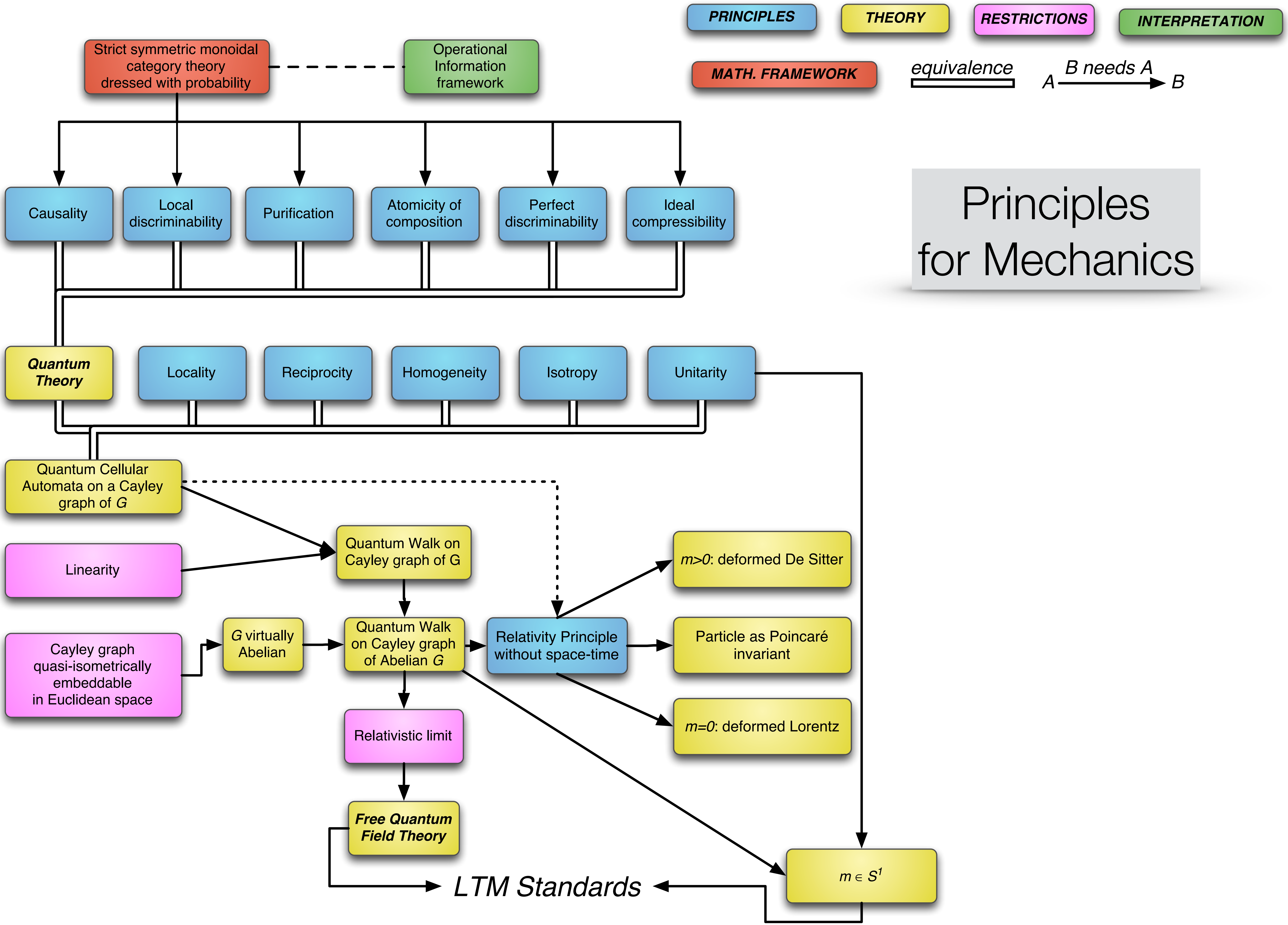
- Mathematical statement:
  irreducible representation of the group of invariance of dynamics (deformed Poincaré group).

- Physical interpretation: particle!



- The Brillouin zone separates into four Poincaré-invariant regions diffeomorphic to balls, corresponding to four different particles.

- *m≠0* De Sitter *SO(1,4)*



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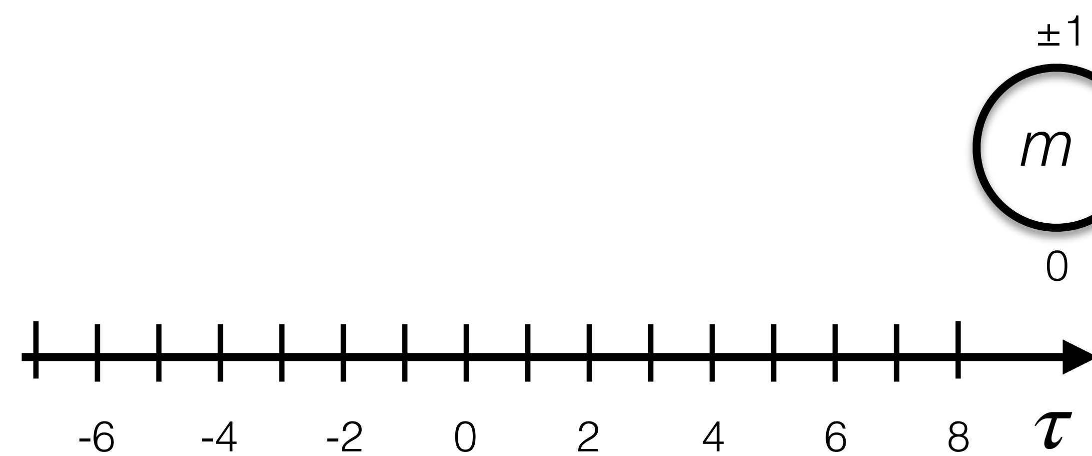
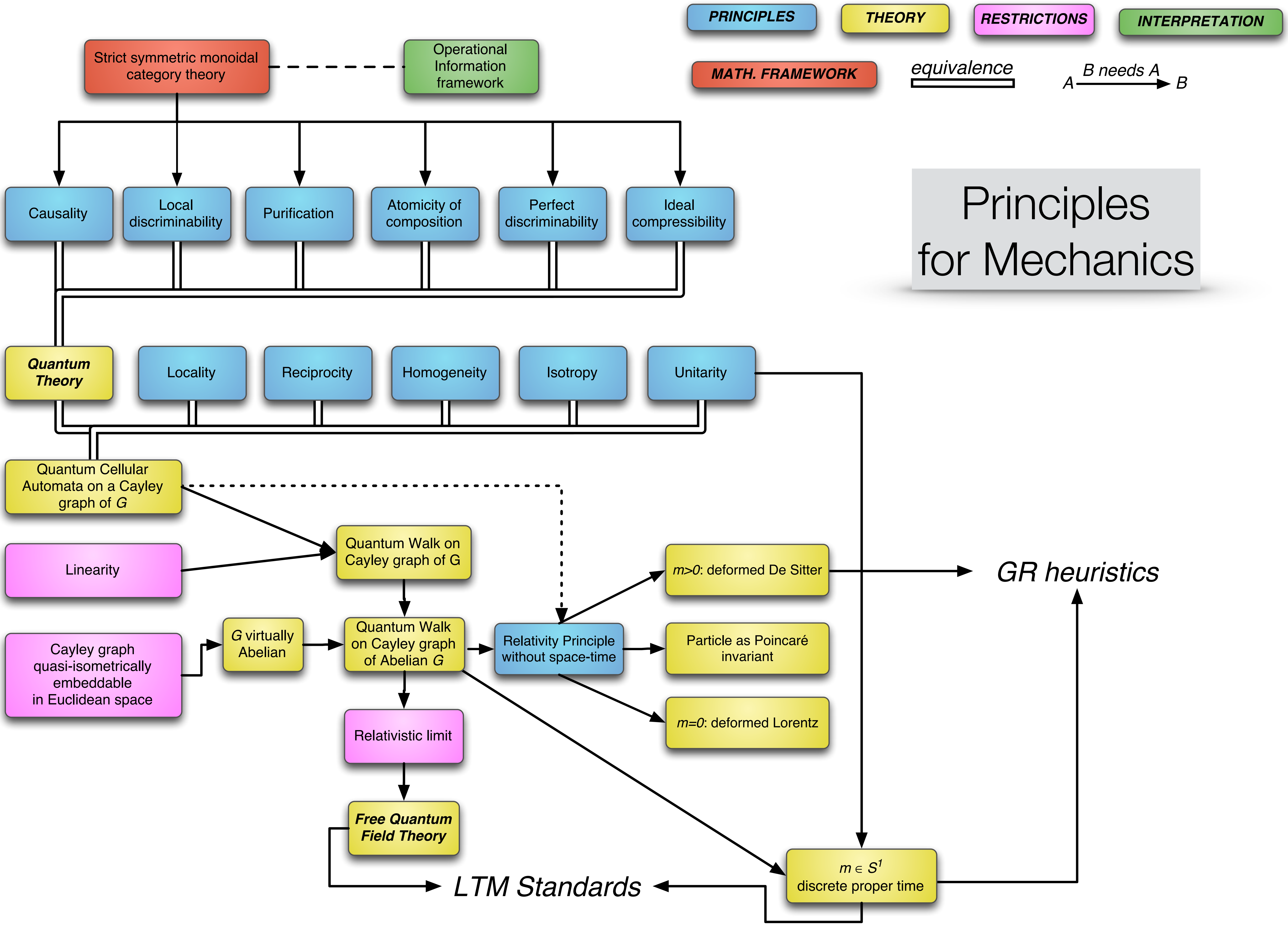
Quantum Walk on Cayley graph of Abelian G
Relativity Principle without space-time
m>0: deformed De Sitter
Particle as Poincaré invariant
m=0: deformed Lorentz
Relativistic limit
Free Quantum Field Theory
m ∈ S¹
LTM Standards

Case of study 4: proper time

- Mathematical statement: topology of domain of the particle mass is a circle
- Physical interpretation: proper time is discrete!

$$H(q_\alpha, p_\alpha, \tau, m) = \sum_\alpha p_\alpha \dot{q}_\alpha + c^2 m \dot{\tau} - L$$





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Particle as Poincaré invariant
$m=0$: deformed Lorentz
GR heuristics
Relativistic limit
Free Quantum Field Theory
LTM Standards
$m \in S^1$
discrete proper time

Interacting theory for d=1: exact solution

First quantum cellular automata *interacting* theory satisfying all principles: massive Hubbard model. Solved analytically by Bethe ansatz. Bounded states established.

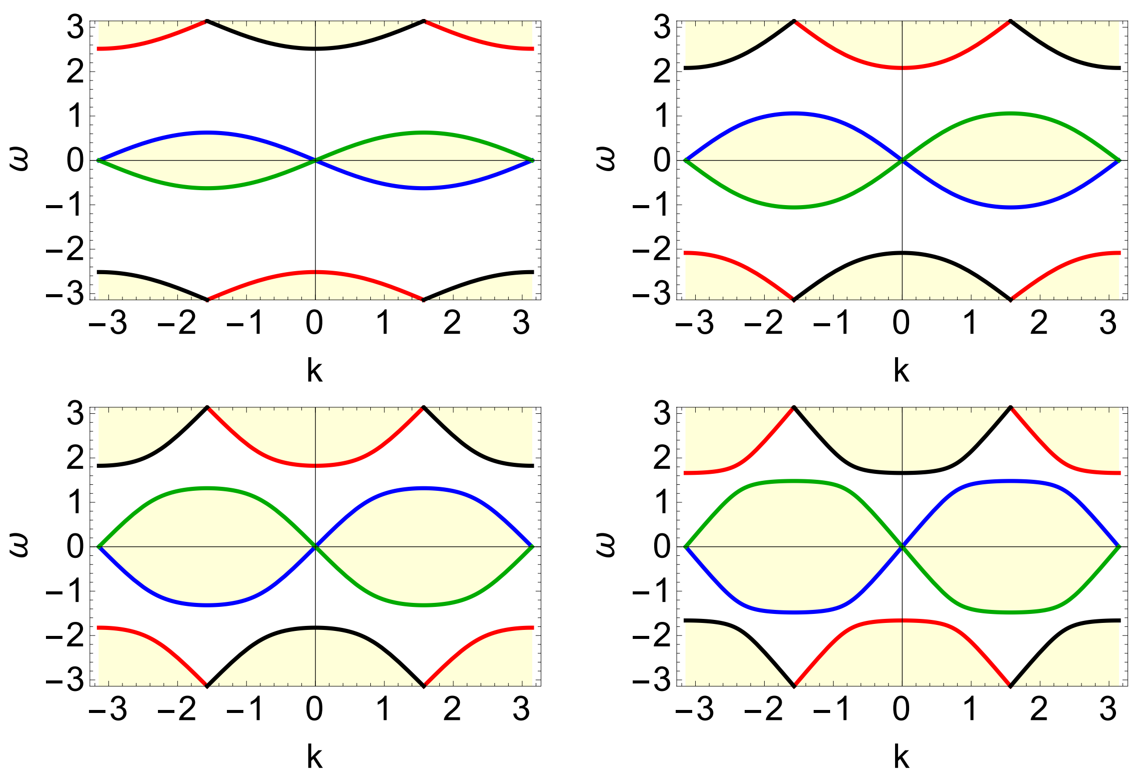

FIG. 1: Dispersion relation of the two particle Dirac Quantum Walk. The eigenvalue of the eigenstates $|++\rangle, |--\rangle, |+-\rangle$ and $|-+\rangle$ are respectively depicted in black, red, blue and green. The eigenvalues are plotted in terms of the relative momentum k, while the mass m and the total momentum p are fixes. The mass and total momentum parameters are $m = 0.9, 0.7, 0.5, 0.3$ and $p = -3\pi/4, -\pi/4, \pi/4, 3\pi/4$ from the top left to the bottom right.

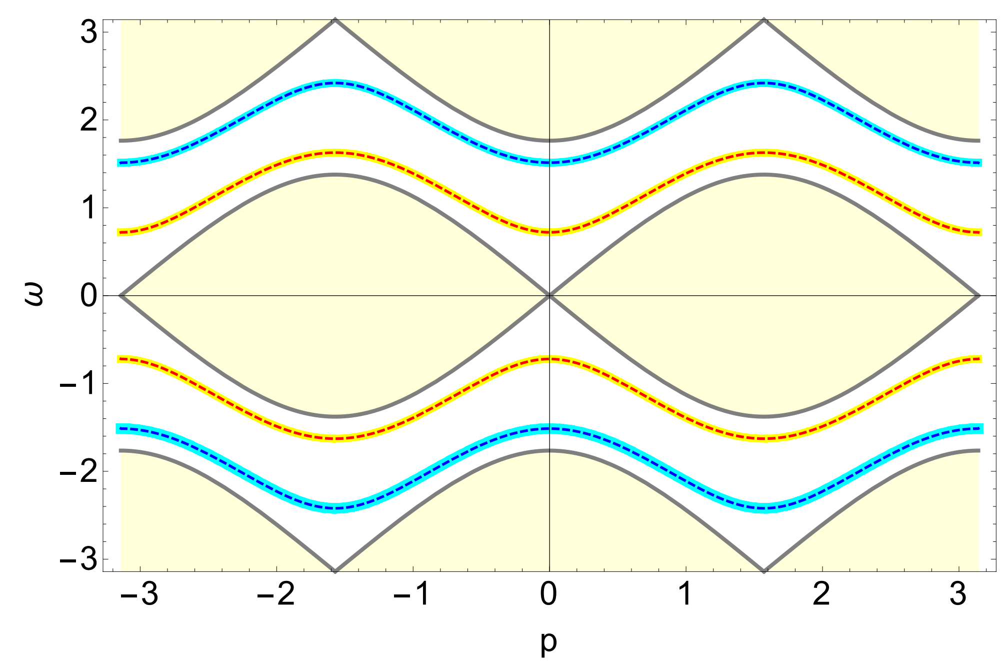

FIG. 2: Discrete spectrum of the Hubbard QCA for $\chi = 1.14$. ω_+ in red and ω_- in blue.

This is more or less what I wanted to say

Thank you for your attention