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QUit quantum information theory group

Toward an operational derivation of Quantum Mechanics

Giacomo Mauro D’Ariano

Pavia University

Quantum Physics and Logic, April 9 2009, Oxford Univ. UK

arXiv:0807.4383: in *Philosophy of Quantum Information and Entanglement*, Eds A. Bokulich and G. Jaeger (Cambridge University Press, Cambridge UK, in press)

QM is a probabilistic theory + *some "principle"*

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A *principle of the Quantumness*, as the *principle of Relativity*

Quantum Mechanics

Lorentz transformations

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Principle of relativity

Existence of a limiting velocity

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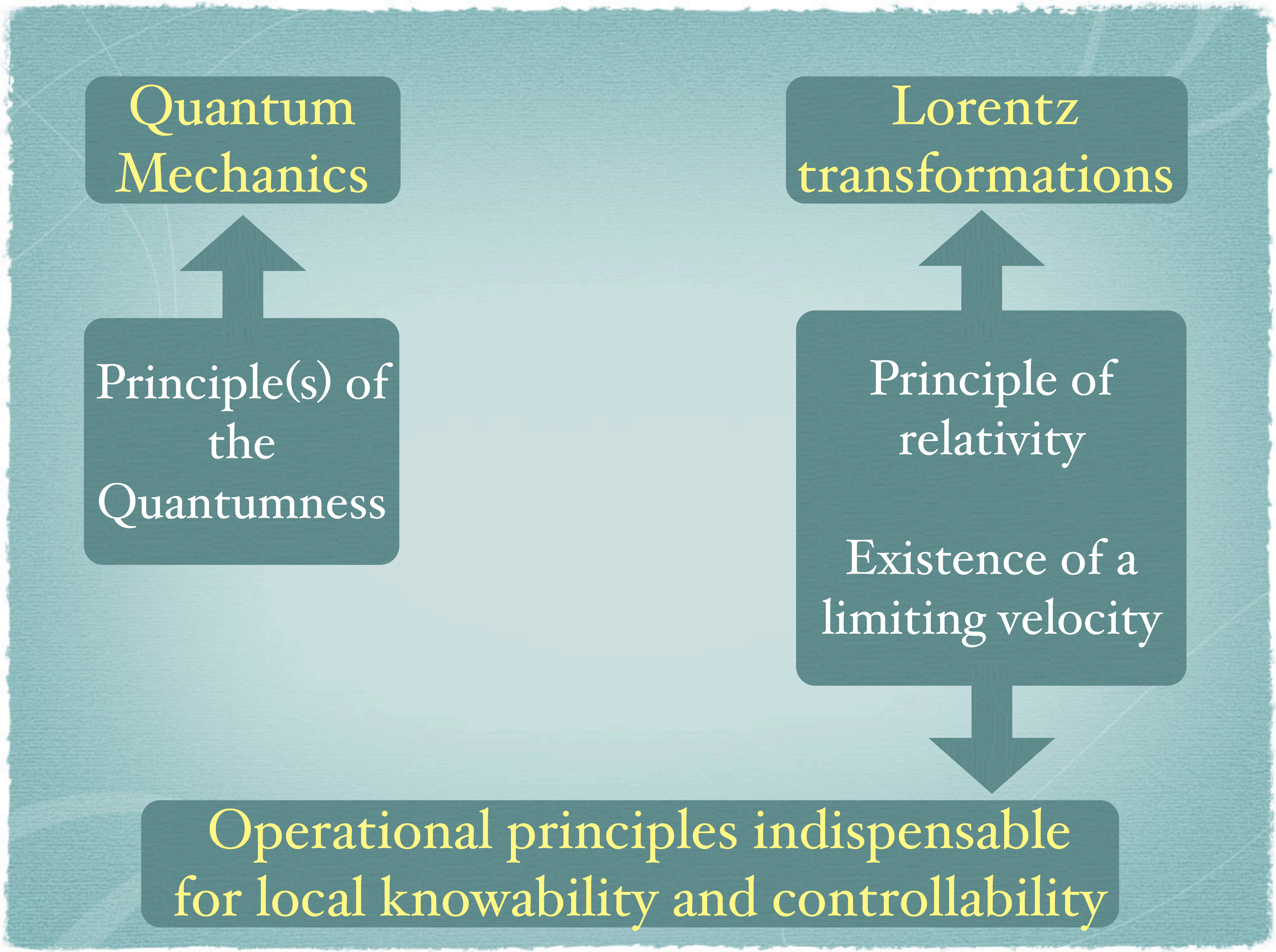
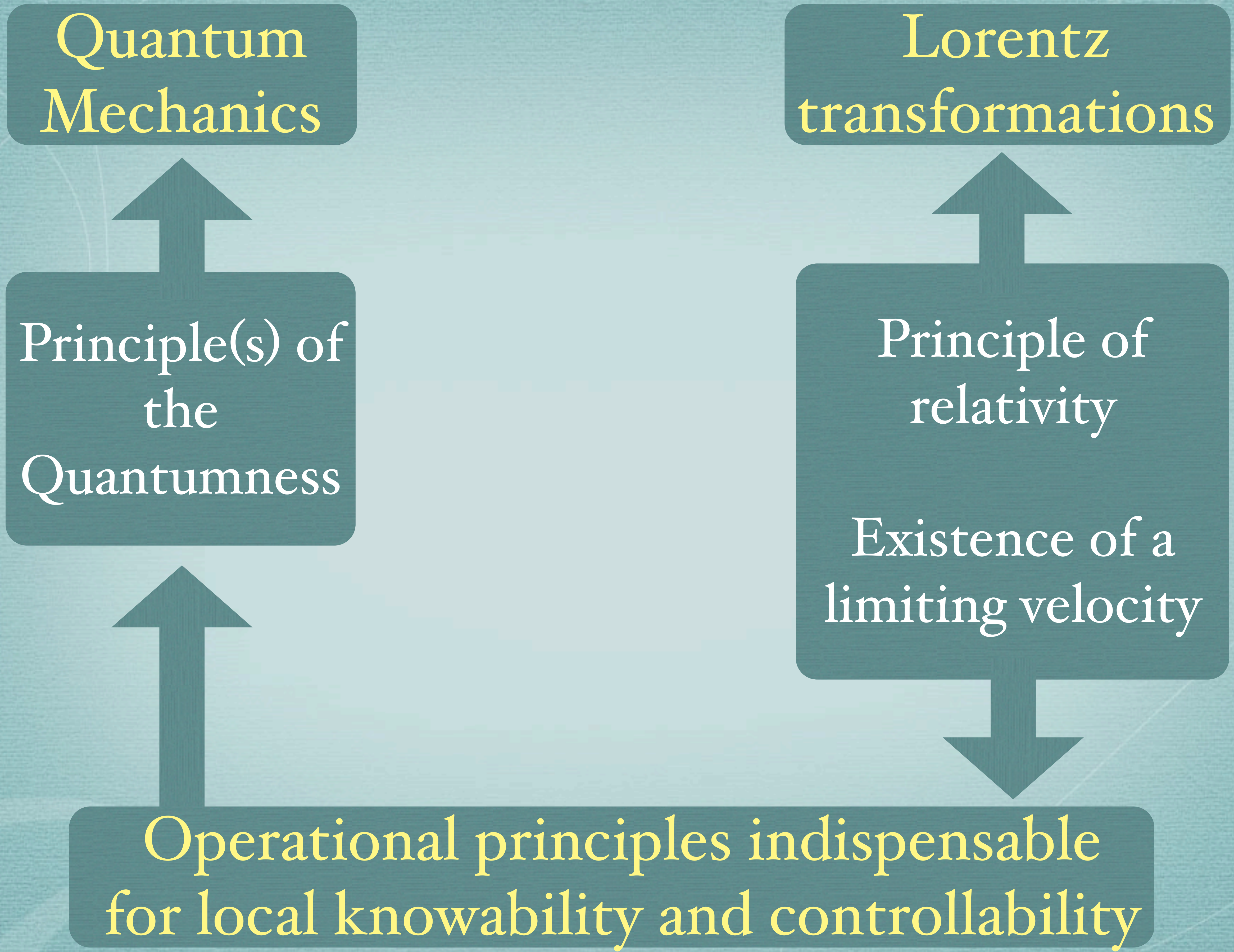


Quantum Mechanics
Principle(s) of the Quantumness
Lorentz transformations
Principle of relativity
Existence of a limiting velocity
Operational principles indispensable for local knowability and controllability


Quantum Mechanics
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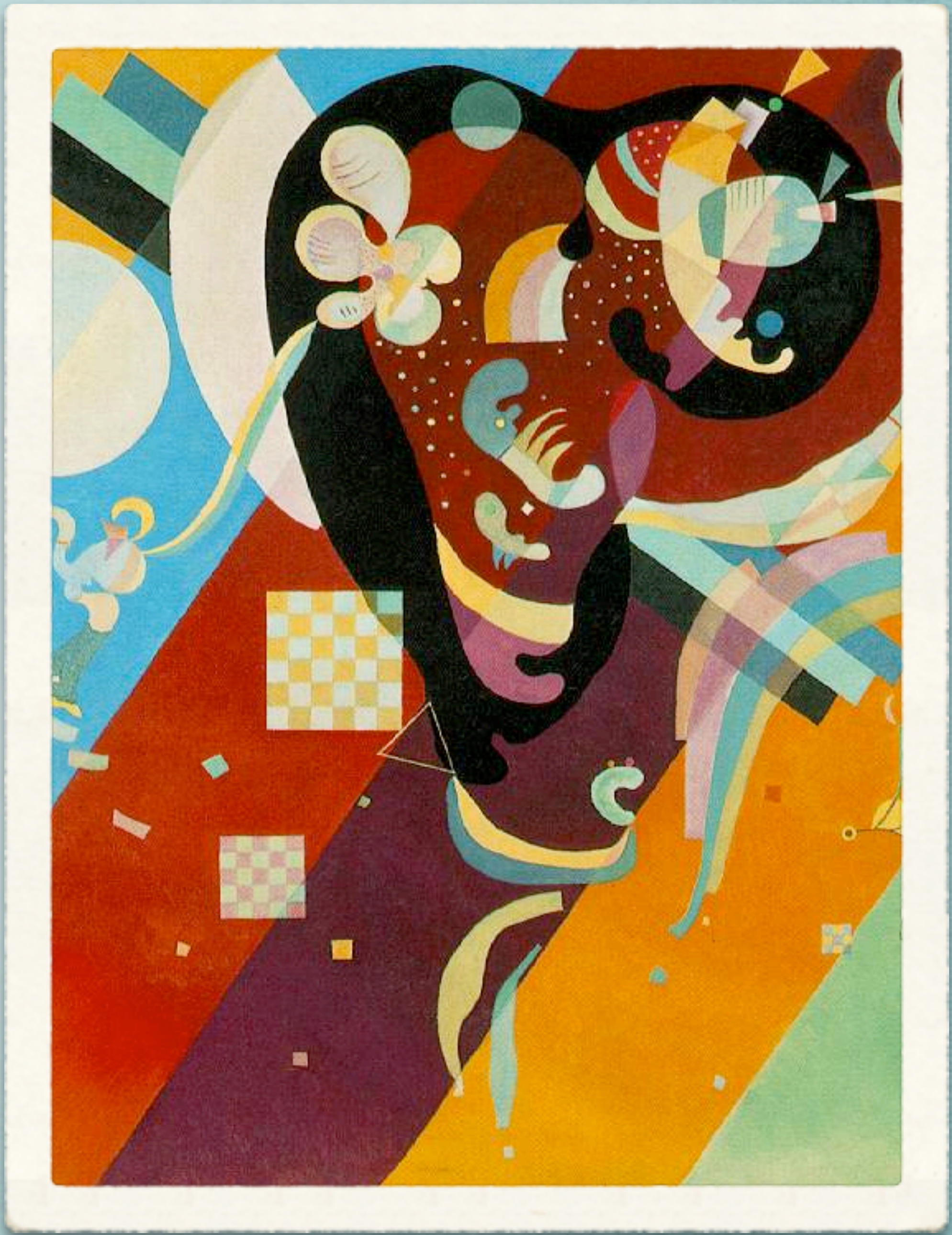
In this talk w.l.g.

* finite dimensions
* only one kind of system



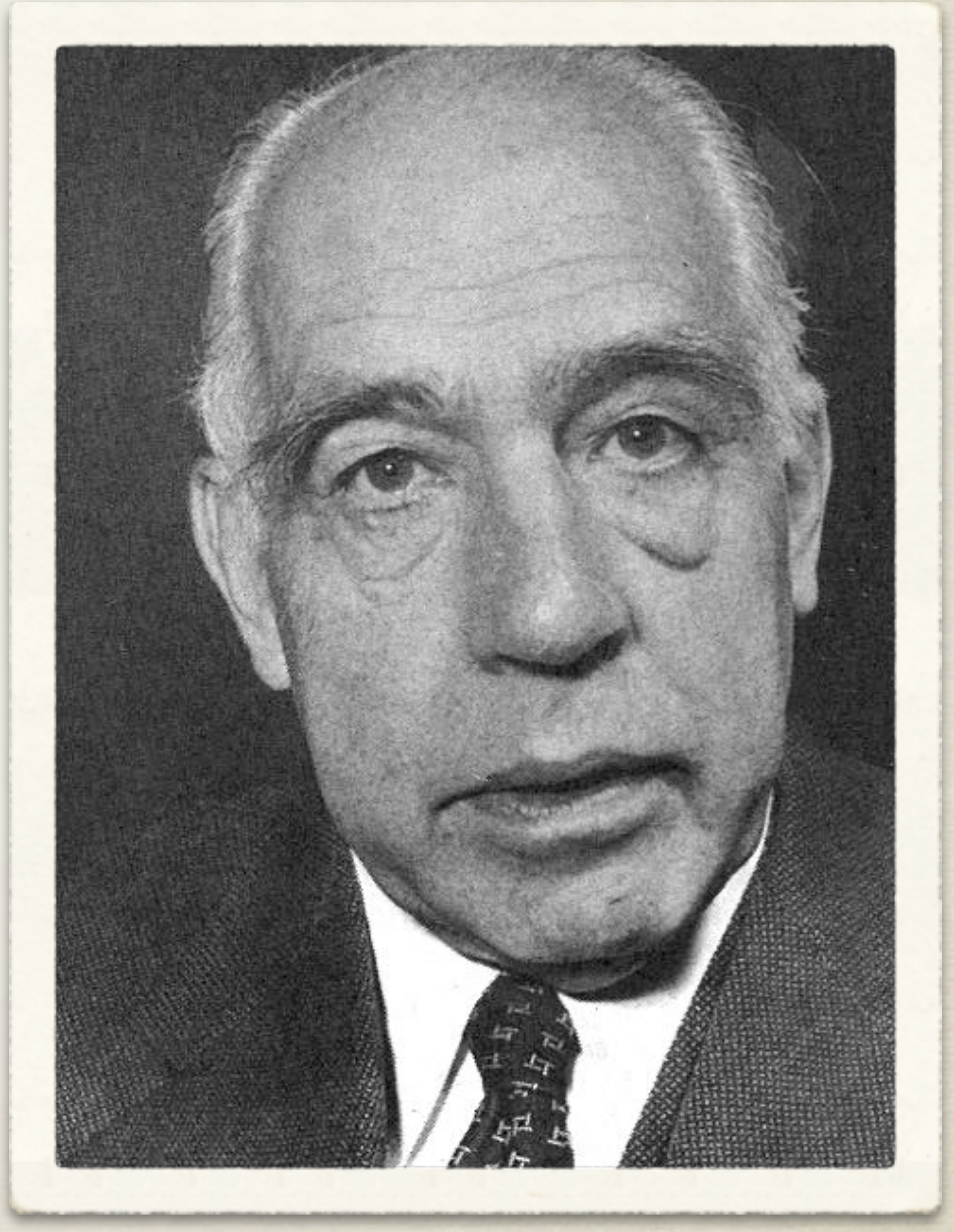
Operational framework

PRIMITIVE NOTIONS

* probability
* events
* ...

General principles

STRONGLY COPENHAGEN



General principles

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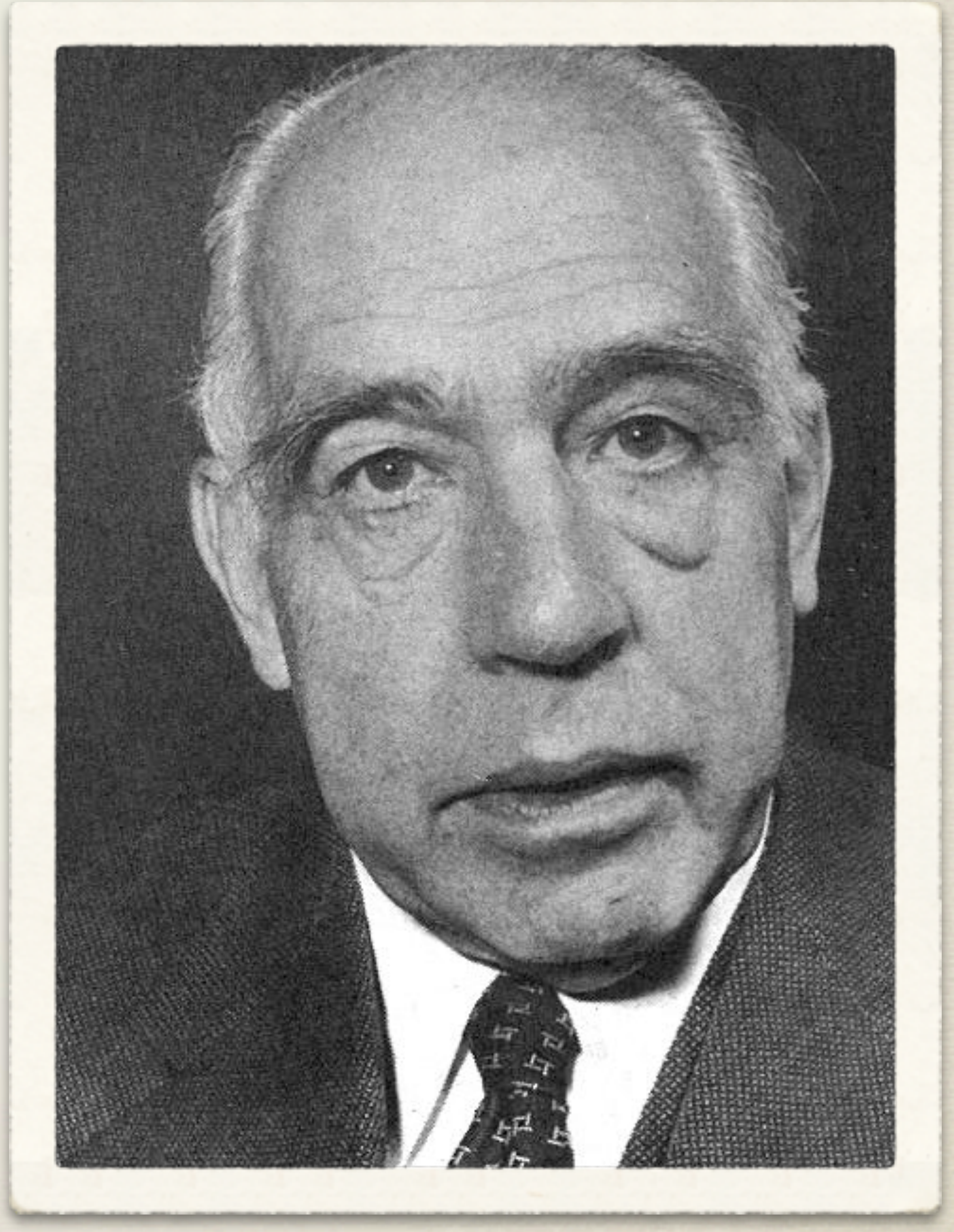
* everything is defined operationally, including all mathematical objects



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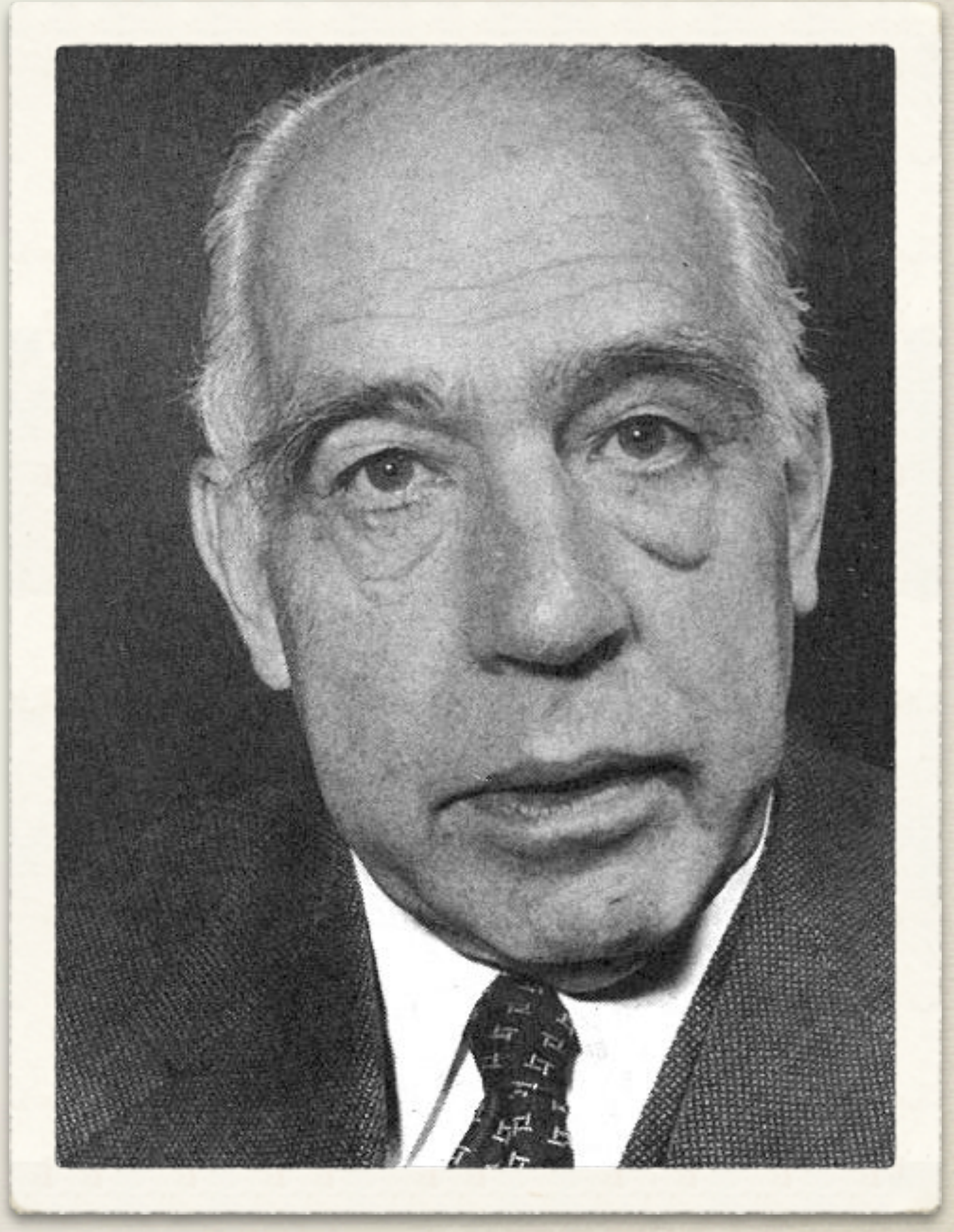

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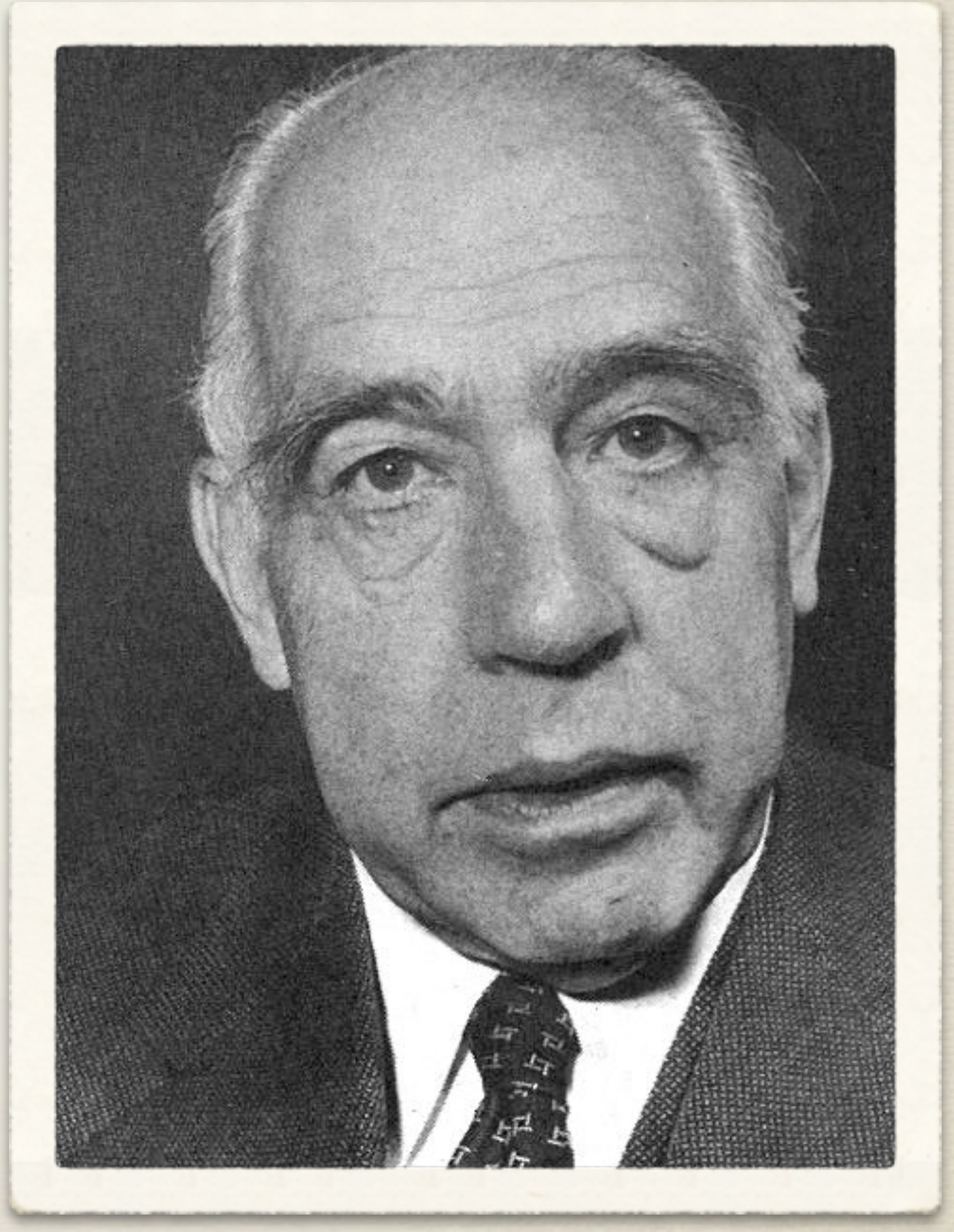
Examples:

* notion of system
* identification of events
* sets of states separating effects



General principles

MATHEMATICAL CLOSURE

* mathematical completion is taken for convenience

General principles

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Examples:

* norm closure
* algebraic closure
* linear span

General principles

OPERATIONAL CLOSURE

* every operational option implicit in the formulation is incorporated in the framework

General principles

OPERATIONAL CLOSURE

* every operational option implicit in the formulation is incorporated in the framework

Examples:

* convex closure
* closure under coarse-graining

Postulates

* NSF: No signaling from the future (=definition of *cascade*)
* NS: No signaling (=definition of *independent systems*)
* PFAITH: There exists preparationally faithful states

Postulates under exploration

* FAITHE: There exists a faithful effect
* PURIFY: There exists a purification for each state
* SUPER-PFAITH

Postulates under exploration

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→ Stinespring dilation

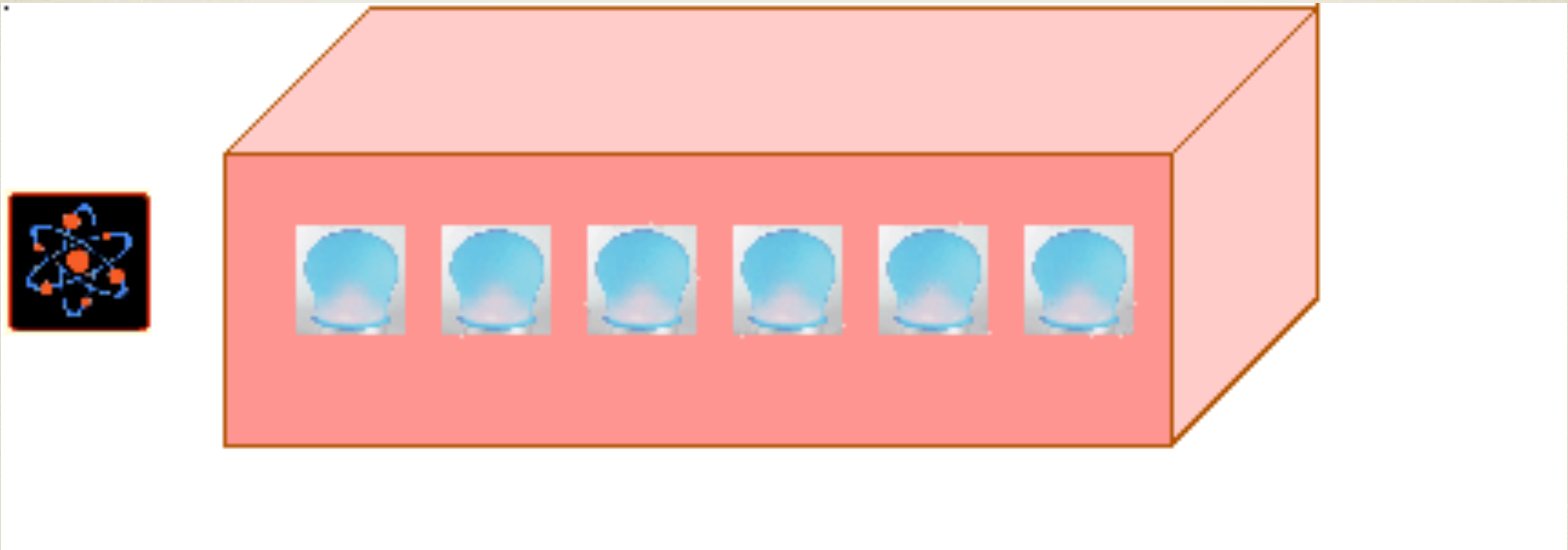
Reconstructing QM from probabilities

Algebra of effects ↔

* AE: Atomicity of evolution
* CJ: Choi-Jamiolkowski isomorphism

TESTS

- Test/experiment: $\mathbb{A} \equiv \{\mathscr{A}_j\}$ set of possible events $\mathscr{A}_j$



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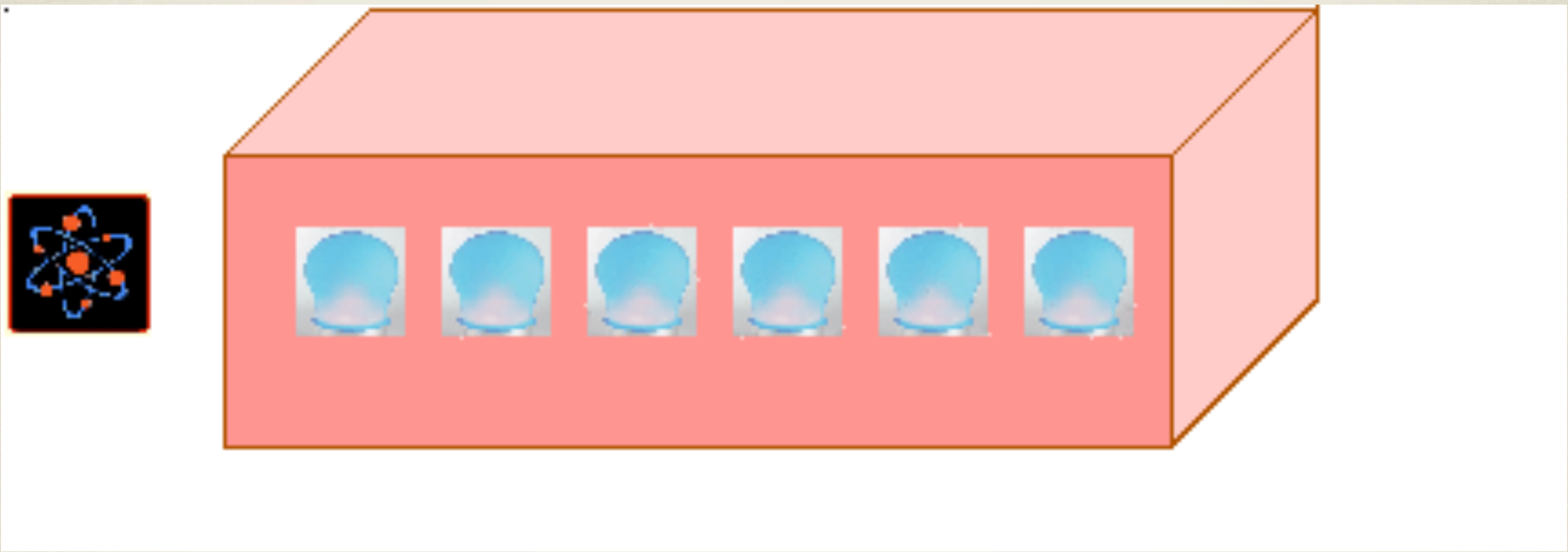

(deterministic test/transformation: $\mathbb{D} = \{\mathscr{D}\}$)

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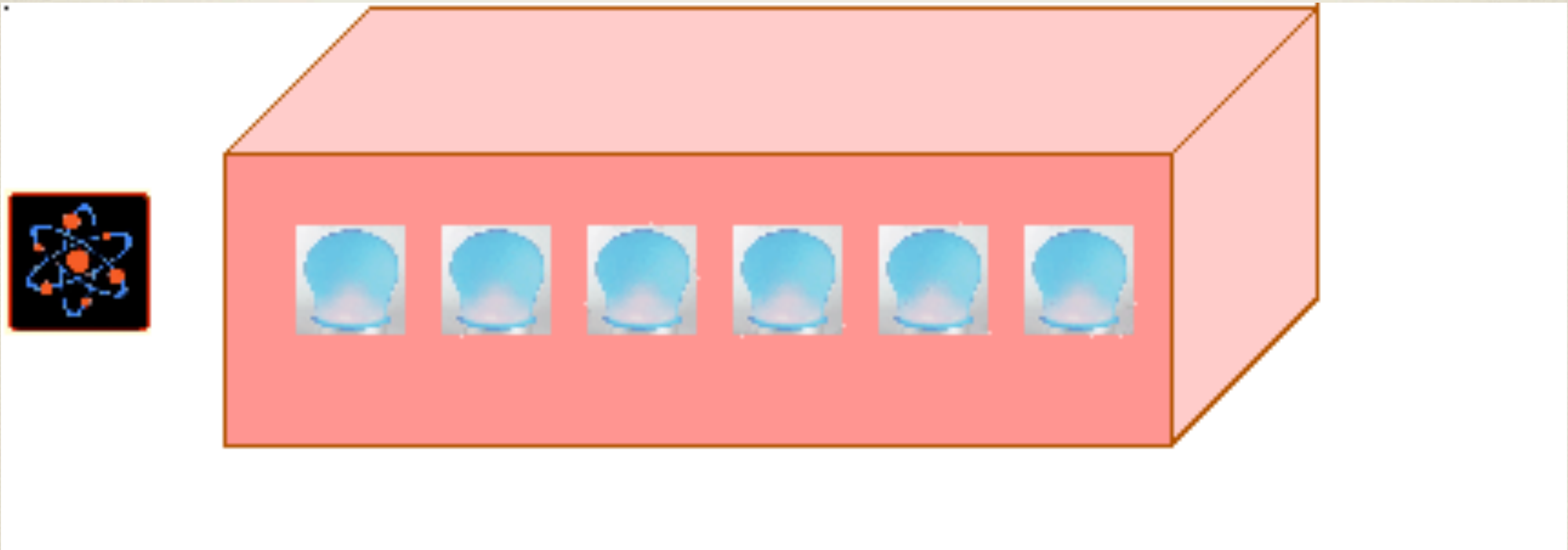

(deterministic test/transformation: $\mathbb{D} = \{\mathscr{D}\}$)

Notice: <u>the same event can occur in different tests</u>

TESTS

Unions of events: $\mathscr{A} \cup \mathscr{B}$

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Coarse-graining

$$\mathbb{A} = \{\mathscr{A}_1, \mathscr{A}_2, \mathscr{A}_3\} \quad \rightleftarrows \quad \mathbb{A}' = \{\mathscr{A}_1, \mathscr{A}_2 \cup \mathscr{A}_3\}$$

Refinement

STATES

State ω: probability rule $\omega(\mathscr{A})$ for any possible event $\mathscr{A}$ in any test

Normalization: $$\sum_{\mathscr{A}_j \in \mathbb{A}} \omega(\mathscr{A}_j) = 1$$

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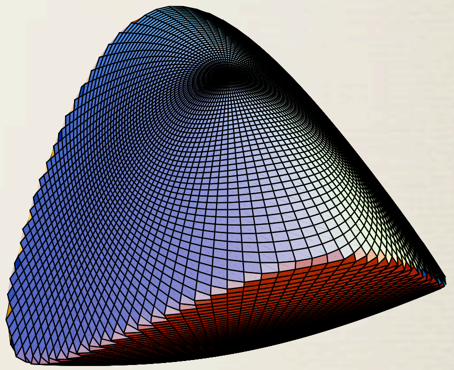
Convex set of states: $\mathfrak{S}$



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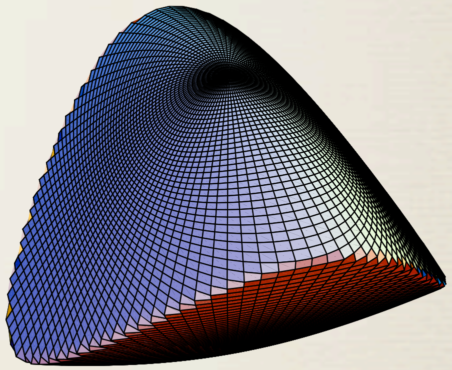

States will also be regarded as tests themselves: "preparation-tests".

CASCADES OF TESTS

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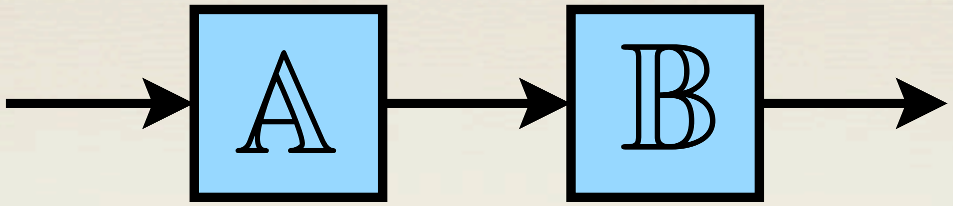
Time-cascade:



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Time-cascade: $\longrightarrow \boxed{\mathbb{A}} \longrightarrow \boxed{\mathbb{B}} \longrightarrow$

$\mathbb{B} \circ \mathbb{A} = \{\mathscr{B}_j \circ \mathscr{A}_i\}$ cascade of tests $\mathbb{A} = \{\mathscr{A}_i\},\ \mathbb{B} = \{\mathscr{B}_j\},$

collection of joined events with the following rule for marginals:

$$\boxed{\textbf{NSF} \sum_{\mathscr{B}_j \in \mathbb{B}} \omega(\mathscr{B}_j \circ \mathscr{A}) =: f(\mathbb{B}, \mathscr{A}) \equiv \omega(\mathscr{A}), \quad \forall \mathbb{B}, \mathscr{A}, \omega}$$

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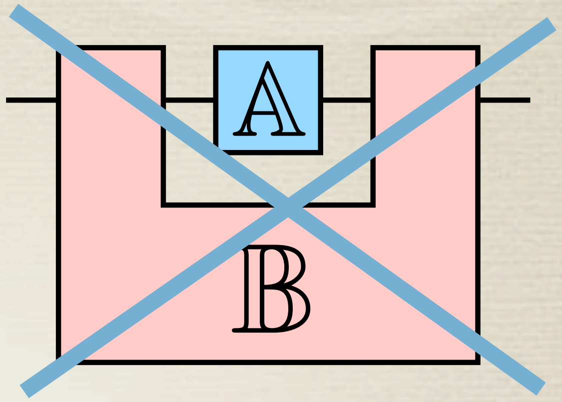

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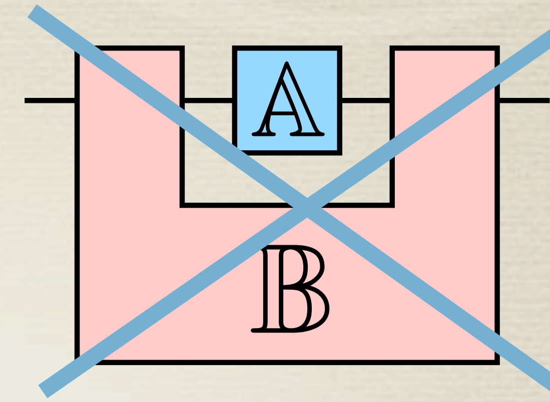

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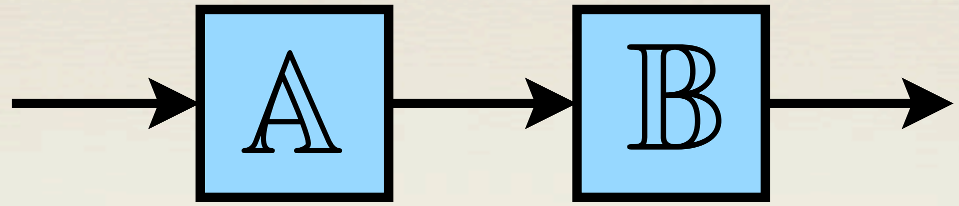

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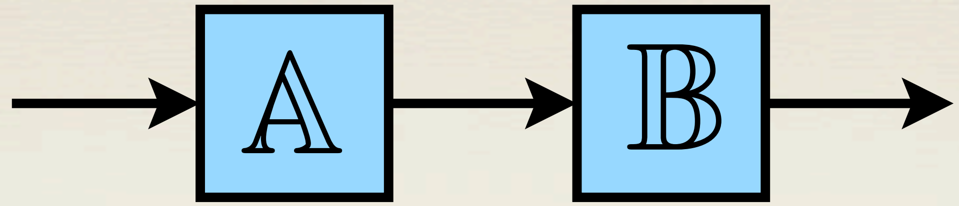
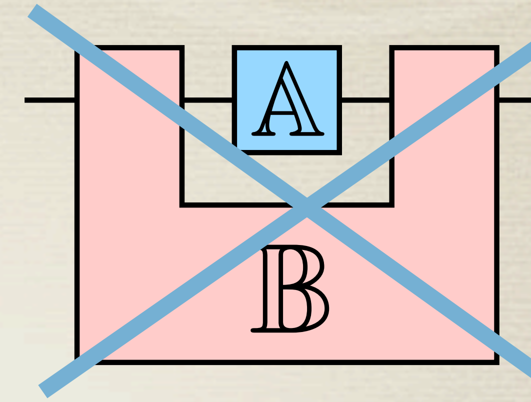
Time-cascade: 



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composition of events: $\mathscr{B} \circ \mathscr{A}$

Convex monoid of events: $\mathscr{T}$

Events ≡ transformations

NSF $$\sum_{\mathscr{B}_j \in \mathbb{B}} \omega(\mathscr{B}_j \circ \mathscr{A}) = \omega(\mathscr{A}), \quad \forall \mathbb{B},\ \forall \mathscr{A},\ \forall \omega$$

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$\Longrightarrow$ conditional probability: $p(\mathscr{B}|\mathscr{A}) = \omega(\mathscr{B} \circ \mathscr{A})/\omega(\mathscr{A})$

$\Longrightarrow$ conditional state: $\omega_{\mathscr{A}} := \omega(\cdot \circ \mathscr{A})/\omega(\mathscr{A})$

variable

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$\Rightarrow$ events ≡ transformations

$\Rightarrow$ linearity of evolution

Logical flow-diagram

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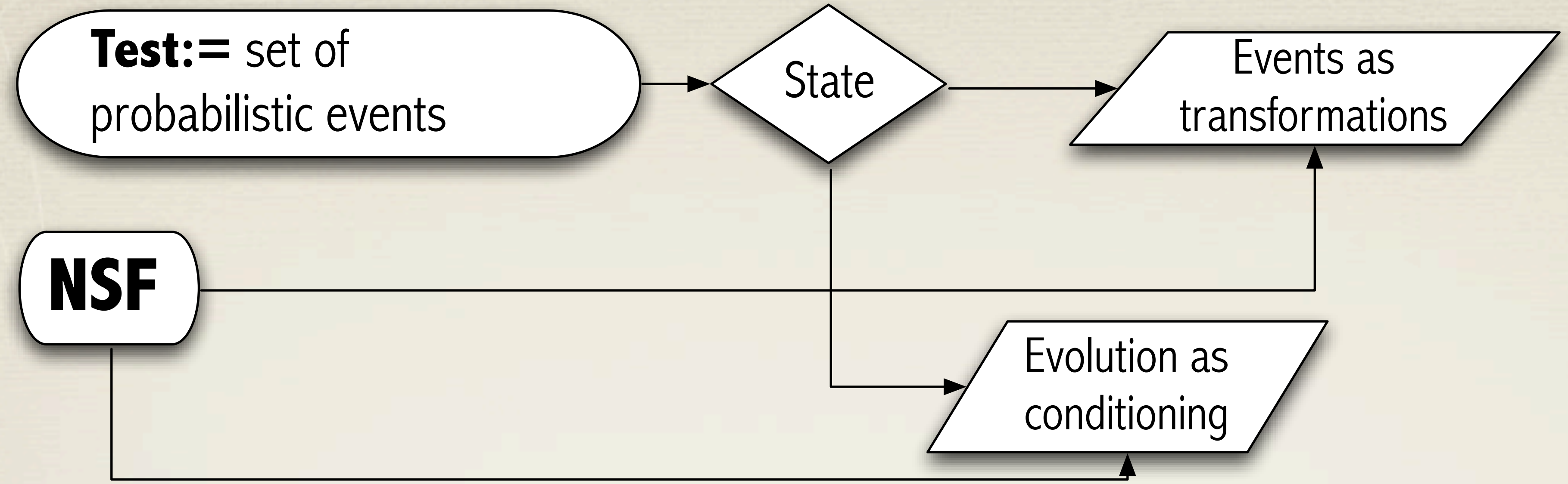


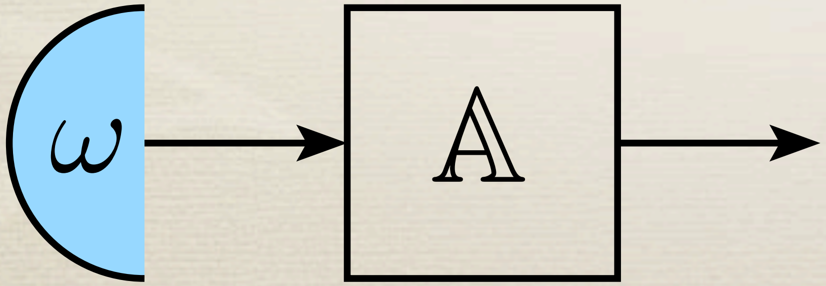
SYSTEM

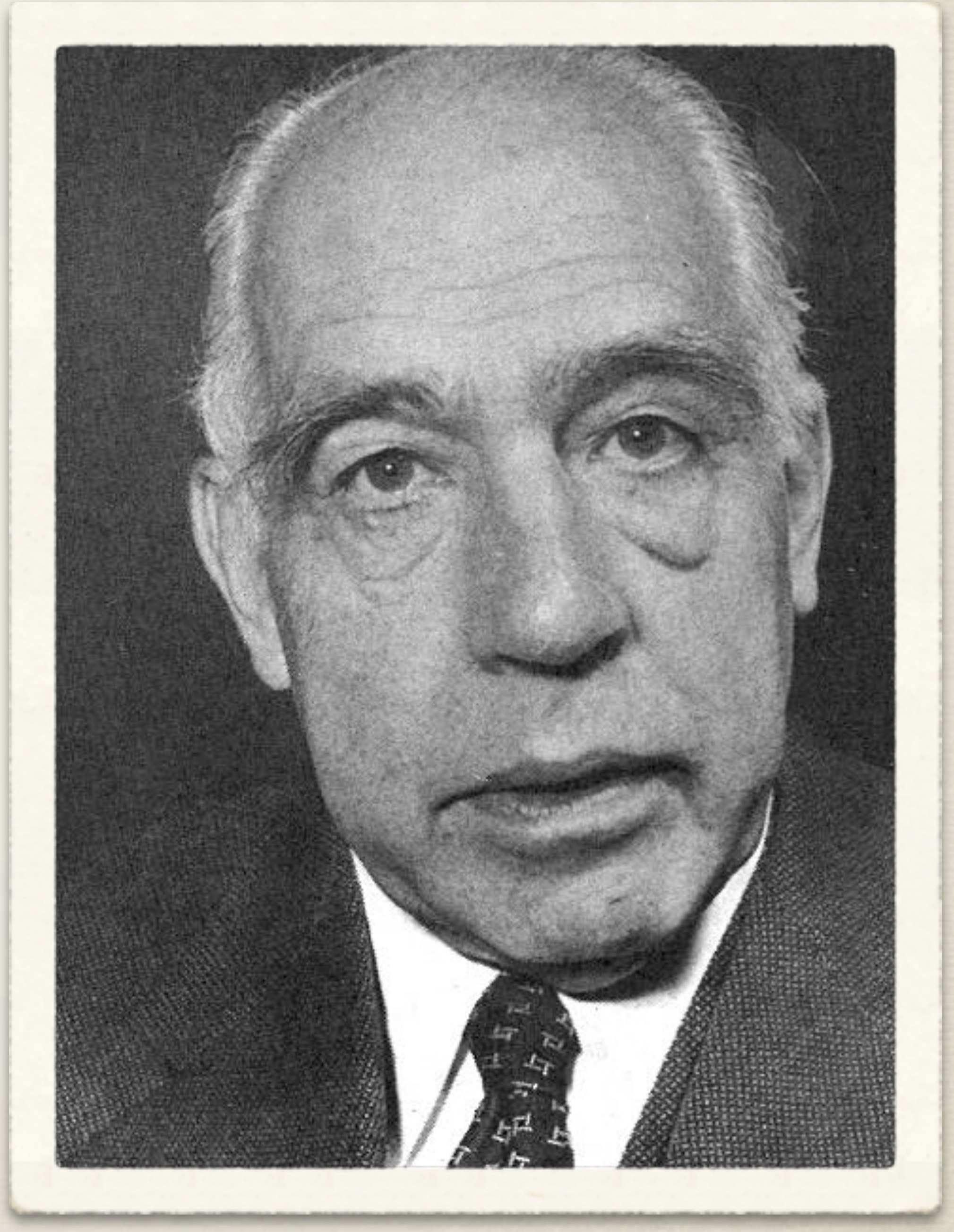
$\mathsf{S} = \{\omega_1, \omega_2, \ldots, \mathbb{A}, \mathbb{B}, \mathbb{C}, \ldots\}$

Copenhagen

collection of tests closed under

* coarse-graining
* conditioning
* cascading
* convex combination





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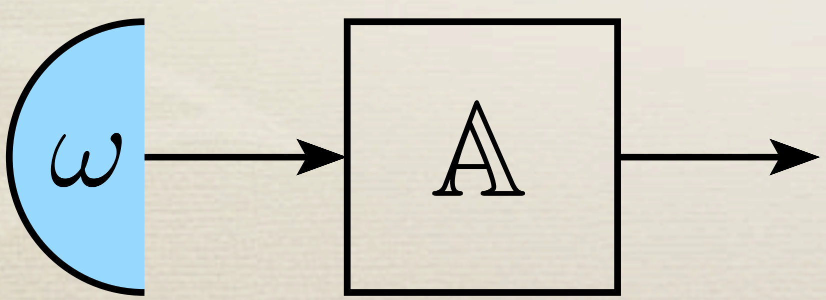
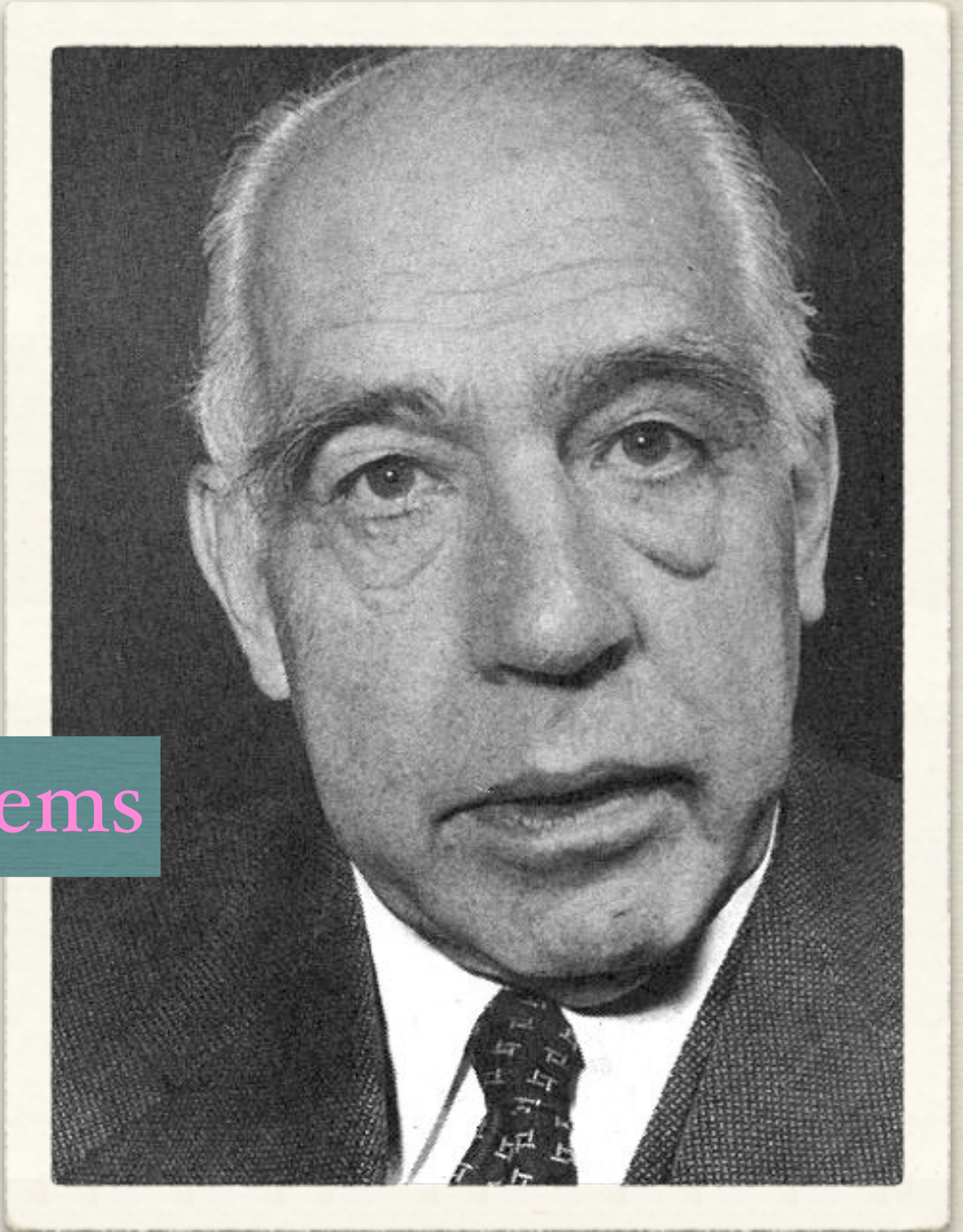
one kind of systems





different systems: tests as letters in a "language"

Two equivalence classes for transformations

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Two transformations $\mathscr{A}$ and $\mathscr{B}$ are conditioning equivalent if

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Conditioning-equivalence class

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Probabilistic-equivalence class

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A transformation is completely specified by the two classes:

$$\boxed{\mathscr{A}\omega = \omega(\mathscr{A})\omega_{\mathscr{A}}}$$

probabilistic

conditioning

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variable

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EFFECTS

Effect $[\mathscr{A}]_{\text{eff}}$: equivalence class of transformations occurring with the same probability as $\mathscr{A}$ for all states.

$$\forall \omega \in \mathfrak{S} : \quad \omega(\mathscr{A}) \equiv \omega([\mathscr{A}]_{\text{eff}})$$

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e deterministic effect i.e. $\omega(e) = 1 \quad \forall \omega \in \mathfrak{S}$

EFFECTS

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State-conditioning $\Rightarrow$ Transformations act linearly over effects:

$$[\mathscr{B}]_{\mathrm{eff}} \circ \mathscr{A} = [\mathscr{B} \circ \mathscr{A}]_{\mathrm{eff}}$$ (Heisenberg picture)

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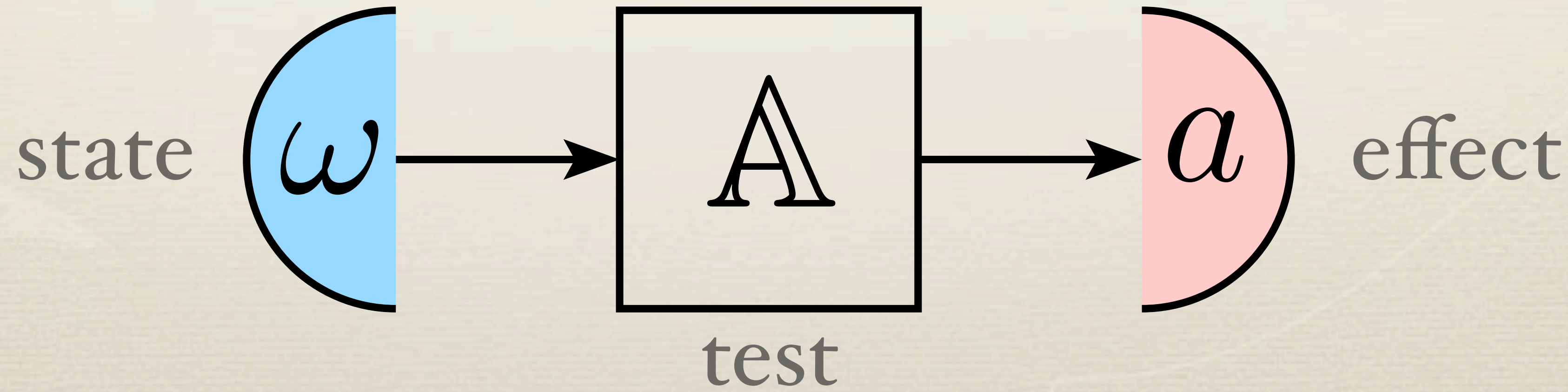
Effects will also be regarded as tests themselves: "effect-tests"

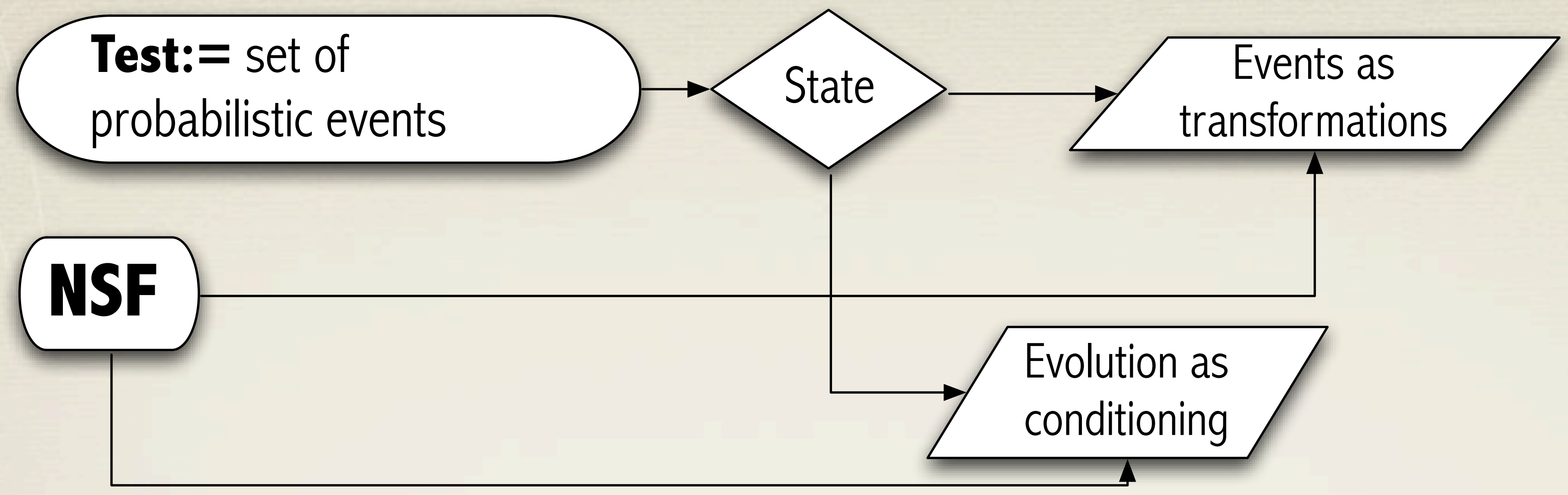


OBSERVABLE

Observable $\mathbb{L} = \{l_i\}$: complete set of effects of a test

Normalization: $\sum_{i \in \mathbb{L}} l_i = e$

Logical flow-diagram



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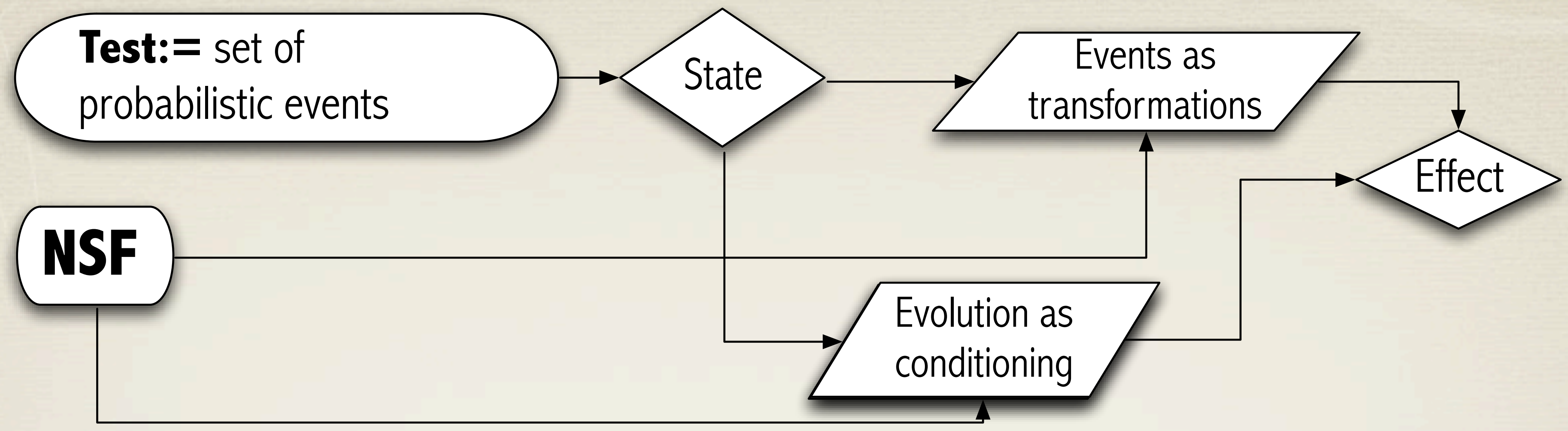

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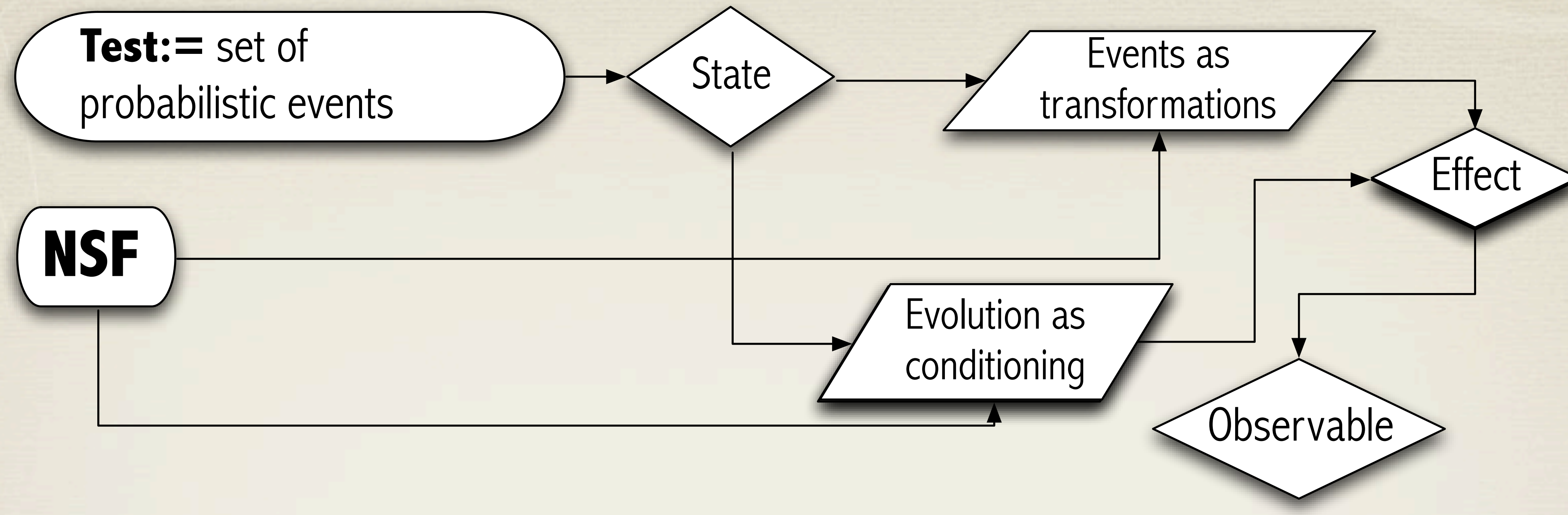

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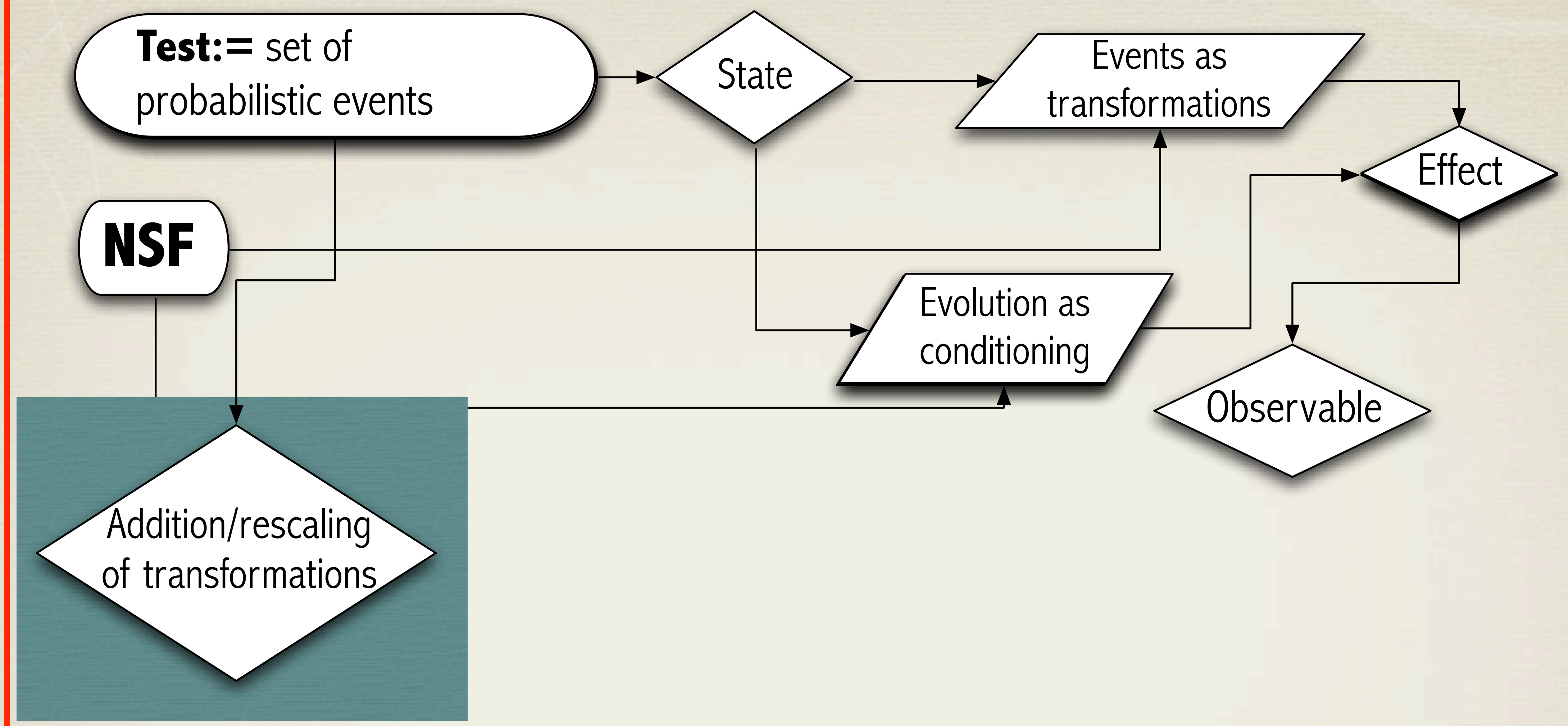

Addition of transformations

Two transformations $\mathscr{A}$ and $\mathscr{B}$ generally occurring in different tests are test-compatible if for every state ω one has

$$\omega(\mathscr{A}) + \omega(\mathscr{B}) \leq 1$$

For any two test-compatible transformations $\mathscr{A}_1$ and $\mathscr{A}_2$ we define the transformation $\boxed{\mathscr{A}_1 + \mathscr{A}_2}$ as the union event $\mathscr{A}_1 \cup \mathscr{A}_2$ as if they belong to the same test

$$\boxed{(\mathscr{A}_1 + \mathscr{A}_2)\omega = \mathscr{A}_1\omega + \mathscr{A}_2\omega}$$

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$$\omega(\mathscr{A}_1 + \mathscr{A}_2) = \omega(\mathscr{A}_1) + \omega(\mathscr{A}_2) \quad \text{(probabilistic class)}$$

$$\omega_{\mathscr{A}_1+\mathscr{A}_2} = \frac{\omega(\mathscr{A}_1)}{\omega(\mathscr{A}_1 + \mathscr{A}_2)}\omega_{\mathscr{A}_1} + \frac{\omega(\mathscr{A}_2)}{\omega(\mathscr{A}_1 + \mathscr{A}_2)}\omega_{\mathscr{A}_2} \quad \text{(conditioning class)}$$

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Rescaling of transformations

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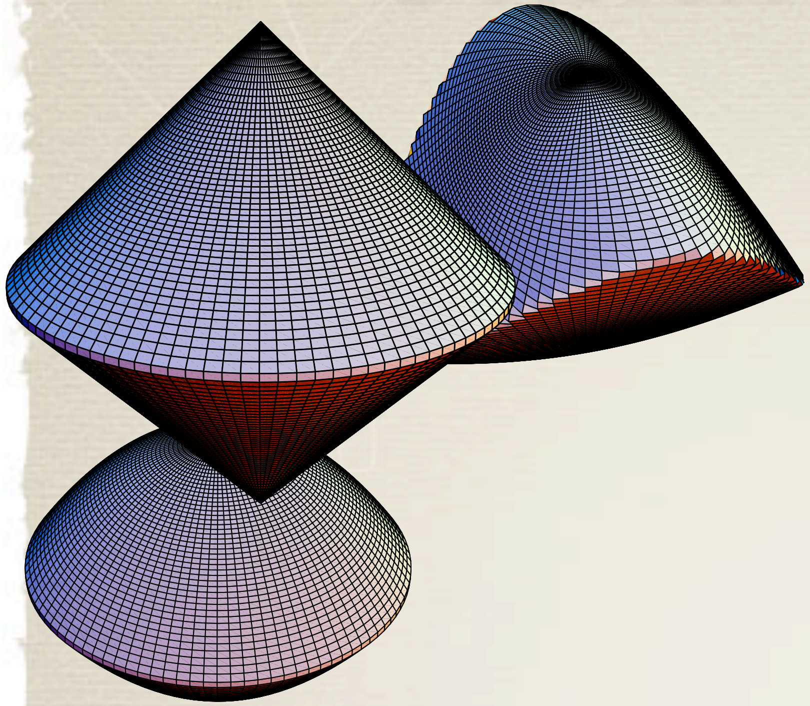
Atomic: a transformation that cannot be "nontrivially" refined in any test, i.e. it cannot be written as $\mathscr{A} = \sum_i \mathscr{A}_i$ with $\mathscr{A}_i \neq \lambda_i \mathscr{A}$ for some i and $0 < \lambda_i < 1$.

[Notice: the identity transformation $\mathscr{I}$ is not necessarily atomic]

Convex sets, Cones and Linear spaces



Convex set of states: $\mathfrak{S}$, cone: $\mathfrak{S}_+$

Convex set of effects: $\mathfrak{E}$, cone: $\mathfrak{E}_+$

Convex monoid of transformations: $\mathfrak{T}$, cone: $\mathfrak{T}_+$

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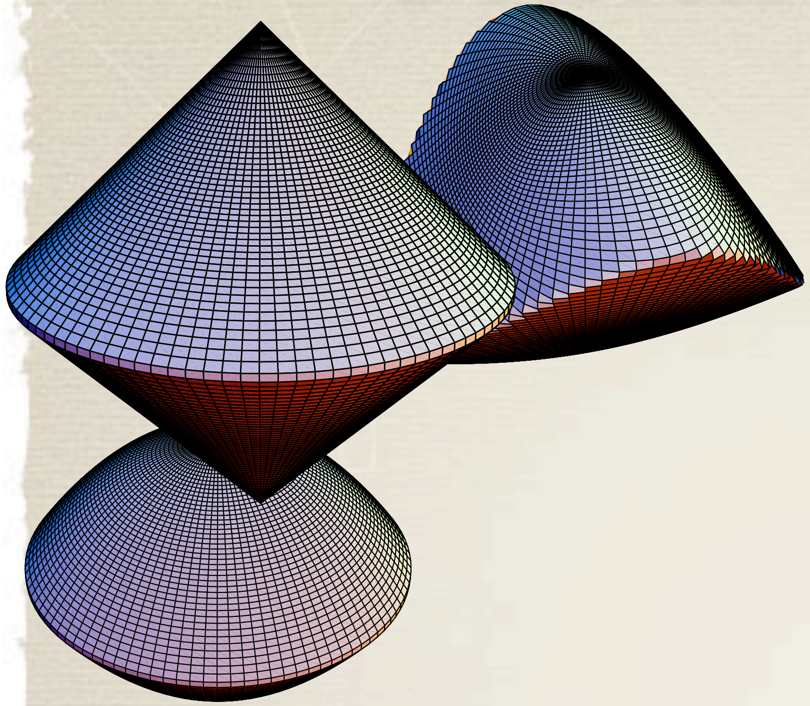

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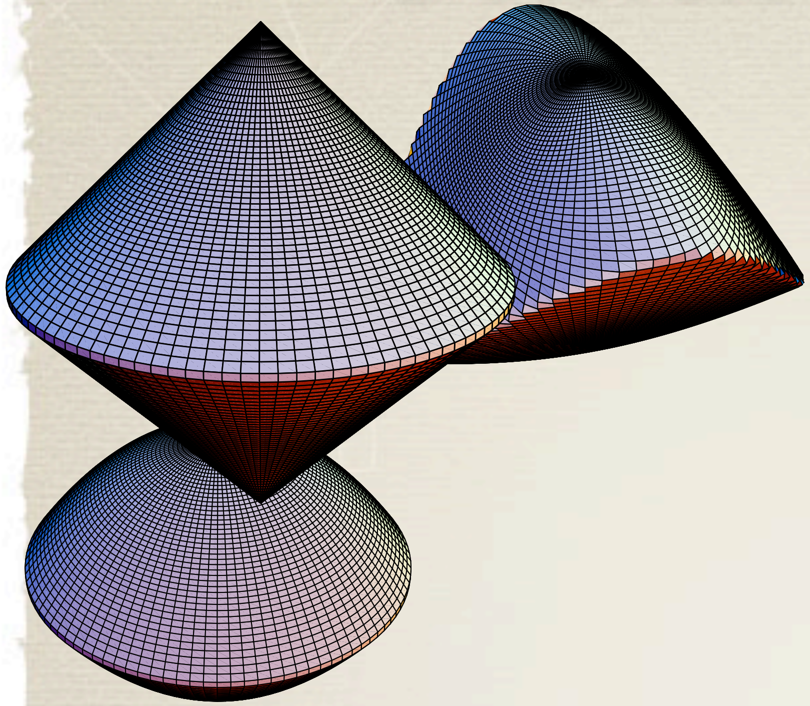

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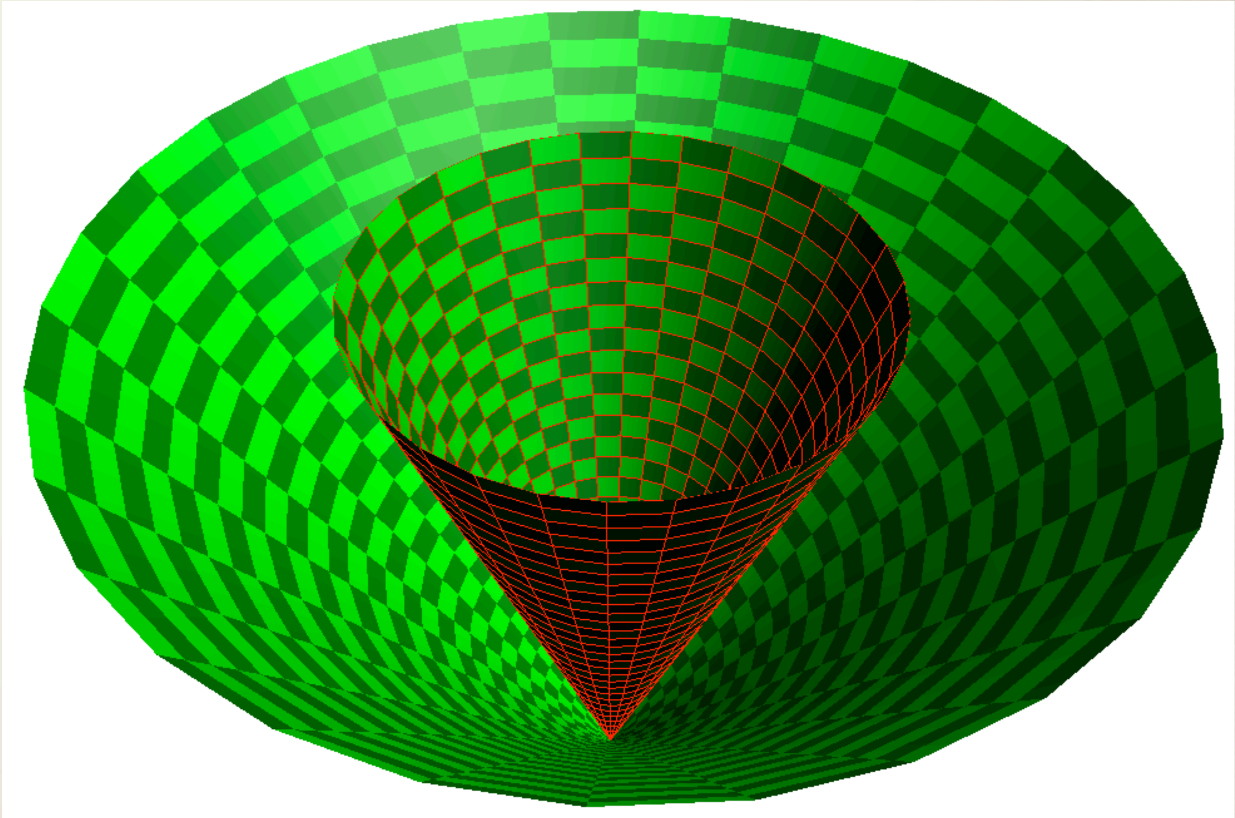

Hypothesis of no limitation to preparability: $\mathfrak{S}_+ = (\mathfrak{E}_+)^*$

Informational completeness

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Informationally complete observable: $\mathbb{L}$

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Quantum Bureau International des Poids et Measures (Fuchs):

$\mathbb{S} = \{\mathscr{S}_i\}$

$\mathscr{S}_i\omega = \omega(\mathscr{S}_i)\omega_i, \ \forall\omega \in \mathfrak{S}, \ \{\omega_i\}$ separating

$\{[\mathscr{S}_i]_{\mathrm{eff}}\}$ informationally complete observable

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C*-algebra of transformations (finite dim.)

Transformations/events are linear maps over effects, i.e. they make a matrix algebra over effects (or over states)

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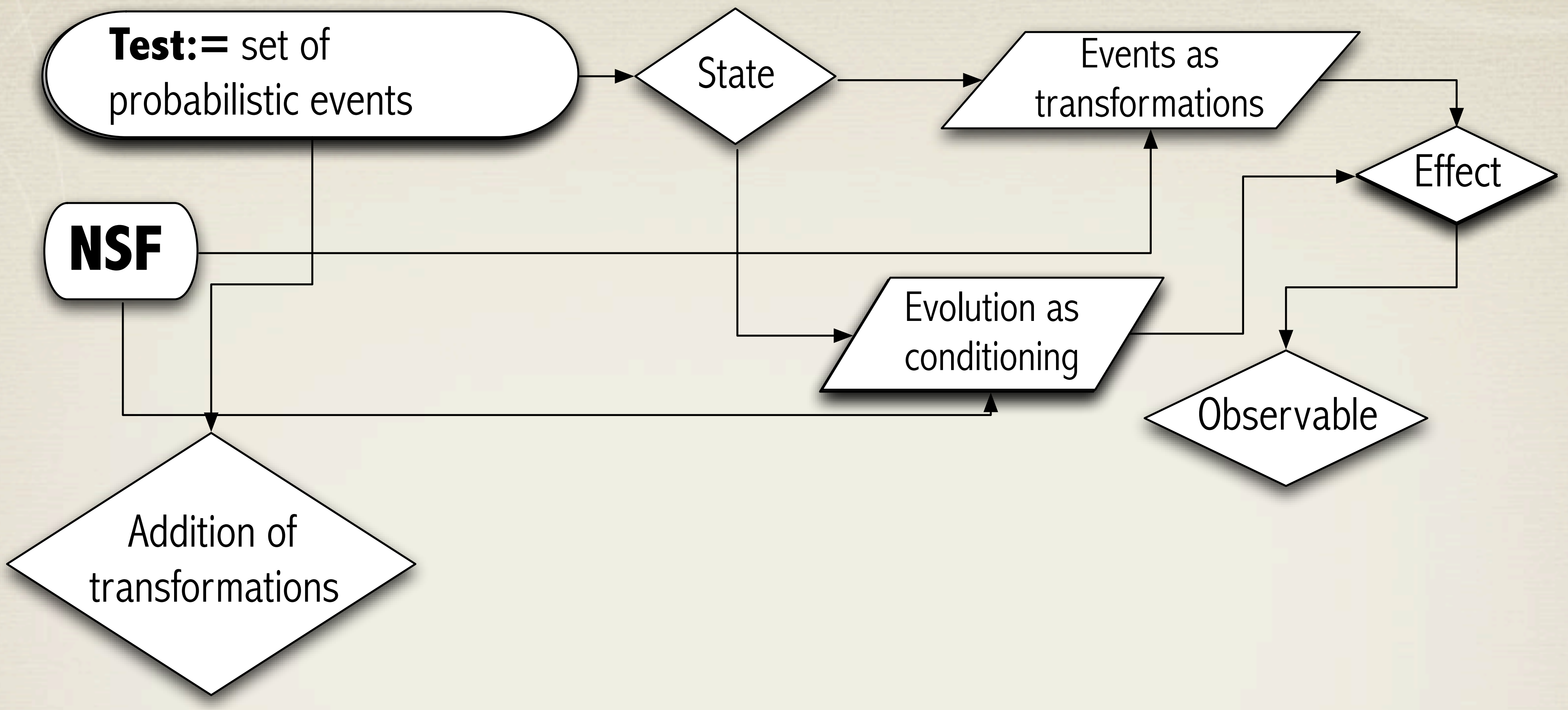
One can introduce a scalar product over effects ...
$\Rightarrow$ transformations become a C*-algebra ...

Logical flow-diagram



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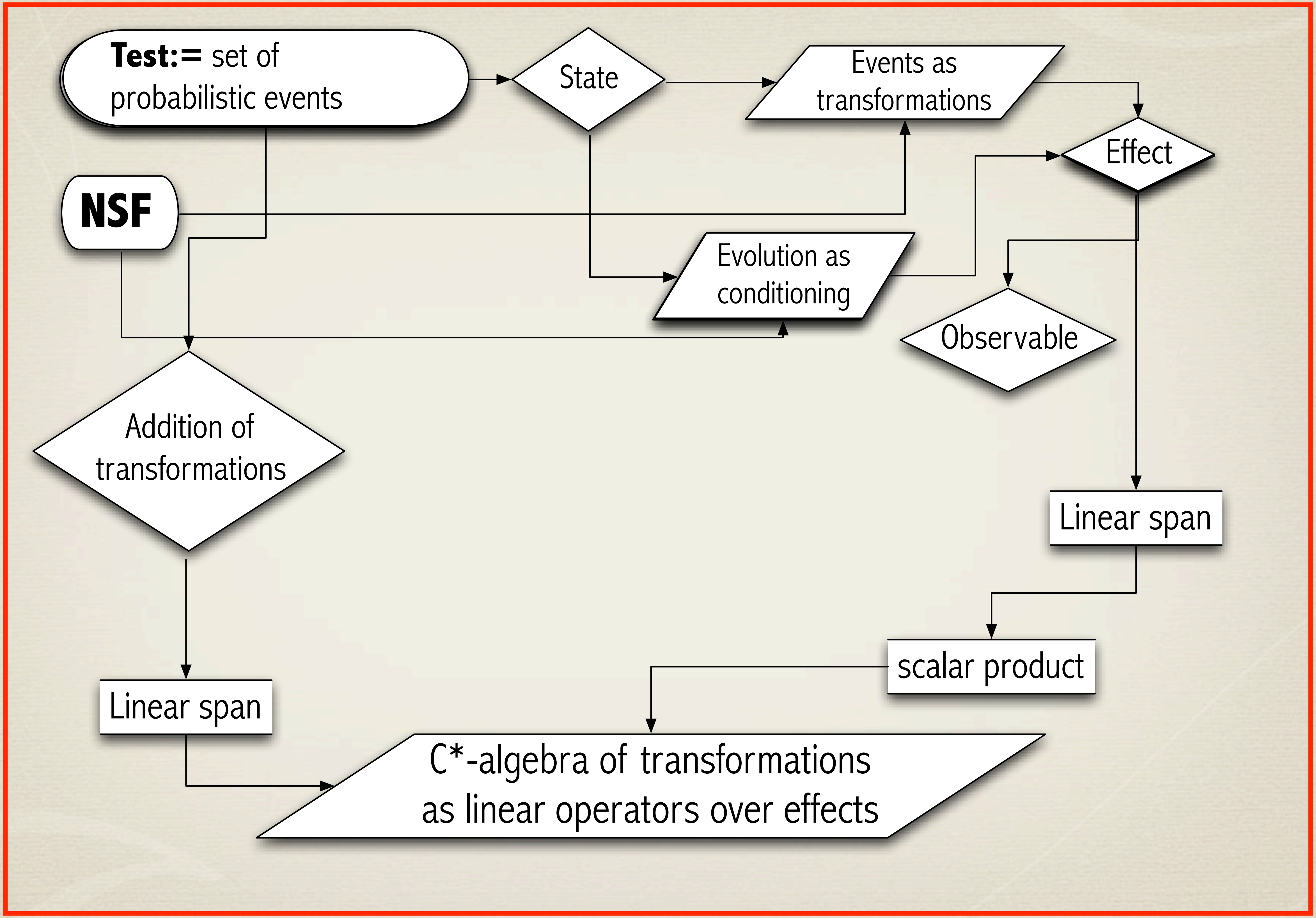
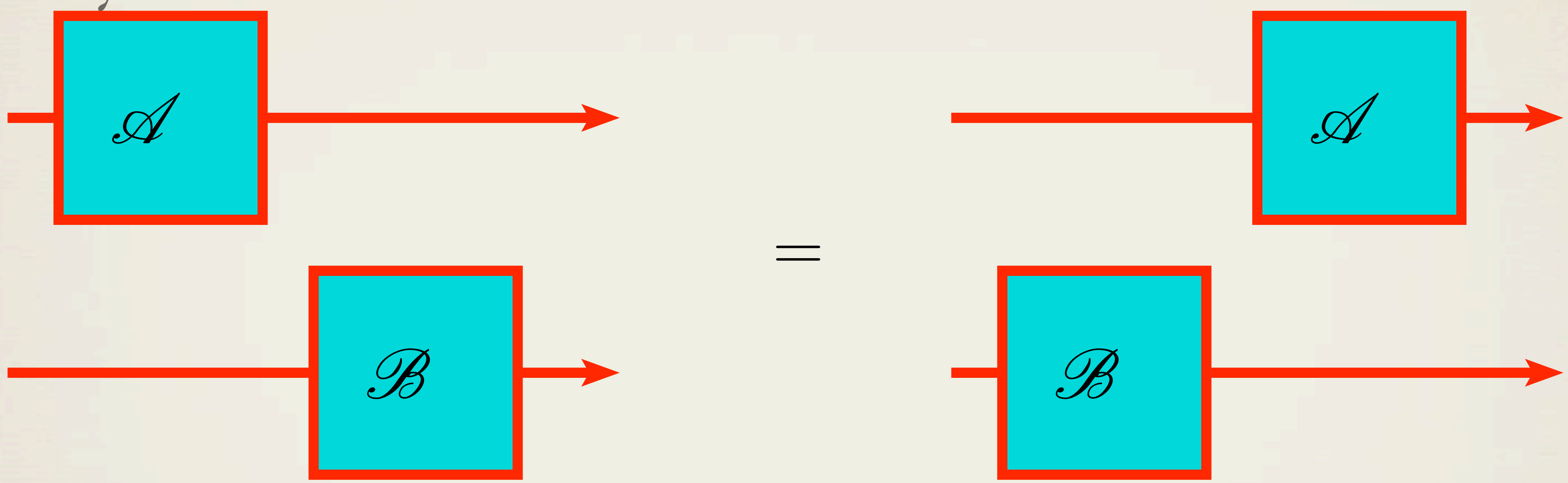


INDEPENDENT SYSTEMS

Two systems are independent if on each system it is possible to perform all their tests as local tests, i.e. such that on every joint state one has the commutativity of the transformations from different systems



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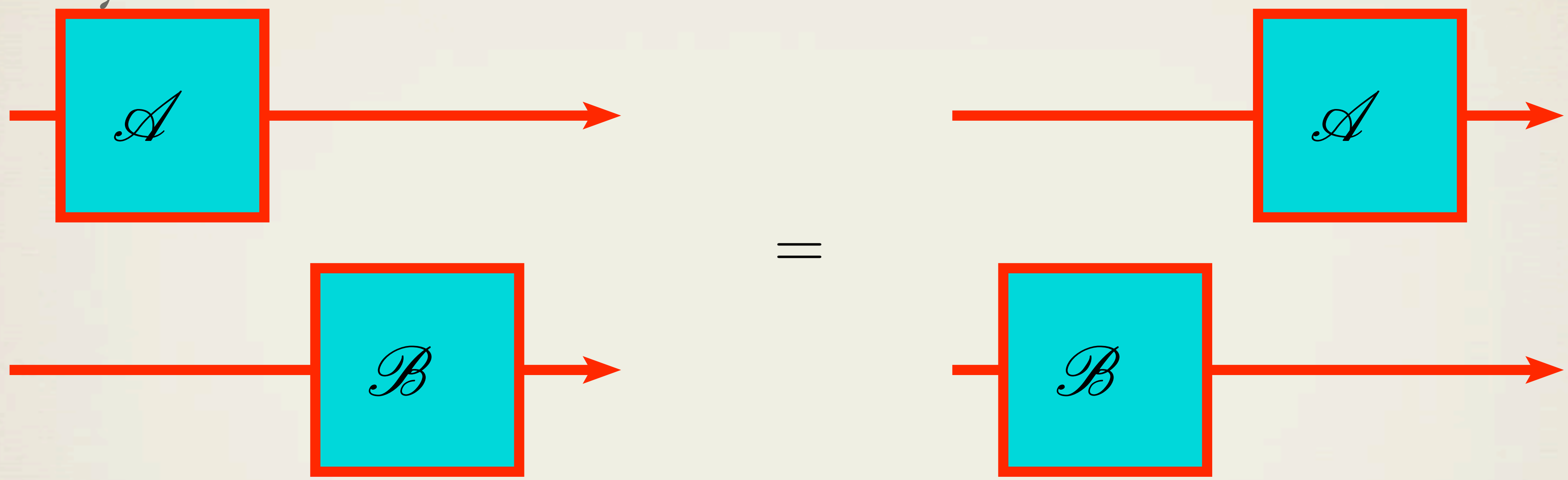

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$$(\mathscr{A}, \mathscr{B}, \mathscr{C}, \ldots) \doteq \mathscr{A}^{(1)} \circ \mathscr{B}^{(2)} \circ \mathscr{C}^{(3)} \circ \ldots \qquad \boxed{\mathsf{S}_1 \times \mathsf{S}_2}$$

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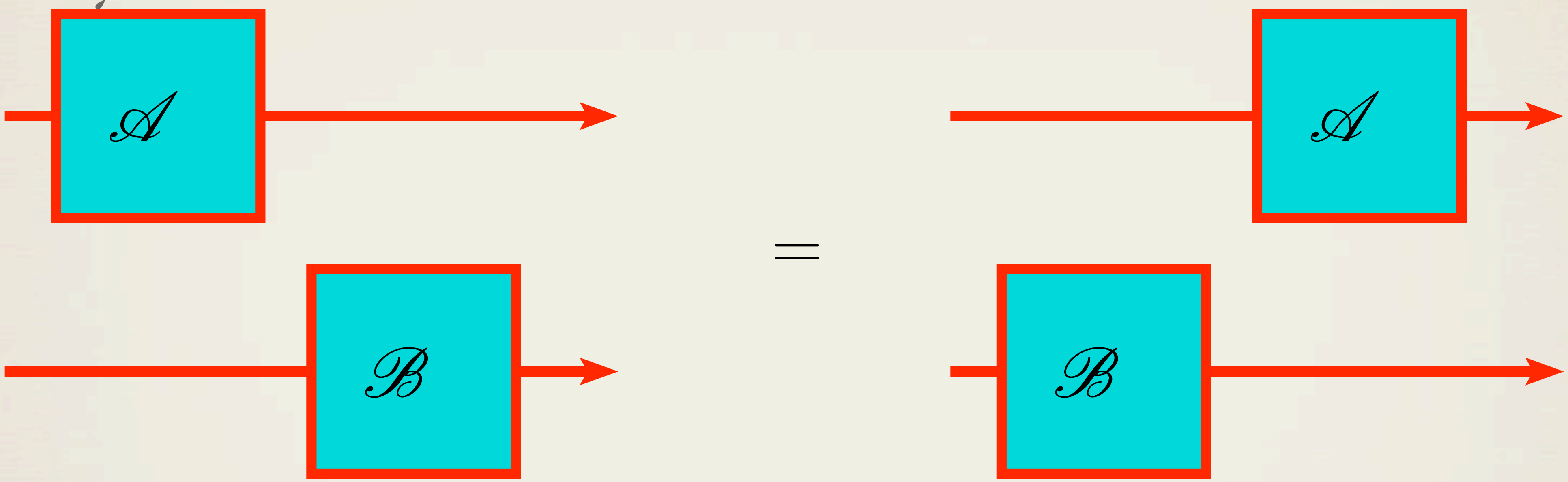

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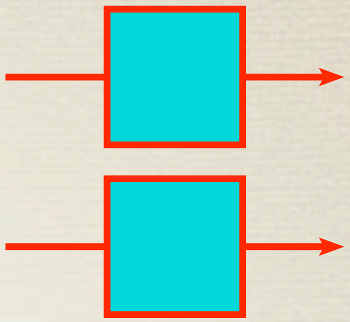
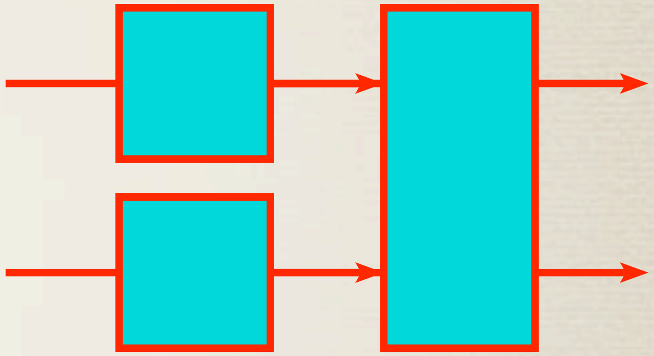
$\boxed{\mathsf{S}_1 \times \mathsf{S}_2}$

$$[(\mathscr{A}, \mathscr{B}, \mathscr{C}, \ldots)]_{\mathrm{eff}} = ([\mathscr{A}]_{\mathrm{eff}}, [\mathscr{B}]_{\mathrm{eff}}, [\mathscr{C}]_{\mathrm{eff}}, \ldots)$$

COMPOSTING SYSTEMS

We compose the two systems S_1 and S_2 into the bipartite system $\mathsf{S}_1 \odot \mathsf{S}_2$ considered as a new system containing all local tests $\mathsf{S}_1 \times \mathsf{S}_2$ plus other tests, and closing w.r.t. coarse graining, convex combination and cascading:





$$\boxed{\mathsf{S}_1 \odot \mathsf{S}_2 \supseteq \mathsf{S}_1 \times \mathsf{S}_2}$$

Nonlocal tests: $\mathsf{S}_1 \odot \mathsf{S}_2 \setminus \mathsf{S}_1 \times \mathsf{S}_2$

MARGINAL STATE

For a multipartite system we define the marginal state $\Omega|_n$ of the n-th system the state that gives the probability of any local transformation $\mathscr{A}$ on the n-th system with all other systems untouched, namely

$$\Omega|_n(\mathscr{A}) := \Omega(\mathscr{I}, \ldots, \mathscr{I}, \underbrace{\mathscr{A}}_{n-th}, \mathscr{I}, \ldots)$$

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$$\Downarrow$$

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NS: (no-signaling) any local test on a system is equivalent to no-test on another independent system.

PROBABILISTIC THEORY?

PROBABILISTIC
EORY?

Matrix algebra of
transformations over effects!

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Independent
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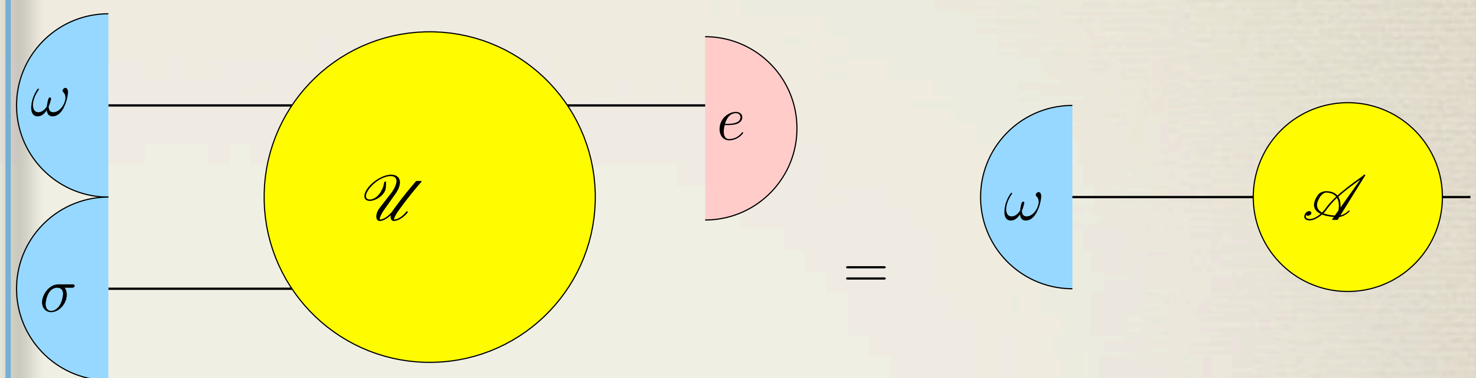
$\mathscr{U}_{12}, (\mathscr{A}, \mathscr{I}) \in \mathfrak{T}^{\odot 2}$

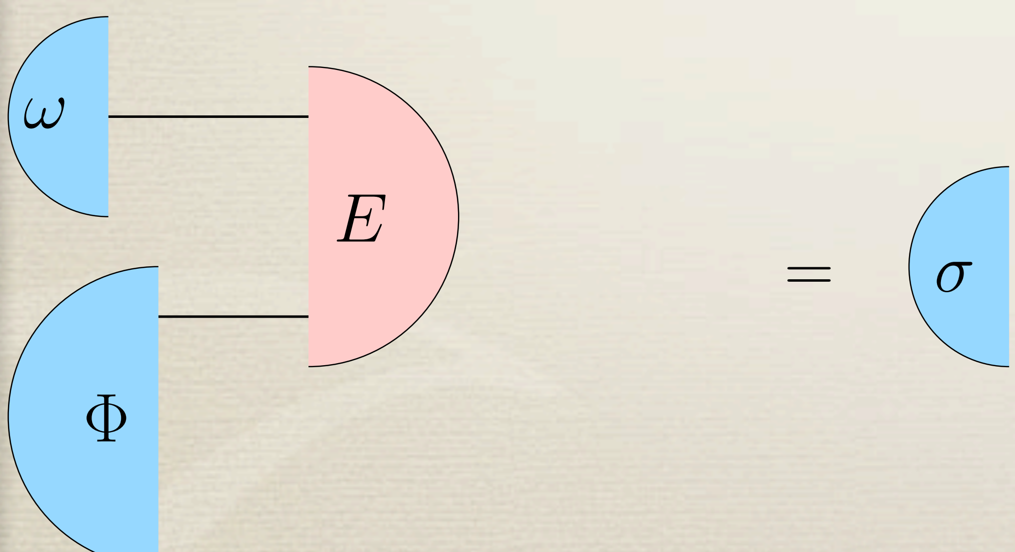
Review of notation

$$\omega(a) \equiv a(\omega)$$

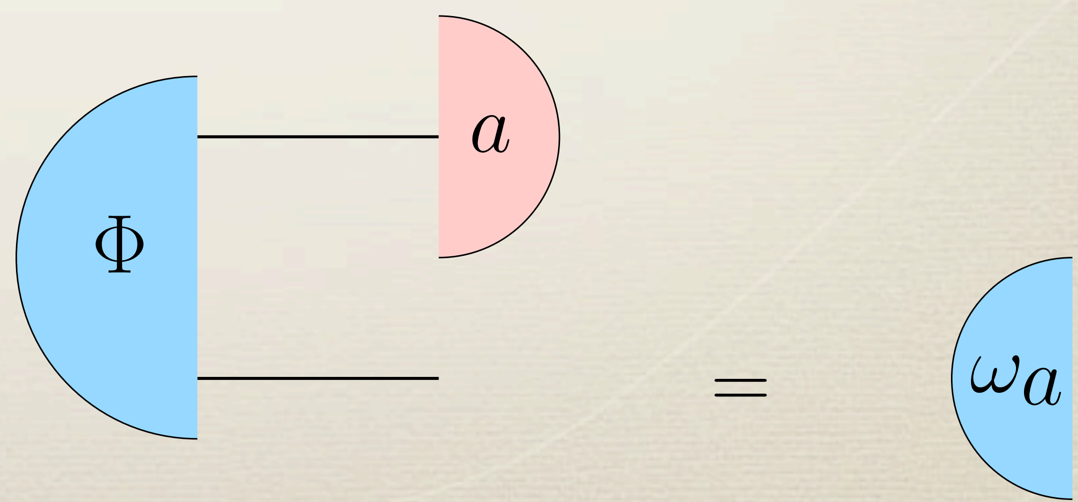
$$\mathscr{U}_{12}(\sigma, \omega)(e, \cdot) = \mathscr{A}\omega$$



$$E_{23}(\Phi, \omega) = \sigma \in \mathfrak{S}$$



$$\Phi(a, \cdot) = \omega_a \in \mathfrak{S}_+$$



FAITHFUL STATES

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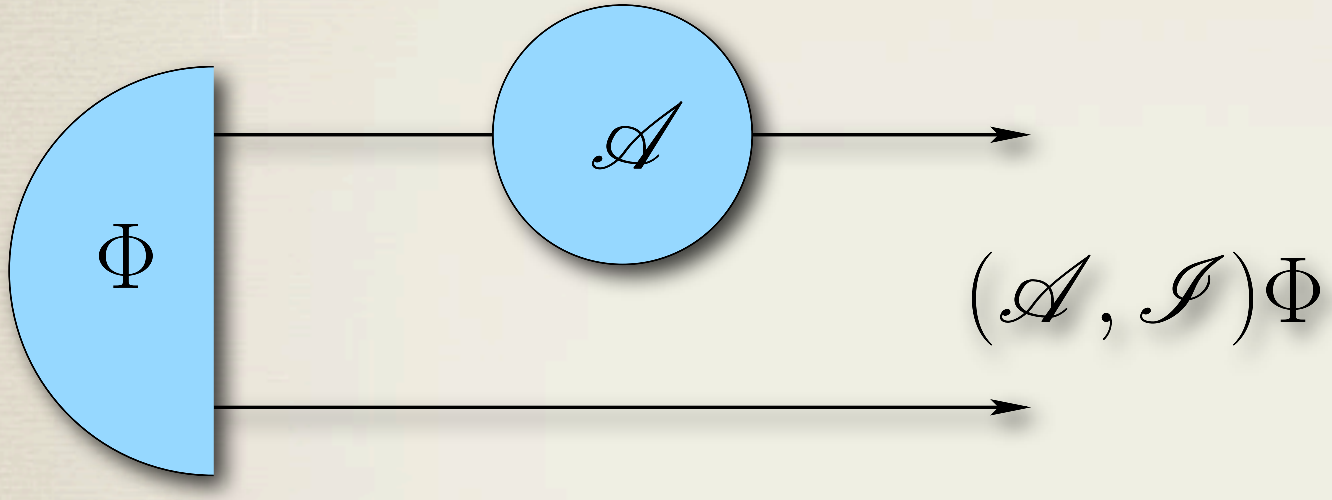

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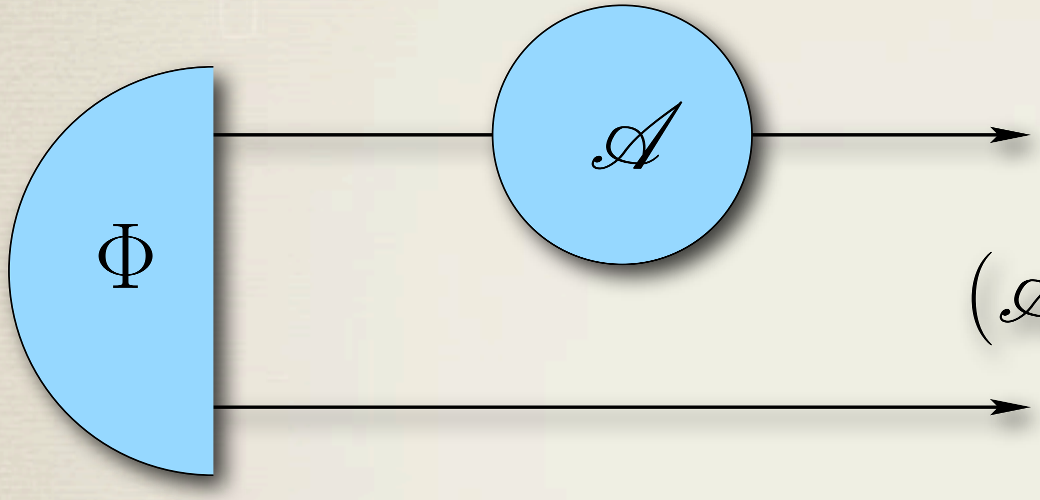

$(\mathscr{A},\mathscr{I})\Phi$

calibrability of tests

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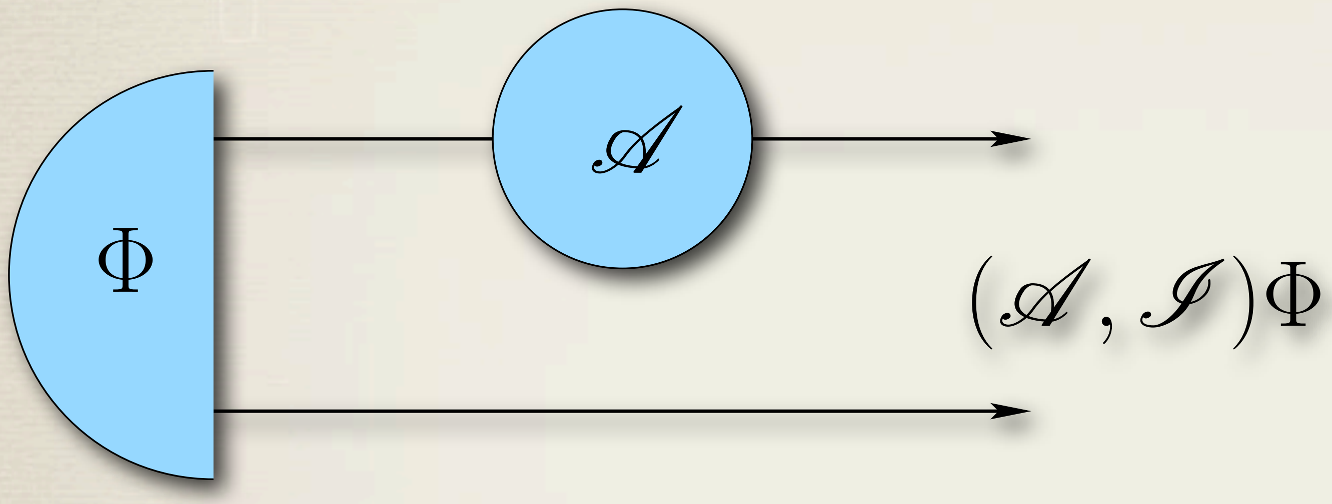
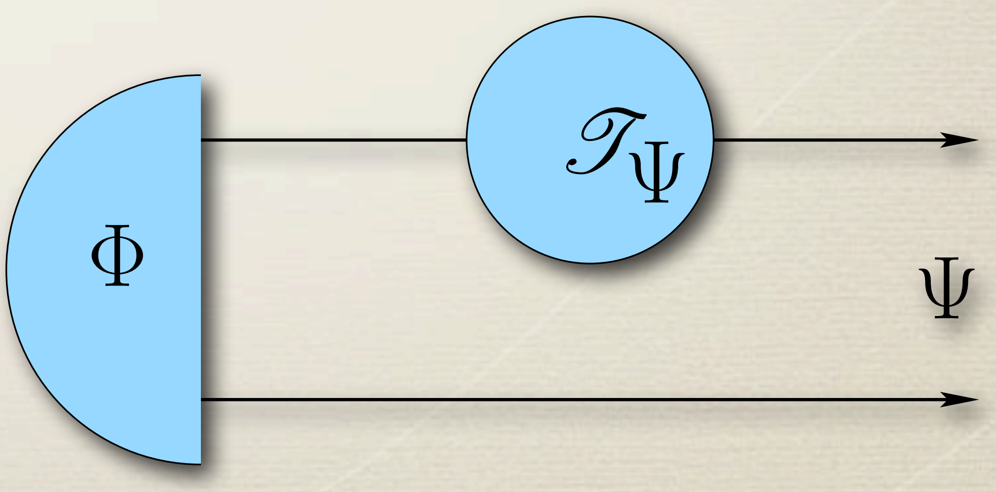


calibrability of tests

A state Φ of a bipartite system is preparationally faithful if every joint state Ψ can be achieved by a suitable local transformation $\mathscr{T}_\Psi$ on one system occurring with nonzero probability



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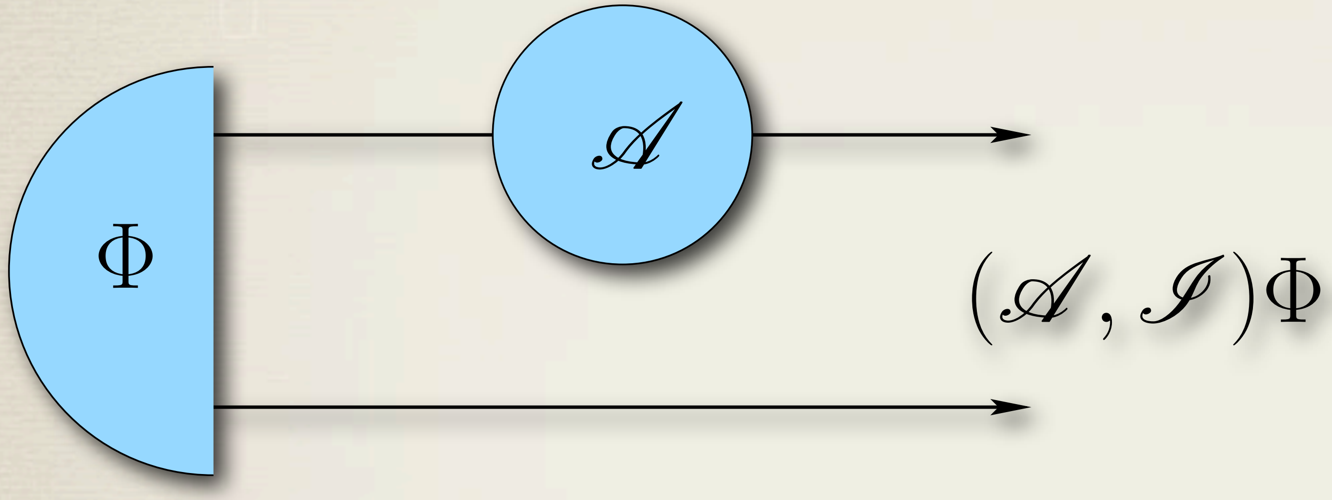

calibrability of tests

local state-preparability

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Postulate PFAITH

PFAITH: For any couple of identical systems, there exist a symmetric* state Φ that is preparationally faithful.

(*) invariant under permutation of the two systems

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Theorem: Φ is also dynamically faithful.

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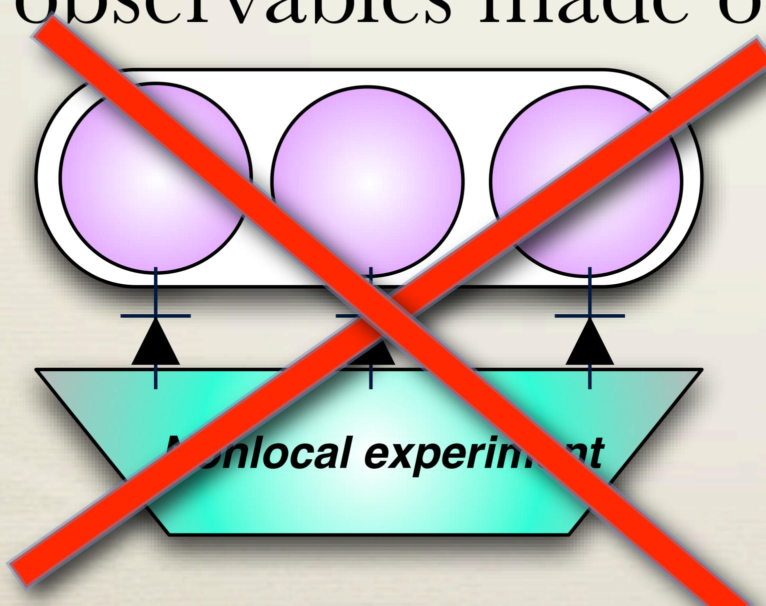
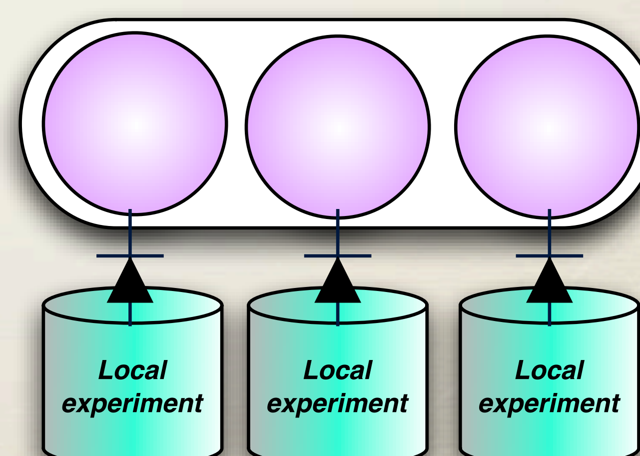
- Local observability: There exist global info-complete observables made of local info-complete





Holism  Reductionism

Consequences of PFAITH

$\boxed{\mathbb{F} = \mathbb{R}, \mathbb{C}}$

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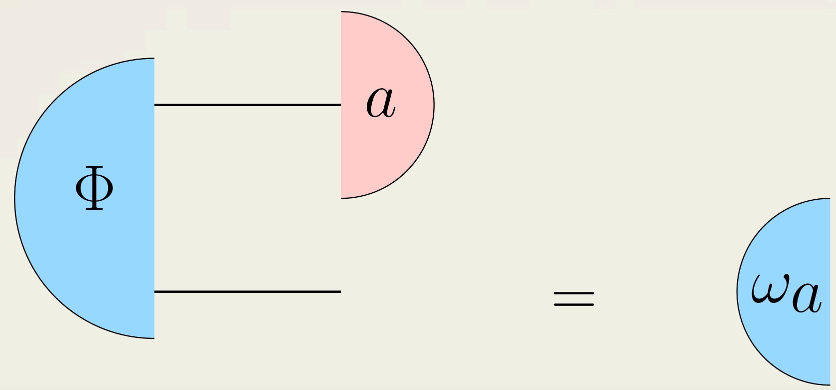
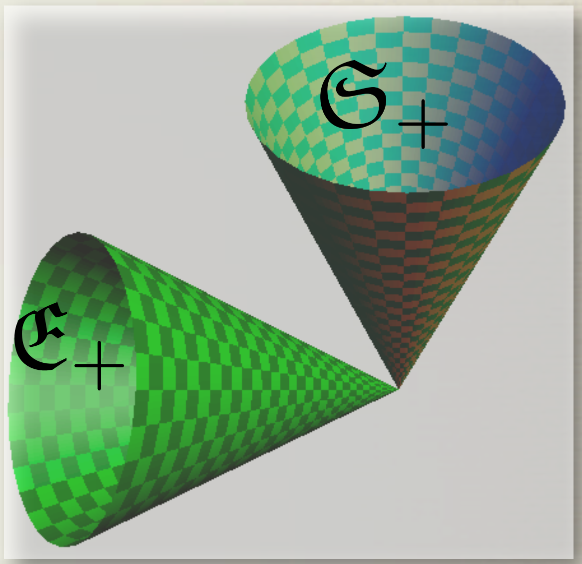
$$\mathfrak{E}_+ \ni a \mapsto \omega_a = \Phi(a, \cdot) \in \mathfrak{S}_+$$





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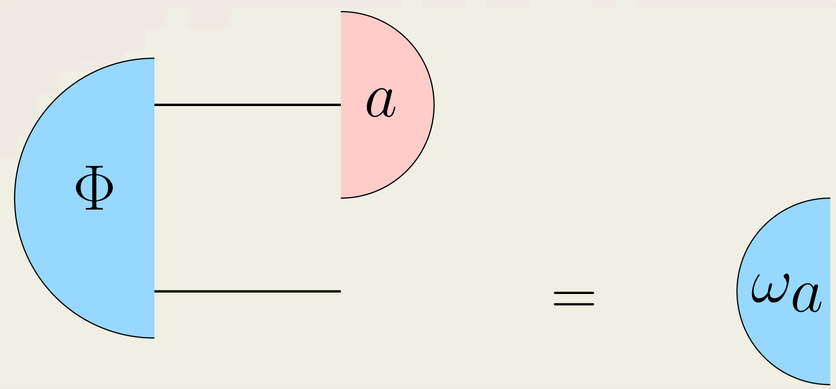

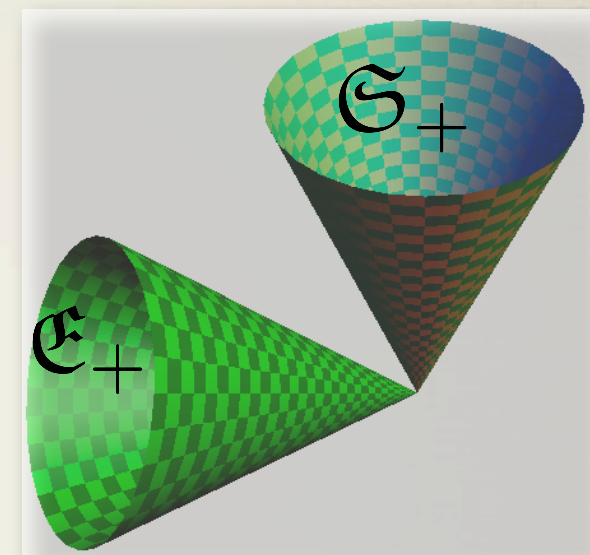

- **Tensor product representation:**

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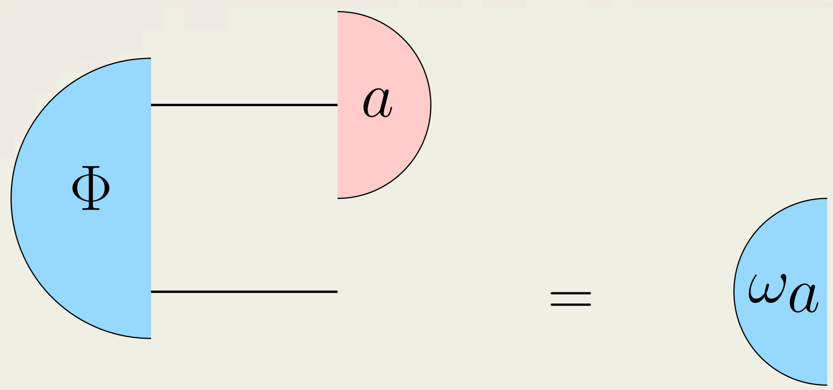

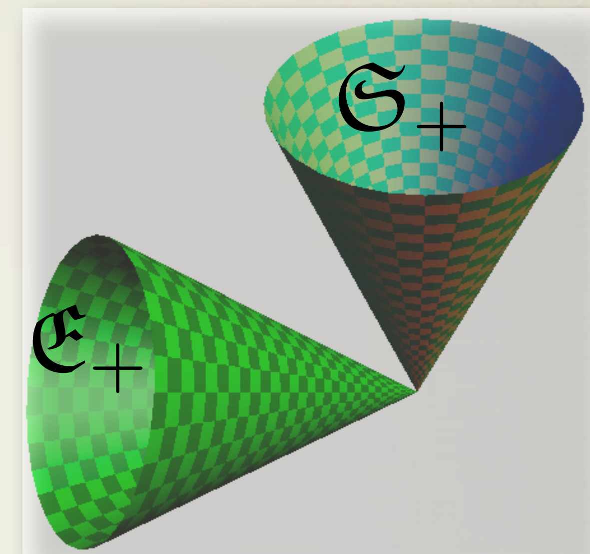

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- **Space of transformations is complete:** $\boxed{\mathfrak{T}_{\mathbb{F}} = \mathsf{Lin}(\mathfrak{E}_{\mathbb{F}})}$

Consequences of PFAITH

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The faithful state Φ provides a non-degenerate scalar product over effects via its Jordan form (ς Jordan involution):

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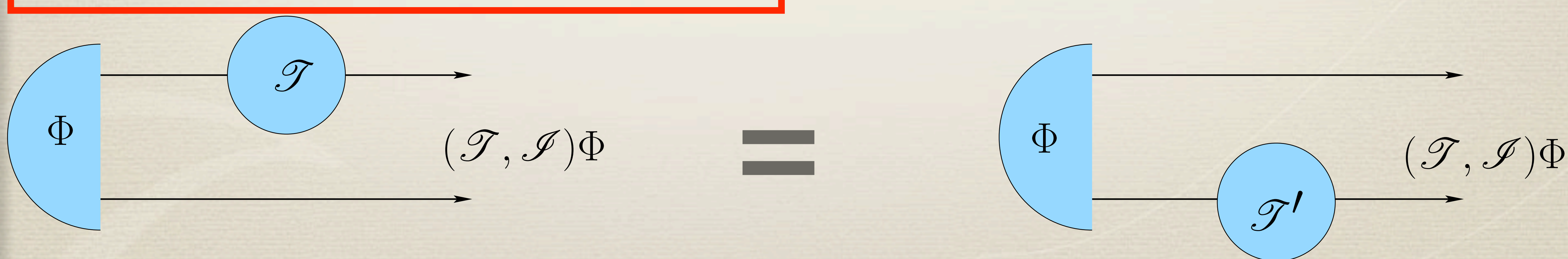
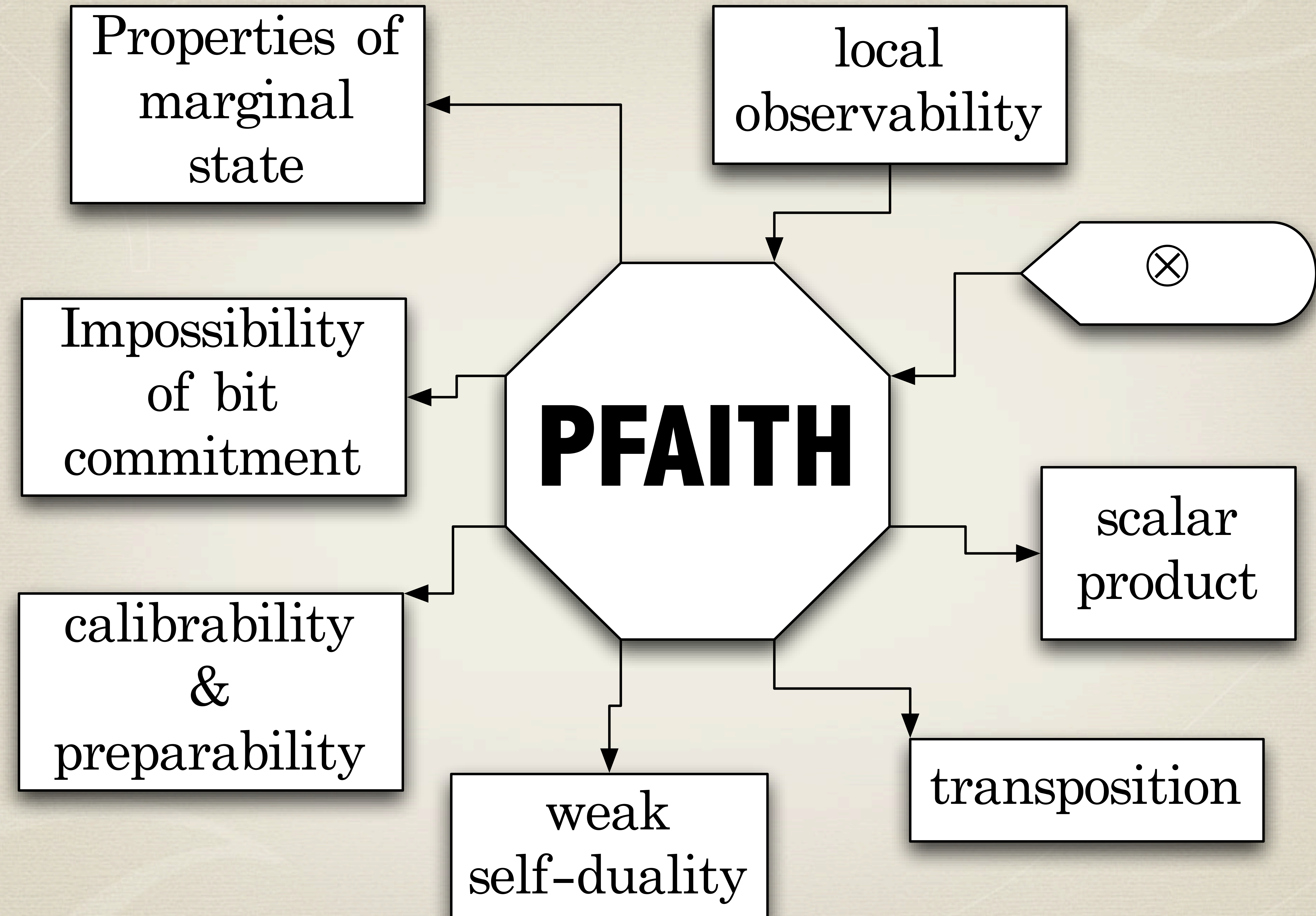
It allows to introduce an operational notion of transposition for transformations:

$$\boxed{(\mathscr{T},\mathscr{I})\Phi = (\mathscr{I},\mathscr{T}')\Phi}$$

1. $(\mathscr{A}+\mathscr{B})' = \mathscr{A}'+\mathscr{B}'$
2. $(\mathscr{A}')' = \mathscr{A}$,
3. $(\mathscr{A}\circ\mathscr{B})' = \mathscr{B}'\circ\mathscr{A}'$

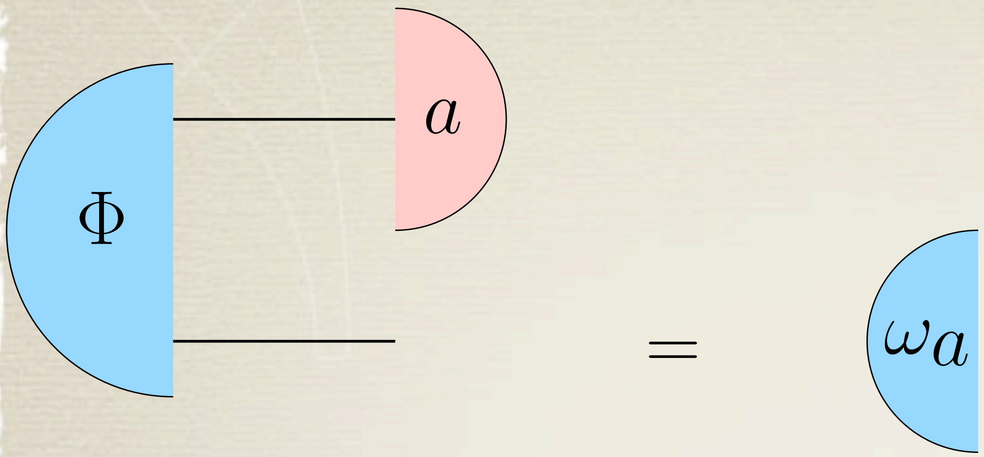



PFAITH
Properties of marginal state
local observability
$\otimes$
Impossibility of bit commitment
scalar product
calibrability & preparability
transposition
weak self-duality

INTERLUDE

Exploring Postulates: **FAITHE** and **PURIFY**

Faithful effect



Remind the cone-isomorphism from the faithful state Φ

$$\boxed{\mathfrak{E}_+ \ni a \mapsto \omega_a = \Phi(a, \cdot) \in \mathfrak{S}_+}$$

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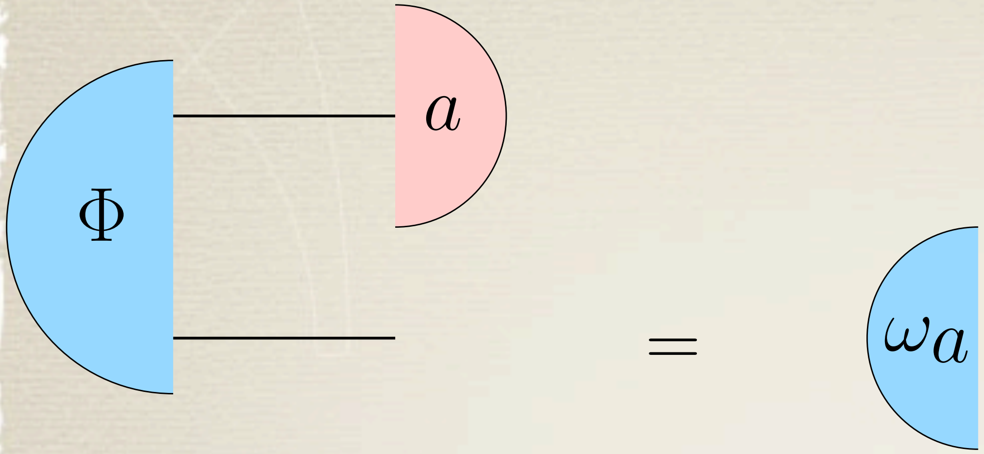
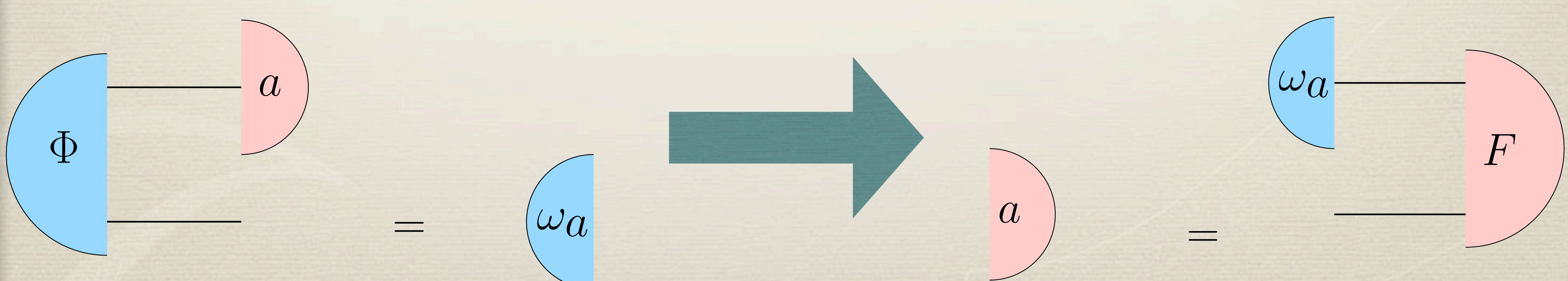


Remind the cone-isomorphism from the faithful state Φ

$$\boxed{\mathfrak{E}_+ \ni a \mapsto \omega_a = \Phi(a, \cdot) \in \mathfrak{S}_+}$$

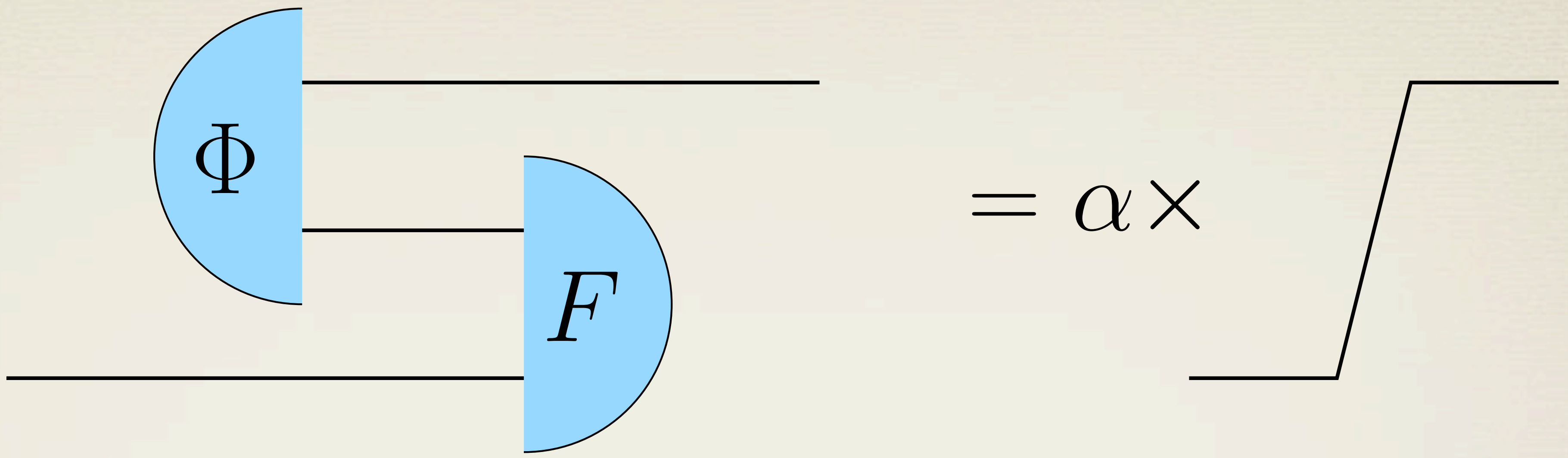
FAITHE: There exist a bipartite effect F achieving the inverse of the isomorphism $a \mapsto \omega_a = \Phi(a, \cdot)$ namely:

$$\boxed{F_{23}(\omega_a)_2 = F_{23}\Phi_{12}(a, \cdot) = \alpha a_3,} \quad 0 < \alpha \leqslant 1$$



Consequences of FAITHE

- Teleportation:



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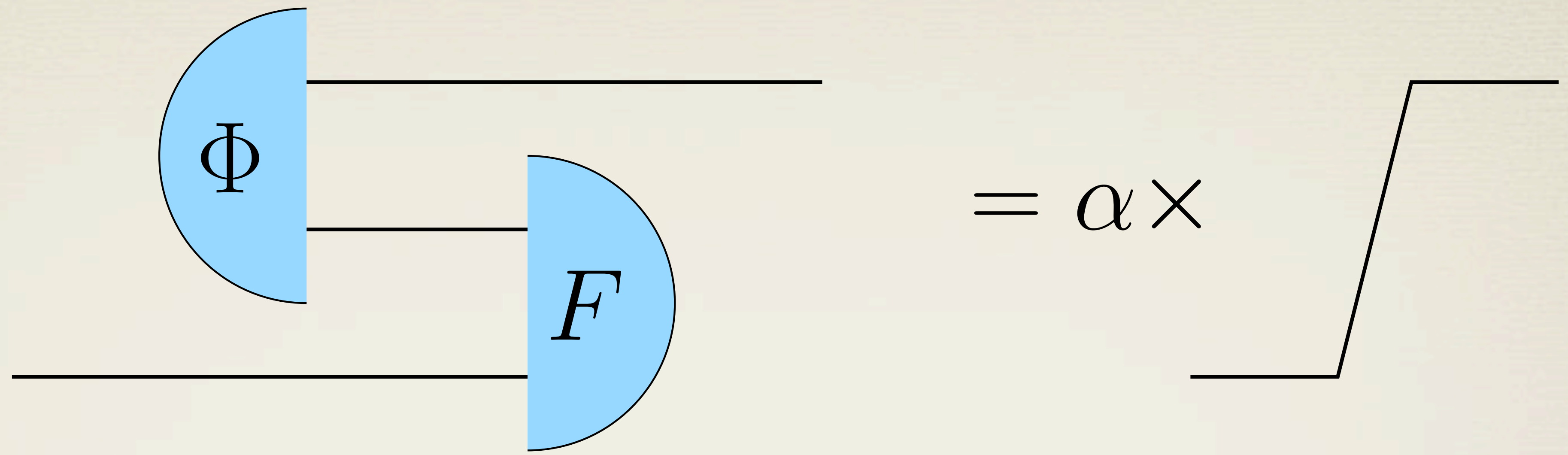


- F is completely faithful, i.e. $F_{\mathscr{A}} := F \circ (\mathscr{I}, \mathscr{A}) \Longleftrightarrow \mathscr{A}$ realizes the cone-isomorphism: $\mathfrak{E}_+(\mathsf{S}^{\odot 2}) \simeq \mathfrak{T}_+(\mathsf{S})$

Consequences of FAITHE

- $\mathfrak{E}_+(\mathsf{S}^{\odot 2}) \ni A \mapsto \Omega_A := A_{23}(\Phi, \Phi) \in \mathfrak{S}_+(\mathsf{S}^{\odot 2})$

is a bijective map between $\mathfrak{S}_{\mathbb{F}}(\mathsf{S}^{\odot 2})$ and $\mathfrak{E}_{\mathbb{F}}(\mathsf{S}^{\odot 2})$

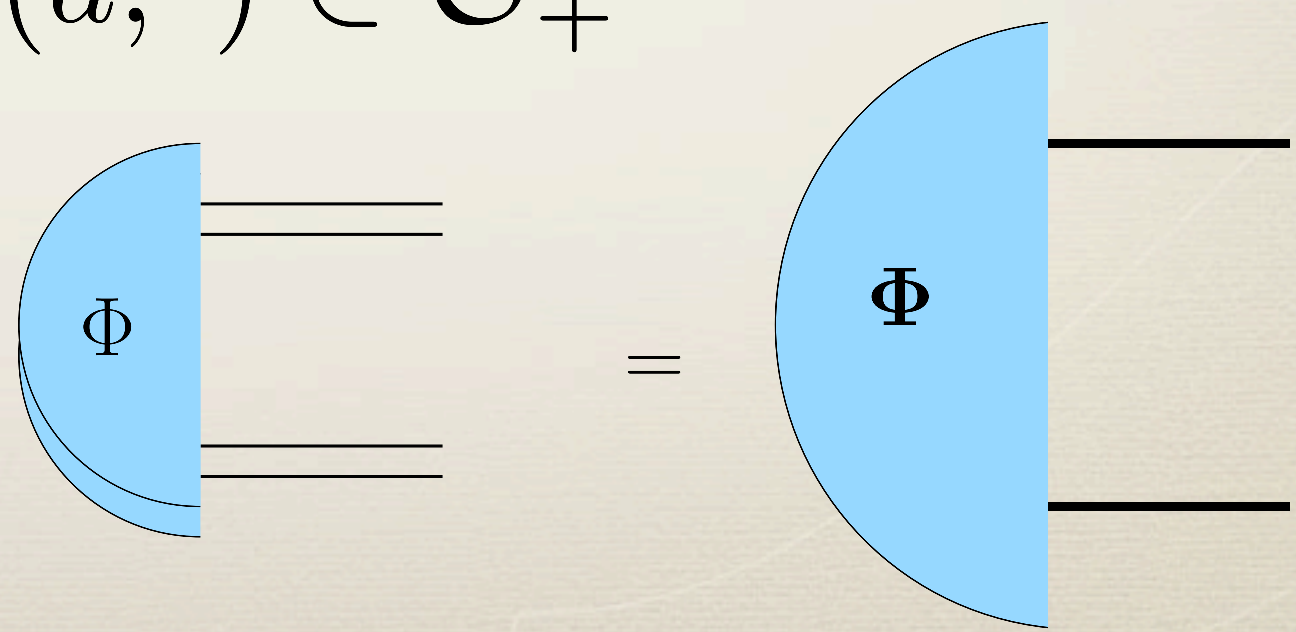
however, it does not necessarily realize the cone-isomorphism:

$$\mathfrak{S}_+(\mathsf{S}^{\odot 2}) \simeq \mathfrak{E}_+(\mathsf{S}^{\odot 2})$$

namely it is not the 4-partite equivalent of:

$$\mathfrak{E}_+ \ni a \mapsto \omega_a = \Phi(a, \cdot) \in \mathfrak{S}_+$$

i.e. we don't have:



Consequences of FAITHE

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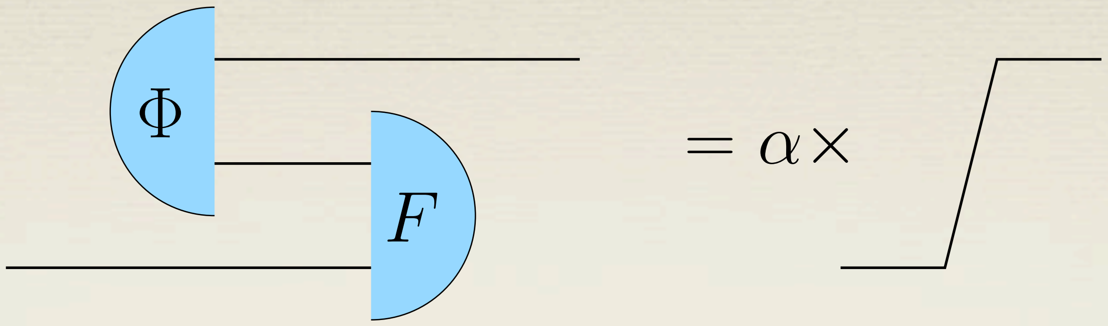

Consequences of FAITHE

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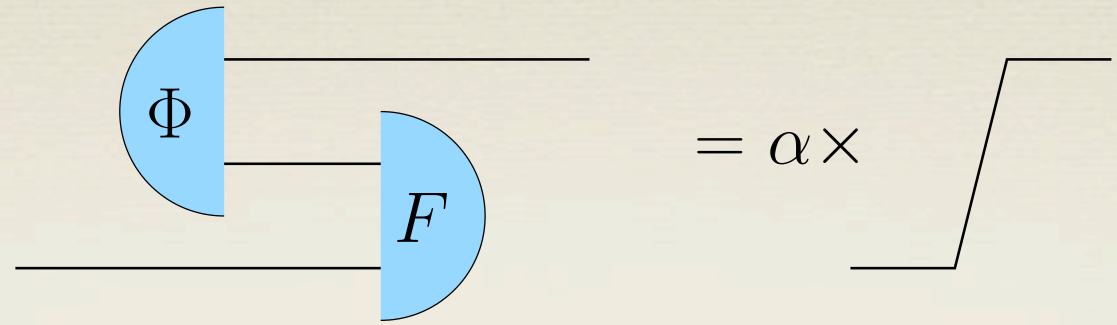

$$\alpha(\mathsf{S}) = \max_{E \in \mathfrak{E}(\mathsf{S}^{\odot 2})} \{(\Phi, \Phi)(e, E, e)\}$$

is a property of the system and depends on the particular probabilistic theory

Consequences of FAITHE

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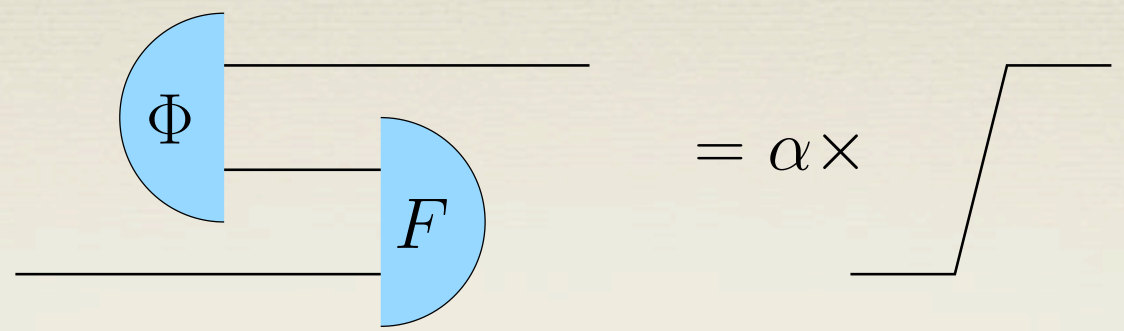

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is a property of the system and depends on the particular probabilistic theory

In Quantum Mechanics: $\alpha = \dim(\mathsf{H})^{-2}$

$$\boxed{\begin{gathered} \omega_a = \sqrt{\alpha}\varsigma(a) \\ (\cdot, F)(\Phi, \cdot) = \sqrt{\alpha}|\Phi| \end{gathered}}$$

Consequences of PURIFY

PURIFY: Every state has a purification on two identical systems.

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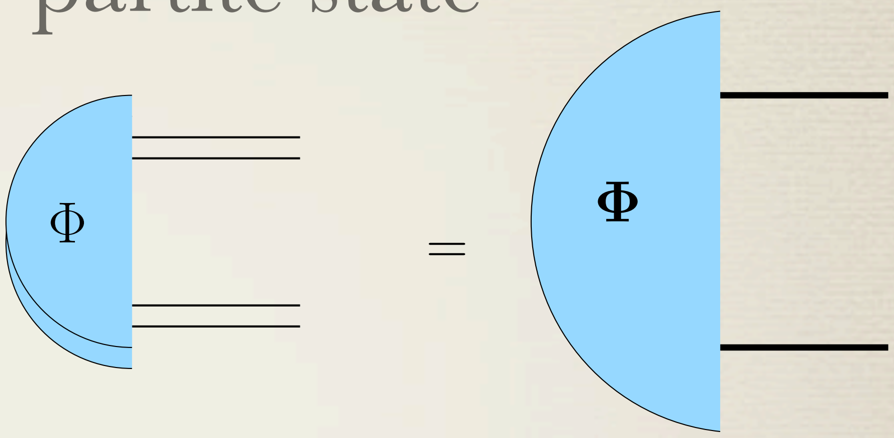
PURIFY: Every state has a purification on two identical systems.

- Each state can be obtained by applying an atomic transformation to the marginal state $\chi = \Phi(e, \cdot)$
- Each effect contains an atomic transformation.
- $\mathscr{I}$ is atomic.
- Φ is pure.

SUPER-PFAITH

SUPERFAITH: There exists a symmetric bipartite state such that for any number N of systems the $2N$-partite state

$$\mathbf{\Phi}_{134\ldots246\ldots} := \Phi_{12}\Phi_{34}\Phi_{56}\cdots$$



is preparationally faithful for $\mathsf{S}^{\odot N}$

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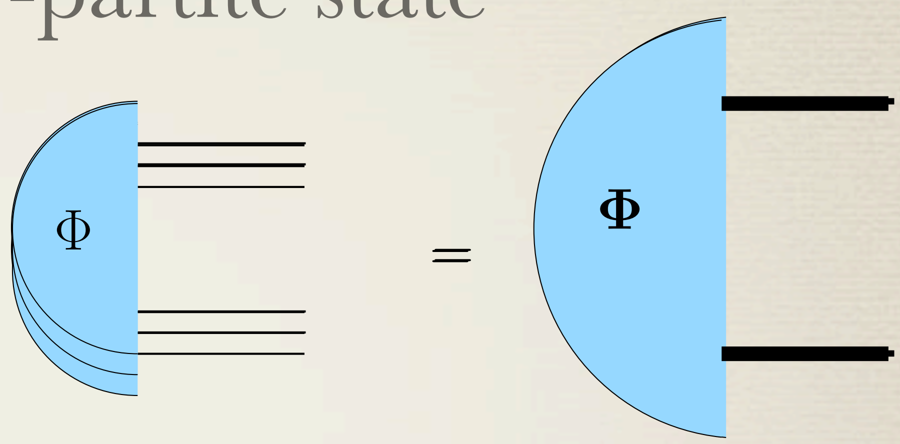

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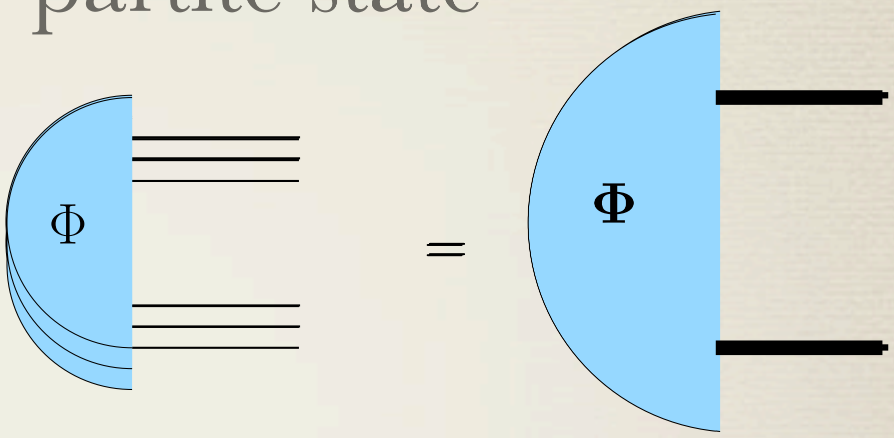

+PURIFY (with purifications connected by local automorphisms) ⟶ **FAITHE**+ Stinespring:

with G. Chiribella and P. Perinotti

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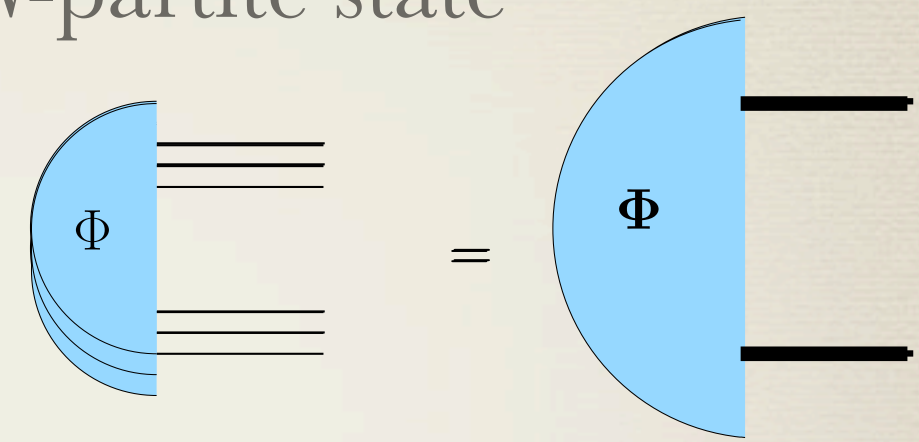
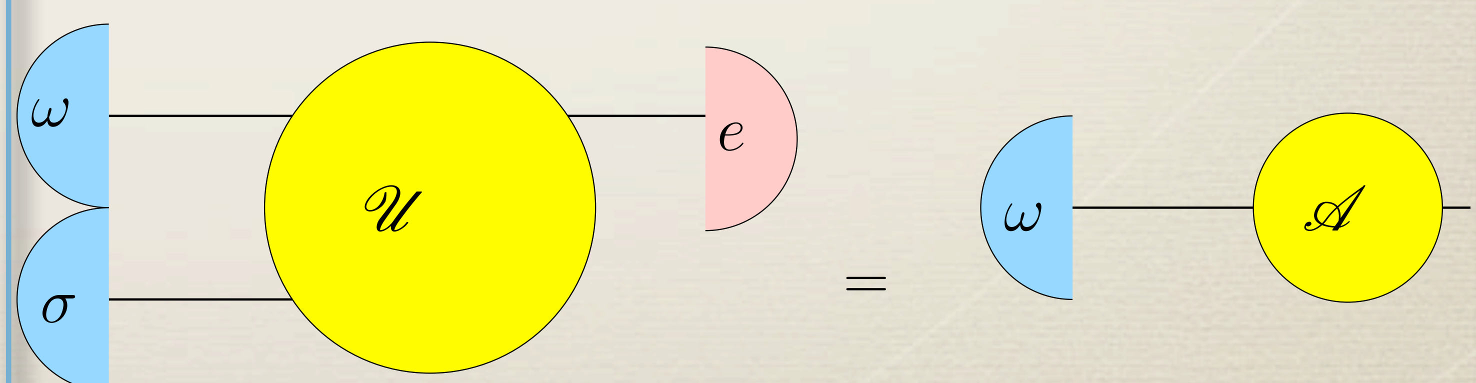


+PURIFY (with purifications connected by local automorphisms) ⟶ **FAITHE**+ Stinespring:

$$\mathscr{U}_{12}(\sigma,\omega)(e,\cdot) = \mathscr{A}\,\omega$$



with G. Chiribella and P. Perinotti

WHAT IS THE *SOMETHING MORE*?

WHAT IS THE *SOMETHING MORE*?

PFAITH
+FAITHE+PURIFY

WHAT IS THE *SOMETHING MORE*?

OR

WHAT IS THE *SOMETHING MORE*?

SUPER-PFAITH +PURIFY?

WHAT IS THE *SOMETHING MORE*?

In any case, it must give that:

effects make a C-algebra*

RECONSTRUCTING QM FROM PROBABILITIES

Algebra of effects ↔ Choi-Jamiolkowski isomorphism
+ atomicity of evolution

Quantum Tomography for Measuring Experimentally the Matrix Elements of an Arbitrary Quantum Operation

G. M. D'Ariano and P. Lo Presti

at our disposal a general method for experimentally determining the quantum operation matrix, using any available quantum-tomographic scheme for the system in consideration, and a single fixed state at the input, which is an entangled (not even maximally) state. In the optical domain we show that one can achieve the tomographic reconstruction of the operation using exactly the same apparatus of the recently performed experiment of Ref. [9].

Let us consider for simplicity a "pure" quantum operation in the form (5). Given an orthonormal basis $\{|j\rangle\}$ corresponding to some physical observable, how can we determine the matrix $A_{ij} = \langle i|A|j\rangle$ experimentally? Instead of acting with the contraction A on an "isolated" system, we perform the map on a system which is entangled in the state $|\psi\rangle\rangle \in \mathcal{H} \otimes \mathcal{H}$ with an identical system; i.e.,

$$|\psi\rangle\rangle \to |\phi\rangle\rangle = \frac{A \otimes I|\psi\rangle\rangle}{\|A\psi\|_{HS}}. \tag{6}$$

With the double ket we denote bipartite vectors $|\psi\rangle\rangle \in \mathcal{H} \otimes \mathcal{H}$, which, keeping the basis $\{|j\rangle\}$ as fixed, are in one-to-one correspondence with matrices as follows:

$$|\psi\rangle\rangle = \sum_{ij} \psi_{ij}|i\rangle \otimes |j\rangle. \tag{7}$$

$$A_{ij} = \kappa\langle E_{ij}(\psi)\rangle, \tag{10}$$

where the operator $E_{ij}(\psi)$ is given by

$$E_{ij}(\psi) = |i_0\rangle\langle i| \otimes |j_0\rangle\langle\psi^{-1*}(j)|, \tag{11}$$

and the proportionality constant is given by

$$\kappa = e^{i\theta}\sqrt{\frac{p_A(\psi)}{\langle|i_0, j_0\rangle\rangle\langle\langle i_0, j_0|\rangle}}. \tag{12}$$

Since A_{ij} is written only in terms of output ensemble averages, it can be estimated through quantum tomography. Quantum tomography [10,11] is a method to estimate the ensemble average $\langle H\rangle$ of any arbitrary operator H on $\mathcal{H}$ by using only measurement outcomes of a *quorum* of observables $\{O(l)\}$. A *quorum* is just a set of operators $\{O(l)\}$ which are observable (i.e., have orthonormal resolution) and span the linear space of operators on $\mathcal{H}$. This means that any operator H can be expanded as $H = \sum_l \mathrm{Tr}[Q^\dagger(l)H]O(l)$, where $\{Q(l)\}$ and $\{O(l)\}$ form a biorthogonal set such that $\mathrm{Tr}[Q^\dagger(i)O(j)] = \delta_{ij}$. Hence, the tomographic estimation of the ensemble average $\langle H\rangle$ is obtained as the double average—over both the ensemble and the quorum—of the unbiased

DIFFICULT PROBLEM:
HOWTO INTRODUCE COMPOSITION OF EFFECTS?

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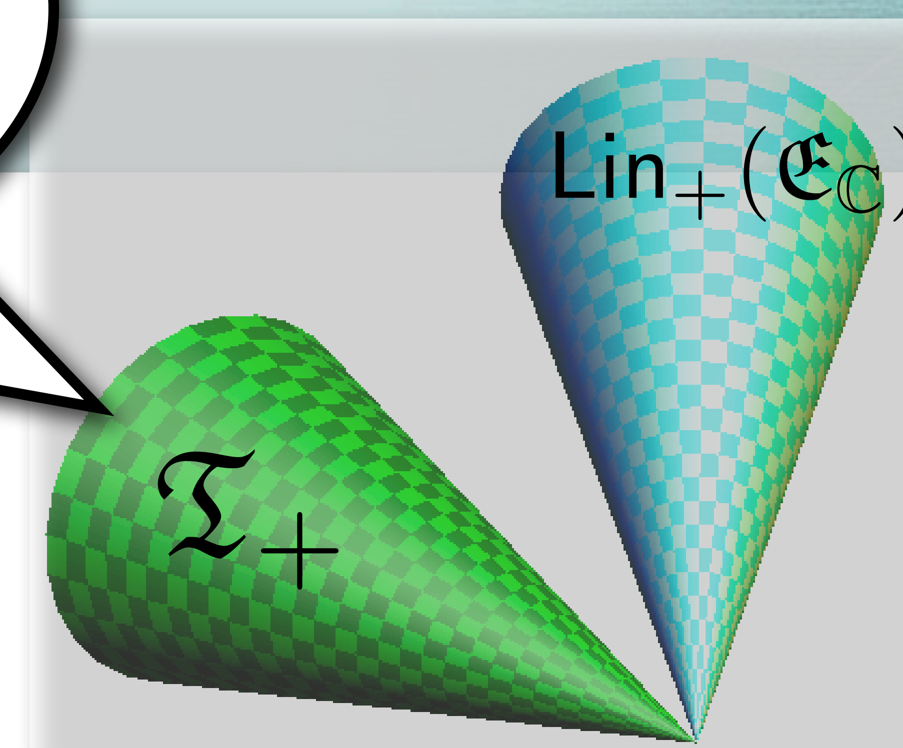
Equivalent to:
Choi-Jamiolkowski isomorphism

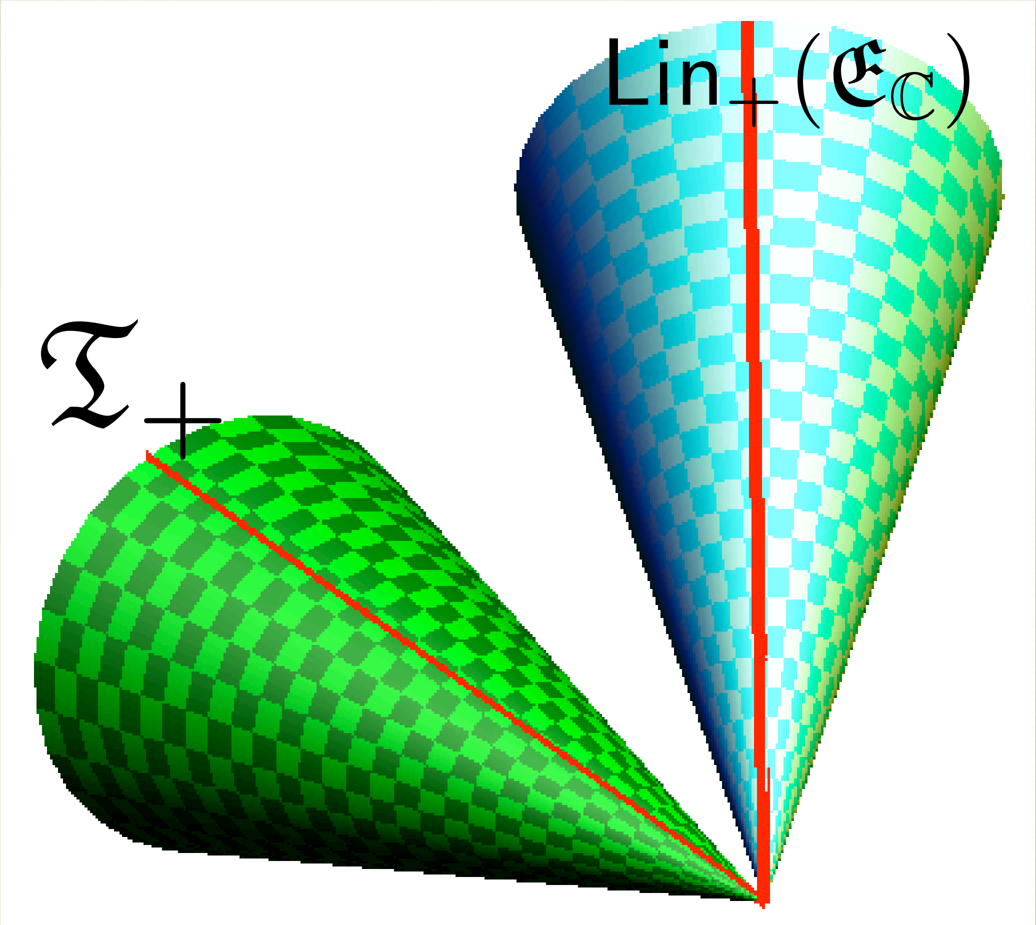
$$\boxed{\mathbf{I}:\mathfrak{T}_+ \simeq \mathsf{Lin}_+(\mathfrak{E}_{\mathbb{C}})}$$



CJ Isomorphism ⇒ composition of effects

Effects are identified with "atomic" events
(apart from a phase) i.e. events that cannot be written as sum of other events

AE (Atomicity of evolution):
the composition of "atomic" events is atomic



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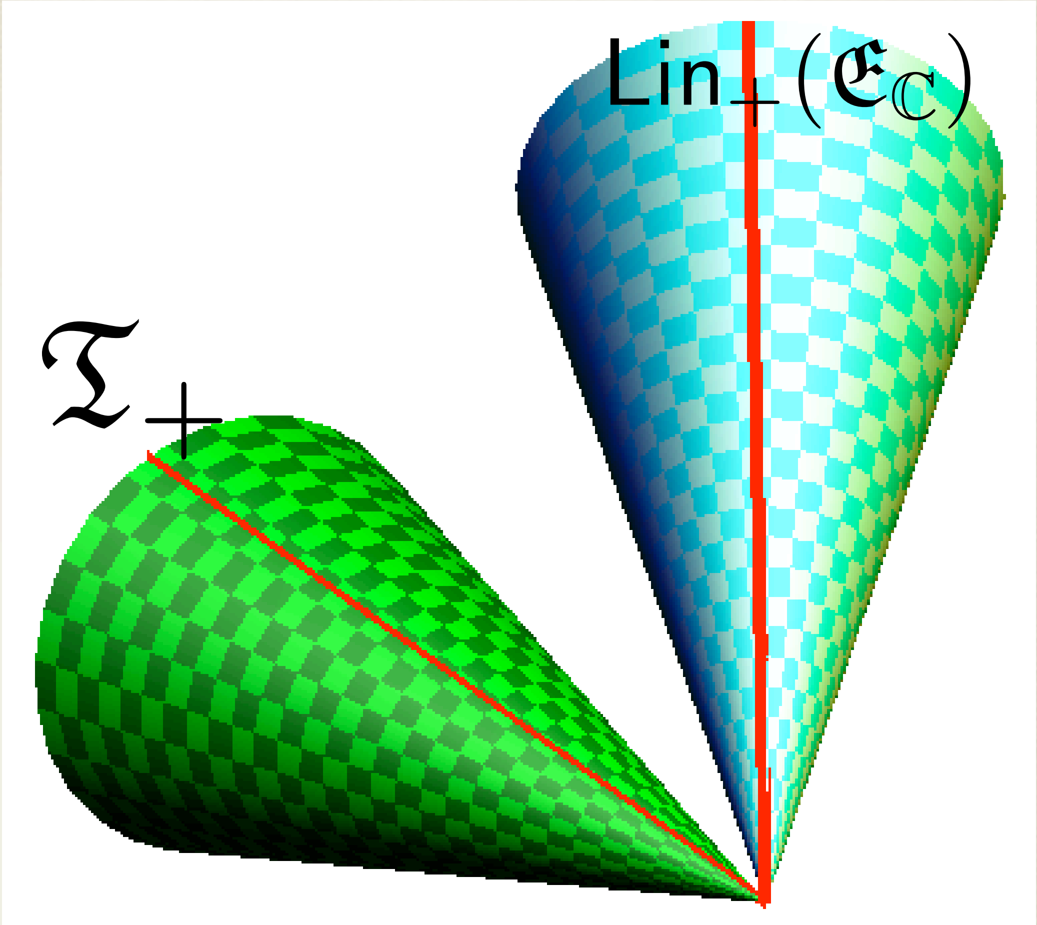

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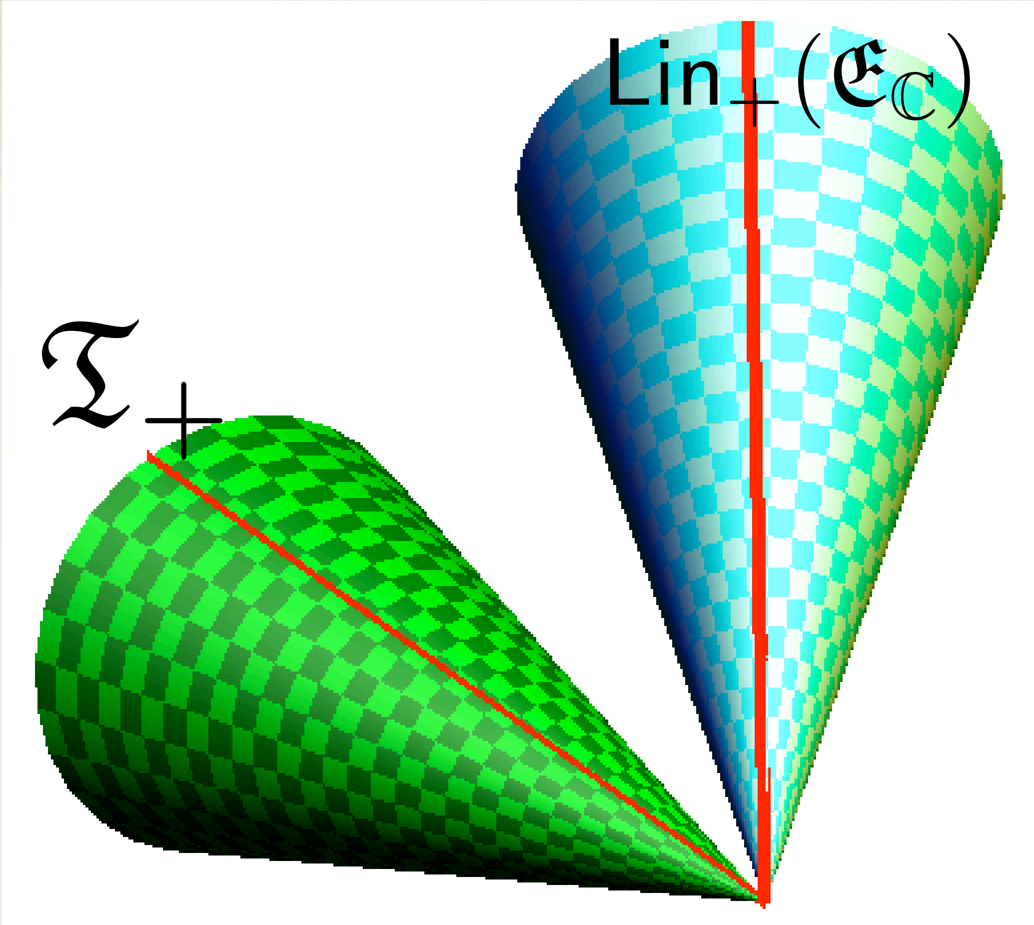

One can prove that the phase (two-cocycle) is trivial.
Introduce the generalized transformation via the polarization identity:

$$\mathscr{T}_{a,b} := \frac{1}{4}\sum_{k=0}^{3} i^k \mathscr{T}_{a+i^k b}$$

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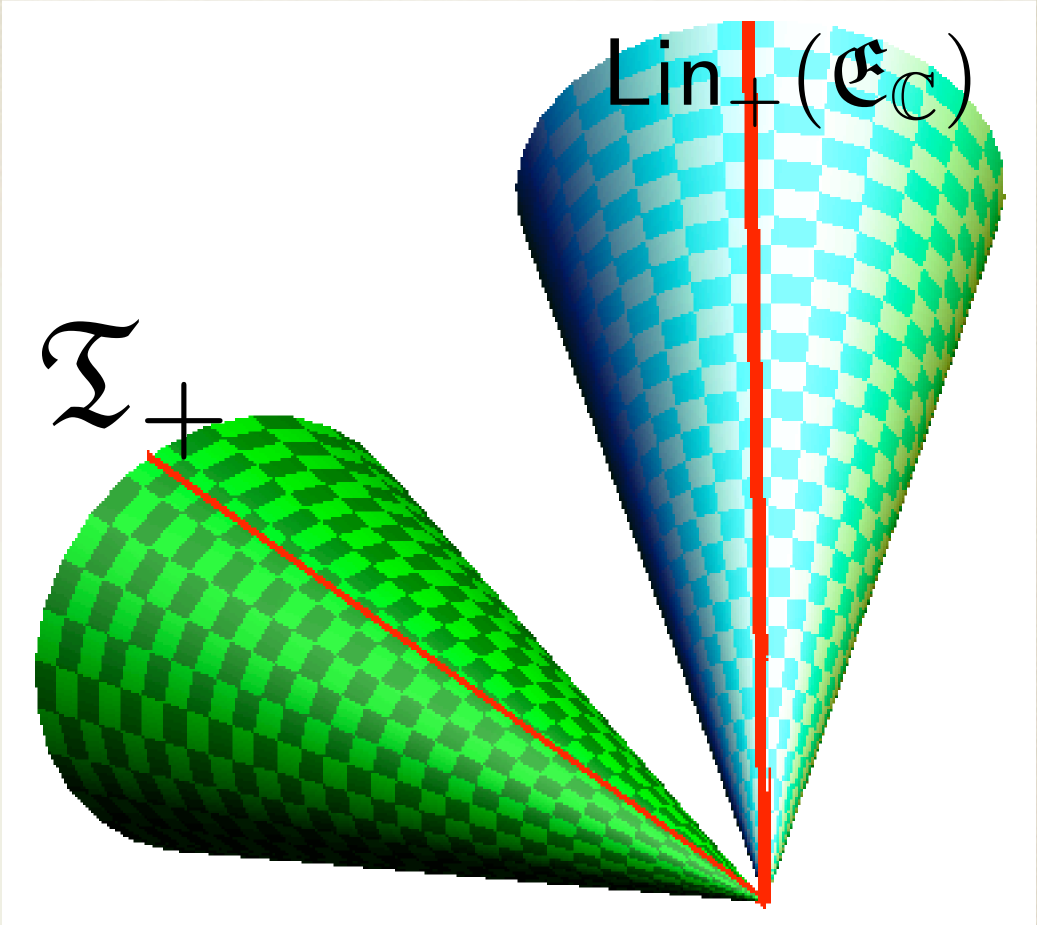

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compositon of effects as: $\quad ab = e \circ \mathscr{T}_{e,a} \circ \mathscr{T}_{e,b}$

SUMMARY

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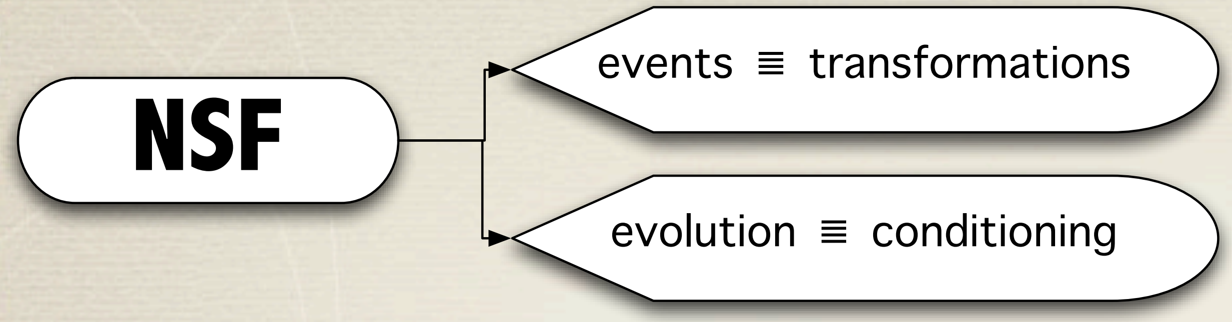

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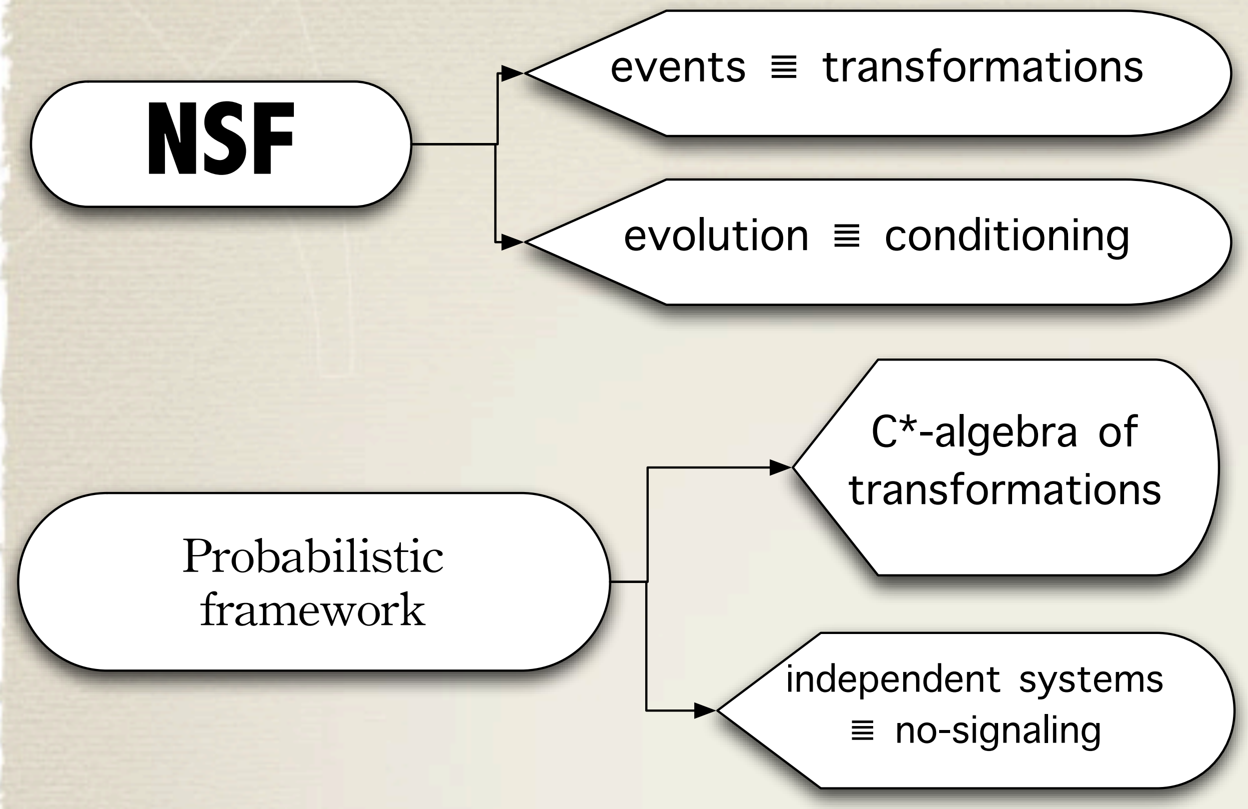
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evolution ≡ conditioning
Probabilistic framework
C*-algebra of transformations
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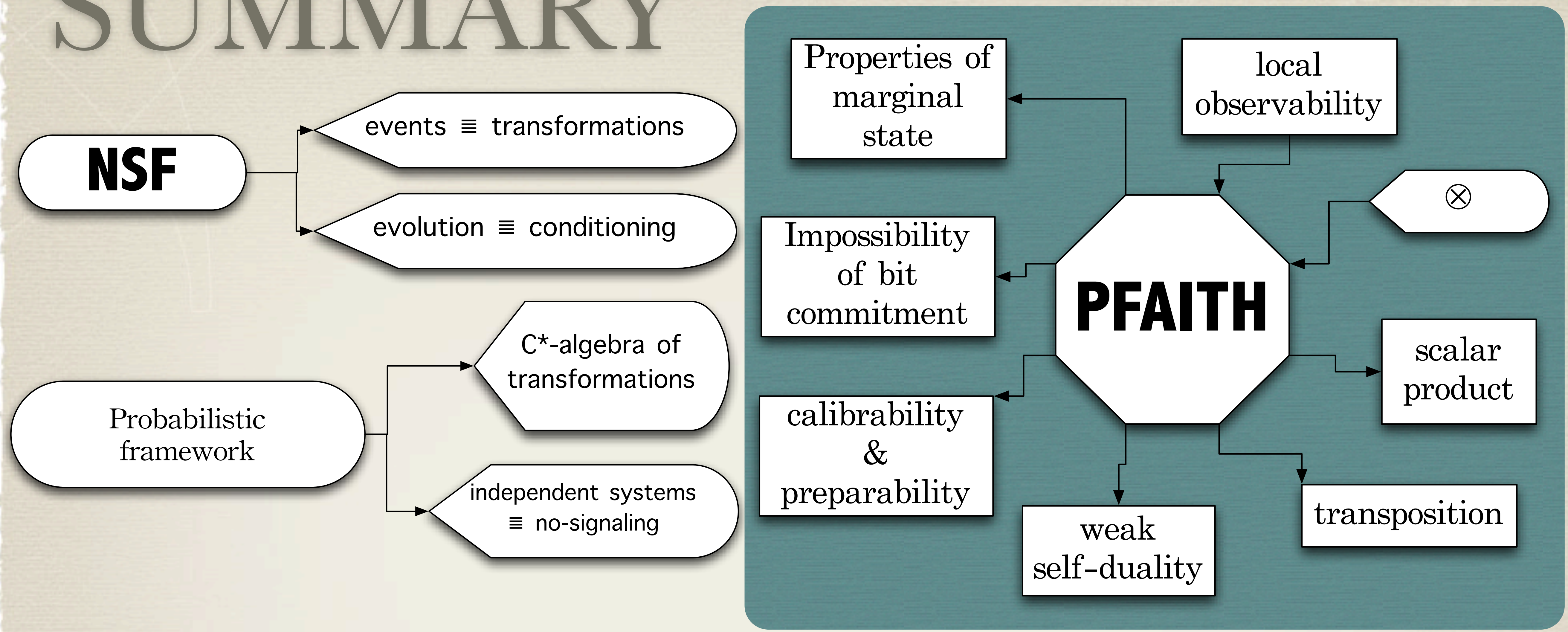
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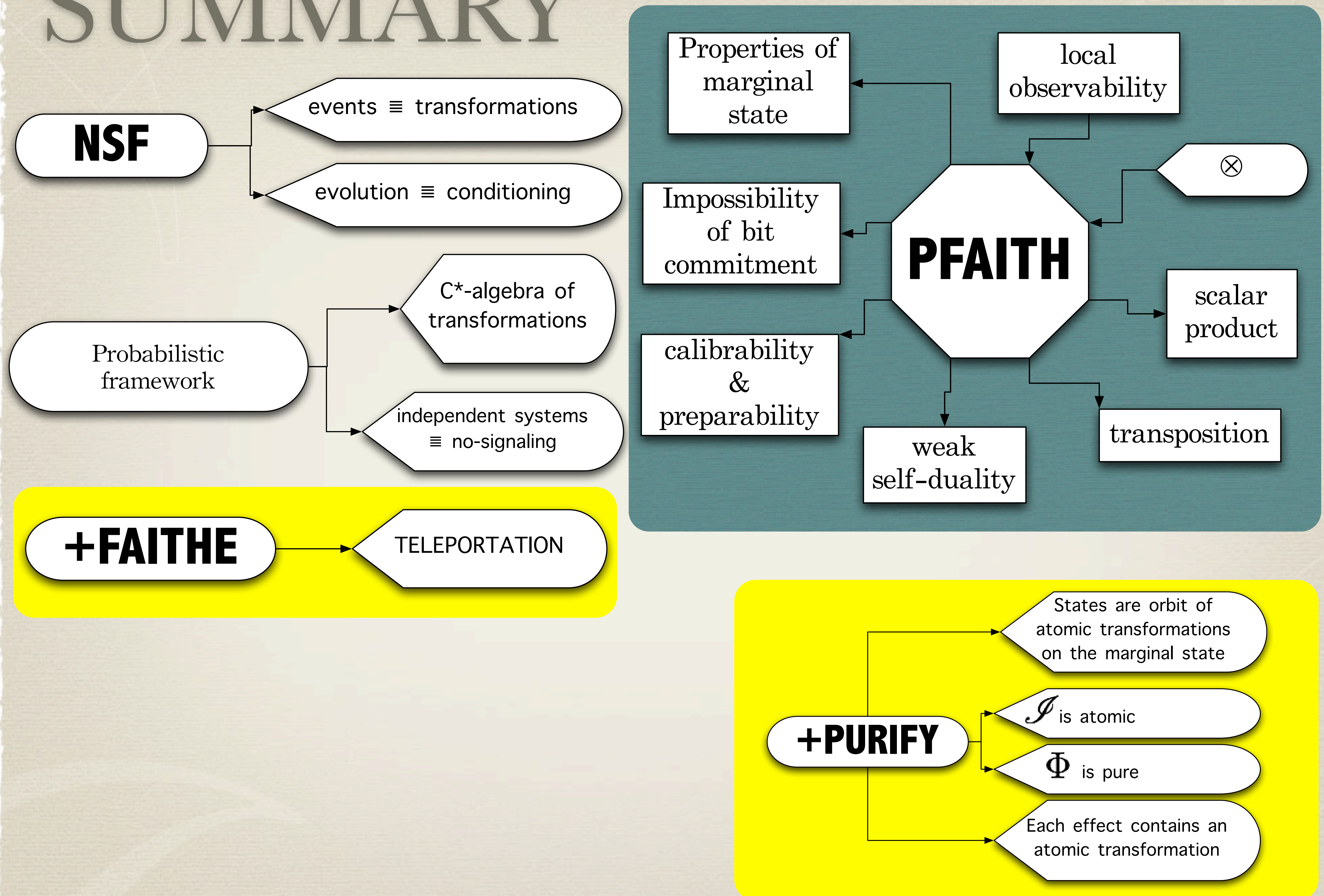
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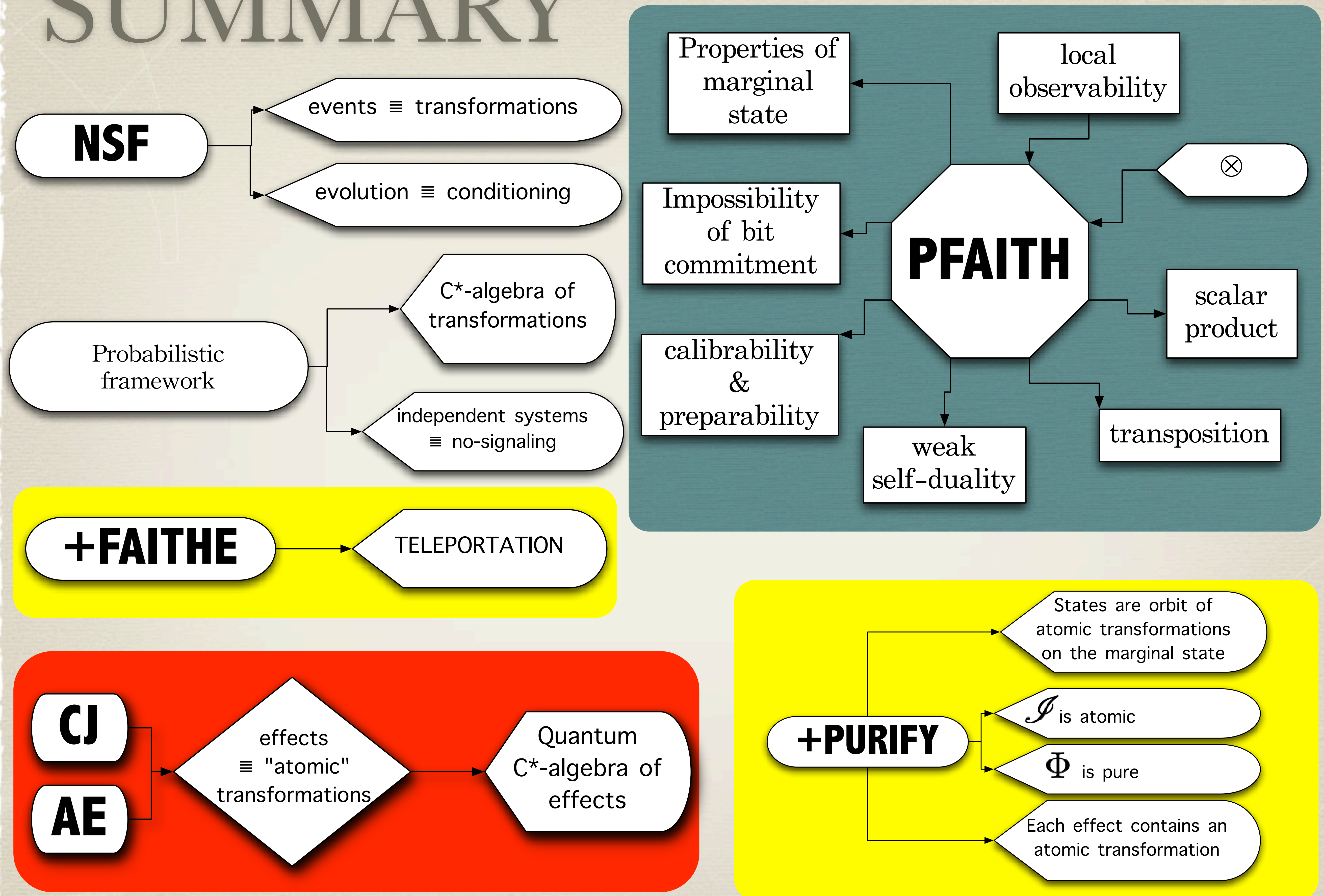

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