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Operational axioms for Quantum Mechanics

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On experimental science

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Since necessarily we work with only partial prior knowledge of both system and experimental apparatus, the rules for the experiment must be given in a probabilistic setting.

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The interaction between object and apparatus produces one of a **set of possible transformations** of the object, each one occurring with some probability.

Postulates

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- **Postulate 1 (Independent systems)** *There exist **independent** systems.*
- **Postulate 2 (Informationally complete observable)** *For each physical system there exists an **informationally complete observable*** (Hardy, Fuchs).
- **Postulate 3 (Local observability principle)** *For every composite system there exist informationally complete observables made only of **local** informationally complete observables.*
- **Postulate 4 (Informationally complete discriminating observable)** *For every system there exists a minimal informationally complete observable that can be achieved using a joint **discriminating observable** on the system + an "ancilla" .*
- **Postulate 5 (Symmetric faithful state)** *For every composite system made of two identical physical systems there exist a **symmetric joint state** that is both **dynamically** and **preparationally faithful**.*

Actions and outcomes

Experiment (or "action"): every experiment is described by a set $\mathbb{A} \equiv \{\mathscr{A}_j\}$ of possible transformations $\mathscr{A}_j$ having overall unit probability, with the apparatus signaling the outcome j labeling which transformation actually occurred.

States

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State: A state ω for a physical system is a rule which provides the probability for any possible transformation within an experiment, namely:

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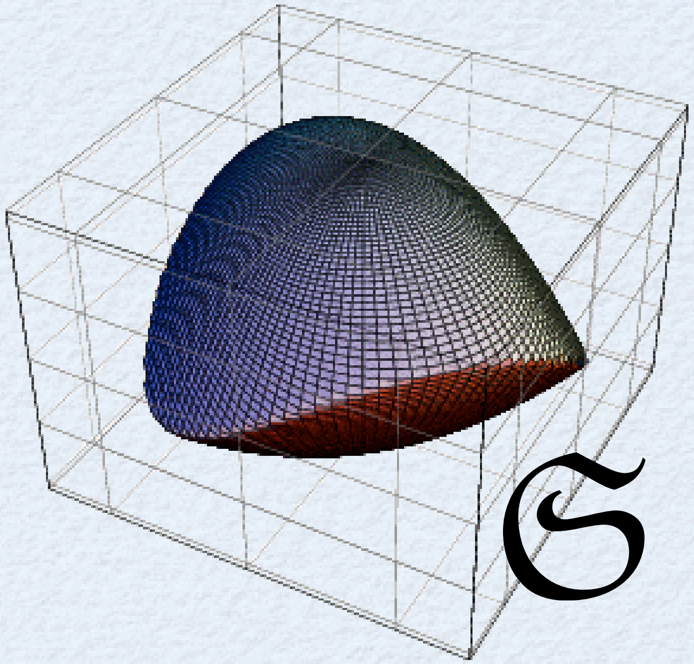
Identity transformation: $$\omega(\mathscr{I}) = 1$$

Convex structure of states

The possible states of a physical system make a convex set $\mathfrak{S}$



$\mathfrak{S}$

ω_1, ω_2 any two states:

$$\omega = \lambda\omega_1 + (1-\lambda)\omega_2, \qquad 0 \le \lambda \le 1,$$

corresponding to the probability rule

$$\omega(\mathscr{A}) = \lambda\omega_1(\mathscr{A}) + (1-\lambda)\omega_2(\mathscr{A})$$

Monoid of transformations

Transformations make a monoid: the composition $\mathscr{A} \circ \mathscr{B}$ of two transformations $\mathscr{A}$ and $\mathscr{B}$ is itself a transformation. Consistency of composition of transformations requires associativity, namely

$$\mathscr{C} \circ (\mathscr{B} \circ \mathscr{A}) = (\mathscr{C} \circ \mathscr{B}) \circ \mathscr{A}$$

There exists the identical transformation $\mathscr{I}$ which leaves the physical system invariant, and which for every transformation $\mathscr{A}$ satisfies the composition rule

$$\mathscr{I} \circ \mathscr{A} = \mathscr{A} \circ \mathscr{I} = \mathscr{A}$$

Independent systems and local transformations

Independent systems and local experiments: two physical systems are "independent" if on each system it is possible to perform "local experiments" for which on every joint state one has the commutativity of the pertaining transformations

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Multipartite system: a collection of independent systems

Local state

For a multipartite system we define the local state $\omega|_n$ of the *n*-th system the state that gives the probability of any local transformation $\mathscr{A}$ on the *n*-th system with all other systems untouched, namely

$$\omega|_n(\mathscr{A}) \doteq \Omega(\mathscr{I}, \ldots, \mathscr{I}, \underbrace{\mathscr{A}}_{nth}, \mathscr{I}, \ldots)$$

Conditional state

When composing two transformations $\mathscr{A}$ and $\mathscr{B}$ the probability that $\mathscr{B}$ occurs conditioned that $\mathscr{A}$ occurred before is given by the **Bayes rule**

$$p(\mathscr{B}|\mathscr{A}) = \frac{\omega(\mathscr{B} \circ \mathscr{A})}{\omega(\mathscr{A})}$$

Conditional state: the conditional state $\omega_{\mathscr{A}}$ gives the probability that a transformation $\mathscr{B}$ occurs on the physical system in the state ω after the transformation $\mathscr{A}$ occurred, namely

$$\boxed{\omega_{\mathscr{A}}(\mathscr{B}) \doteq \frac{\omega(\mathscr{B} \circ \mathscr{A})}{\omega(\mathscr{A})}}$$

Weights and Operations

Weight: un-normalized state $\boxed{\omega = \frac{\tilde{\omega}}{\tilde{\omega}(\mathscr{I})}}$

$$0 \leqslant \tilde{\omega}(\mathscr{A}) \leqslant \tilde{\omega}(\mathscr{I}) < +\infty$$

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Action of a transformation over a state ("Schrödinger picture"):

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$$(\mathscr{A}\,\omega)(\mathscr{B}) := \omega(\mathscr{B} \circ \mathscr{A})$$

Evolution as conditioning

Axioms

Theorems

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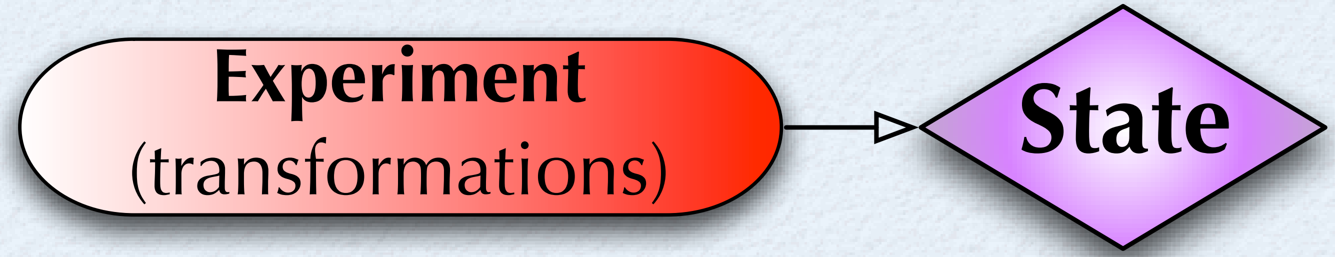

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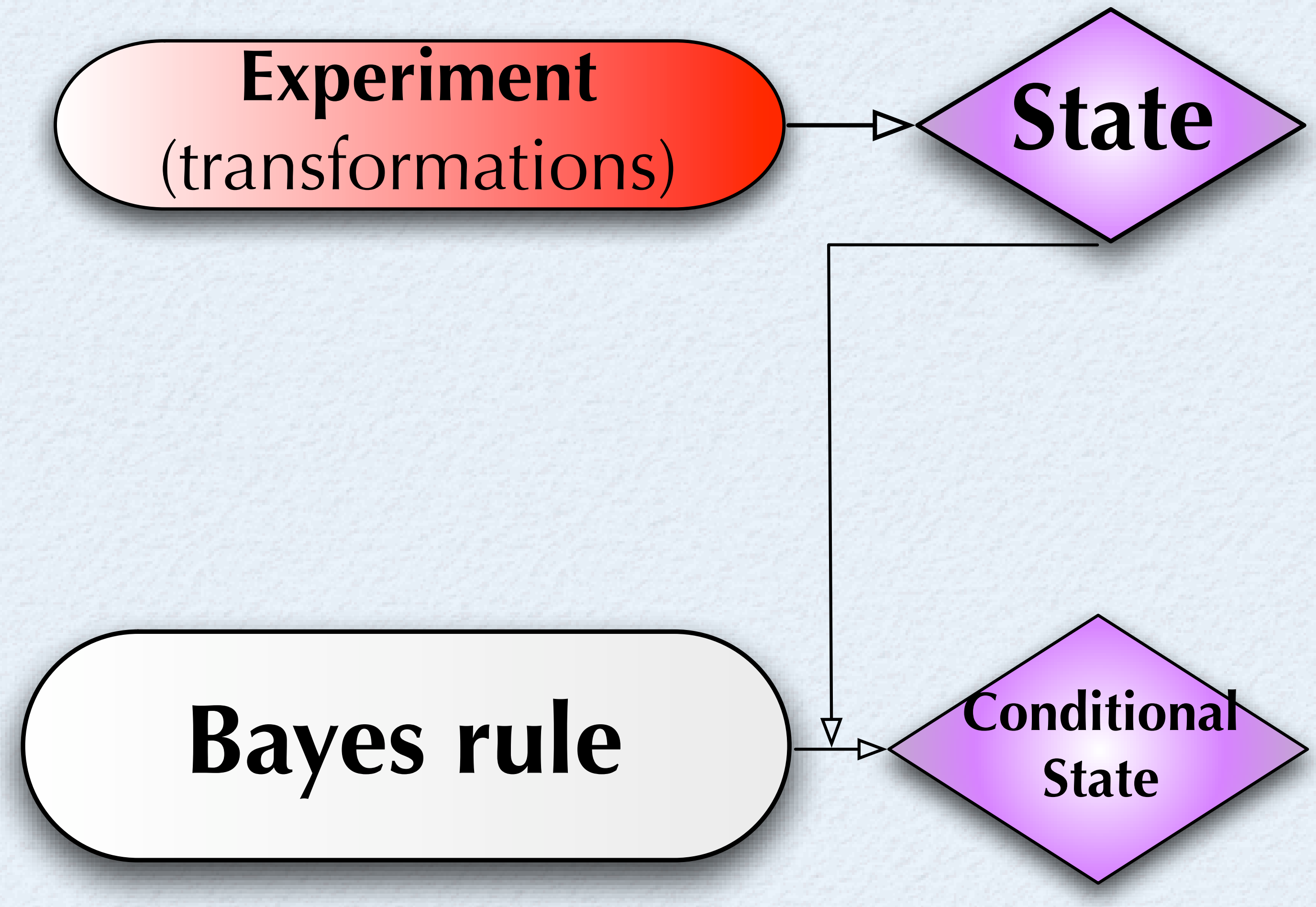

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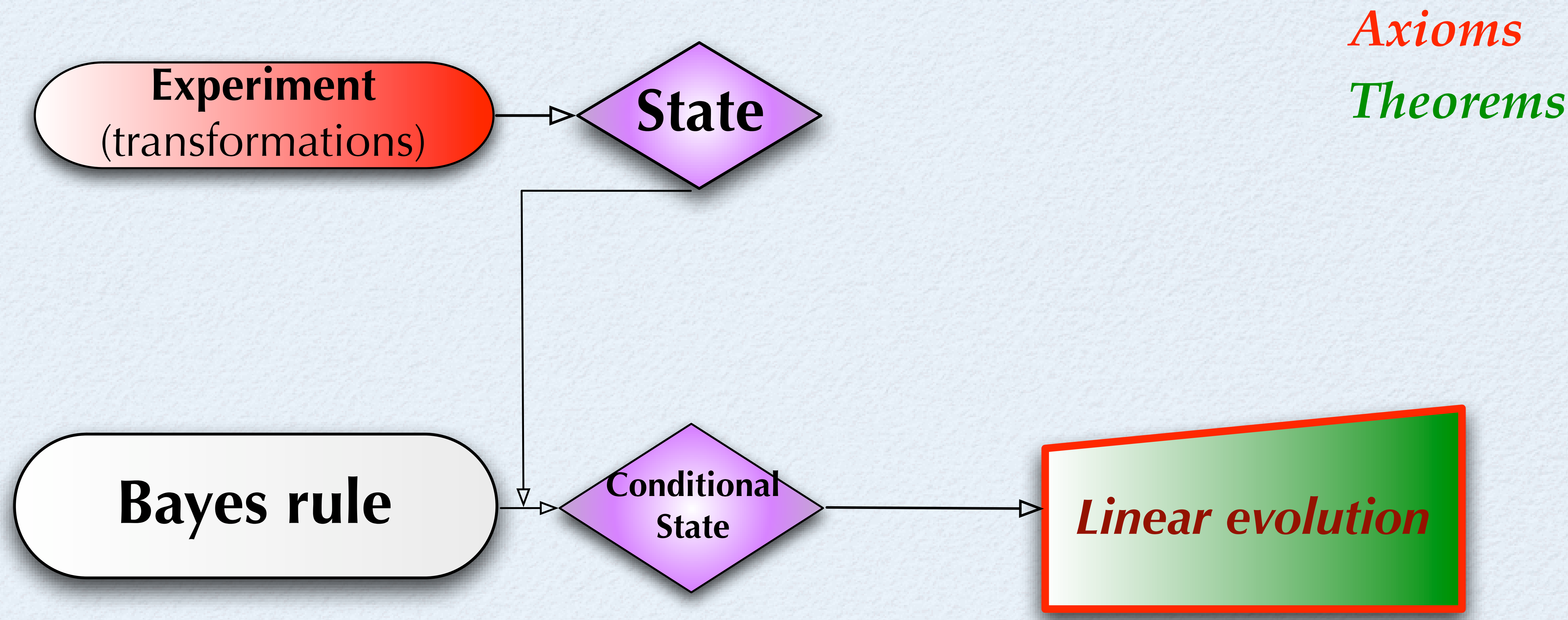

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A transformation is completely specified by the two classes

Addition of transformations

Two transformations $\mathscr{A}$ and $\mathscr{B}$ are ***informationally compatible*** (or coexistent) if for every state ω one has

$$\omega(\mathscr{A}) + \omega(\mathscr{B}) \leq 1$$

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$$\forall \omega \in \mathfrak{S} \qquad \omega(\mathscr{A}_1 + \mathscr{A}_2) = \omega(\mathscr{A}_1) + \omega(\mathscr{A}_2) \qquad \textbf{\textit{(info-class)}}$$

$$\forall \omega \in \mathfrak{S} \qquad \omega_{\mathscr{A}_1 + \mathscr{A}_2} = \frac{\omega(\mathscr{A}_1)}{\omega(\mathscr{A}_1 + \mathscr{A}_2)} \omega_{\mathscr{A}_1} + \frac{\omega(\mathscr{A}_2)}{\omega(\mathscr{A}_1 + \mathscr{A}_2)} \omega_{\mathscr{A}_2}$$

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$$\forall \omega \in \mathfrak{S} \qquad \omega(\mathscr{A}_1 + \mathscr{A}_2) = \omega(\mathscr{A}_1) + \omega(\mathscr{A}_2)$$ ***(info-class)***

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$$\boxed{\circ, + \ \text{distributive}}$$

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The occurrence of the transformation $\mathscr{B}$ on system 1 generally affects the local state on system 2, i. e.

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However a local action $\mathbb{A} \equiv \{\mathscr{A}_j\}$ on system 2 does not affect the local state on system 1, more precisely:

acausality of local actions: any local action on a system is equivalent to the identity transformation on another independent system.

$$\boxed{\mathscr{S}(\mathbb{A}) := \sum_{\mathscr{A}_j \in \mathbb{A}} \mathscr{A}_j}$$

$$\forall \Omega \in \mathfrak{S}^{\times 2}, \forall \mathbb{A}, \qquad \Omega_{\mathscr{S}(\mathbb{A}),\mathscr{I}}|_2 = \Omega|_2$$

No-signaling

Theorem 2 (No signaling, i. e. acausality of local actions) *Any local "action" (i. e. experiment) on a system does not affect another independent system. More precisely, any local action on a system is equivalent to the identity transformation when viewed from another independent system. In equations one has*

$$\forall \Omega \in \mathfrak{S}^{\times 2}, \forall \mathbb{A}, \qquad \Omega_{\mathscr{S}(\mathbb{A}),\mathscr{I}}|_2 = \Omega|_2. \tag{25}$$

Proof. By definition, for $\mathscr{B} \in \mathfrak{T}$ one has $\Omega|_2(\mathscr{B}) = \Omega(\mathscr{I},\mathscr{B})$, and using Eq. (24) according to Rule 4 one has

$$\Omega(\mathscr{S}(\mathbb{A}),\mathscr{B}) = \sum_{\mathscr{A}_j \in \mathbb{A}} \Omega(\mathscr{A}_j,\mathscr{B}) = \Omega(\mathscr{I},\mathscr{B}) =: \Omega|_2(\mathscr{B}). \tag{26}$$

On the other hand, we have

$$\Omega_{\mathscr{S}(\mathbb{A}),\mathscr{I}}|_2(\mathscr{B}) = \Omega((\mathscr{I},\mathscr{B})\circ(\mathscr{S}(\mathbb{A}),\mathscr{I}) = \Omega(\mathscr{S}(\mathbb{A}),\mathscr{B}), \tag{27}$$

namely the statement. ■

Notice the consistency with Rule 4:

$$\begin{aligned}\Omega_{\mathscr{S}(\mathbb{A}),\mathscr{I}}|_2(\mathscr{B}) =&\Omega_{\mathscr{S}(\mathbb{A}),\mathscr{I}}(\mathscr{I},\mathscr{B}) = \sum_{\mathscr{A}_j \in \mathbb{A}} \Omega_{\mathscr{A}_j,\mathscr{I}}(\mathscr{I},\mathscr{B}) \frac{\Omega(\mathscr{A}_j,\mathscr{I})}{\sum_{\mathscr{A}_j \in \mathbb{A}} \Omega(\mathscr{A}_j,\mathscr{I})} \\ &= \sum_{\mathscr{A}_j \in \mathbb{A}} \frac{\Omega(\mathscr{A}_j,\mathscr{B})}{\Omega(\mathscr{A}_j,\mathscr{I})} \frac{\Omega(\mathscr{A}_j,\mathscr{I})}{\Omega(\mathscr{I},\mathscr{I})} = \sum_{\mathscr{A}_j \in \mathbb{A}} \Omega(\mathscr{A}_j,\mathscr{B}) = \Omega(\mathscr{I},\mathscr{B}).\end{aligned} \tag{28}$$

No-signaling from dynamical independence

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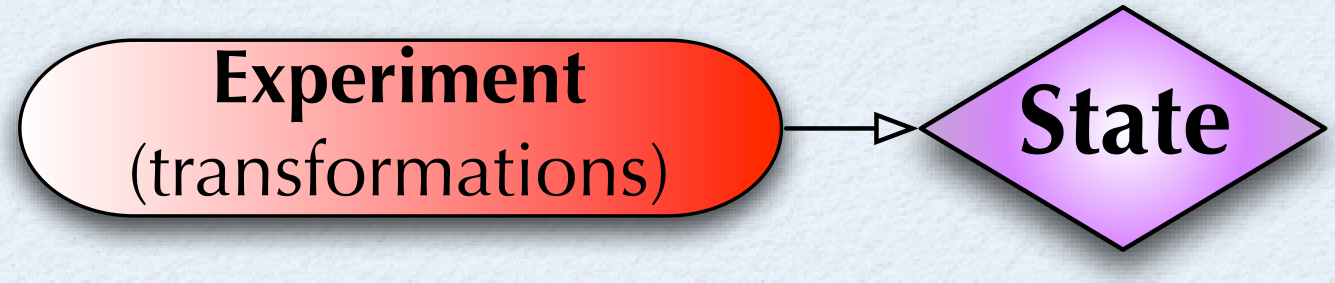
No-signaling from dynamical independence



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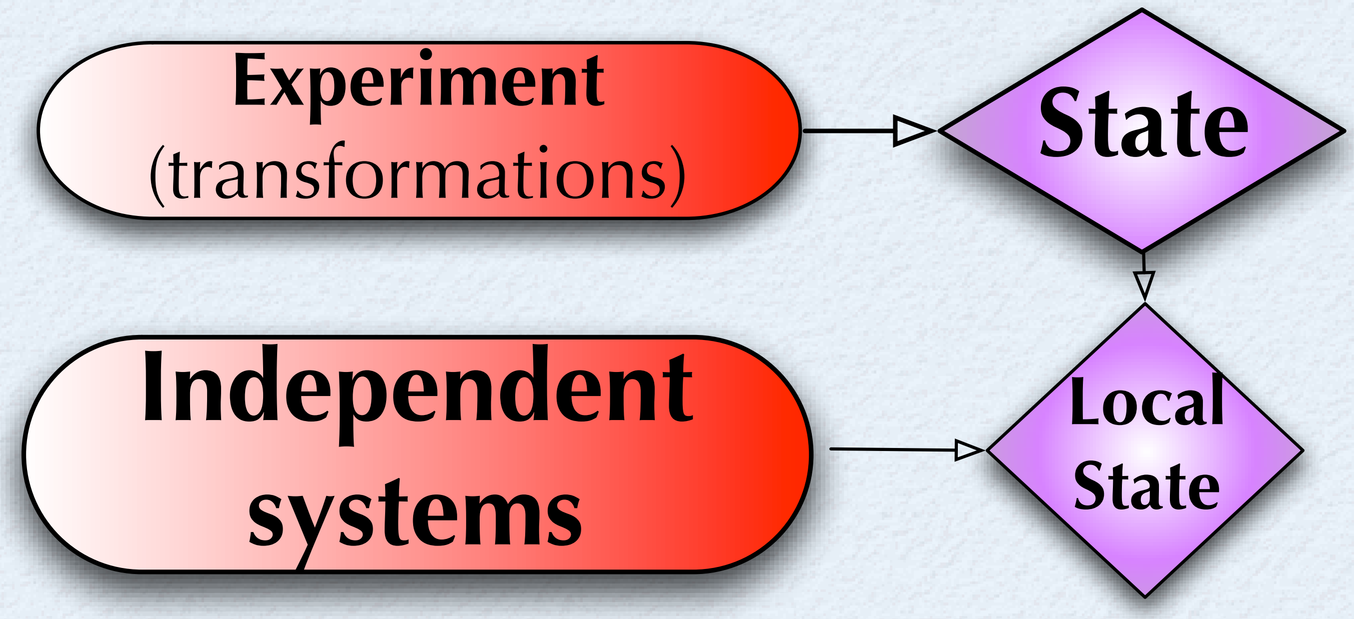

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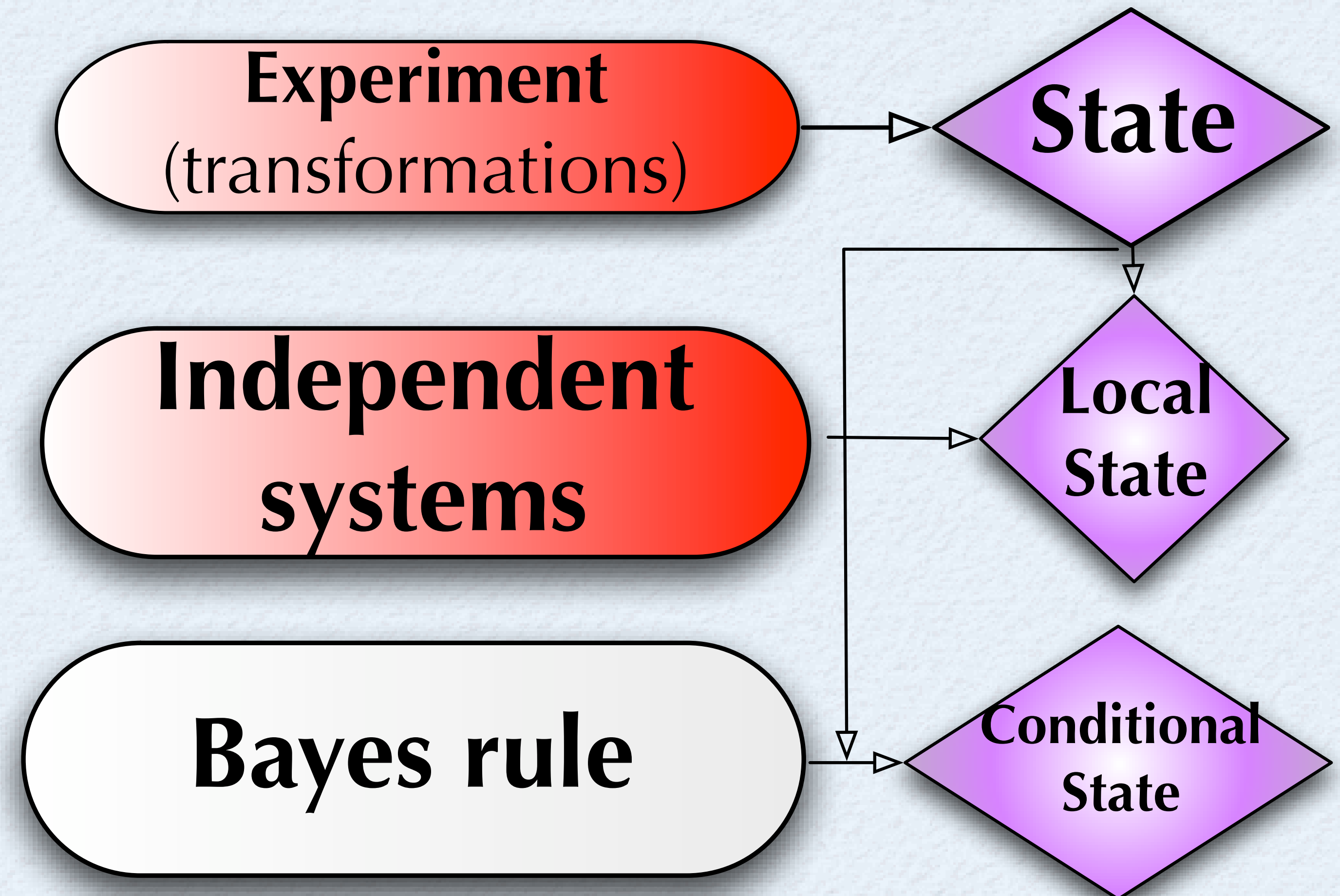

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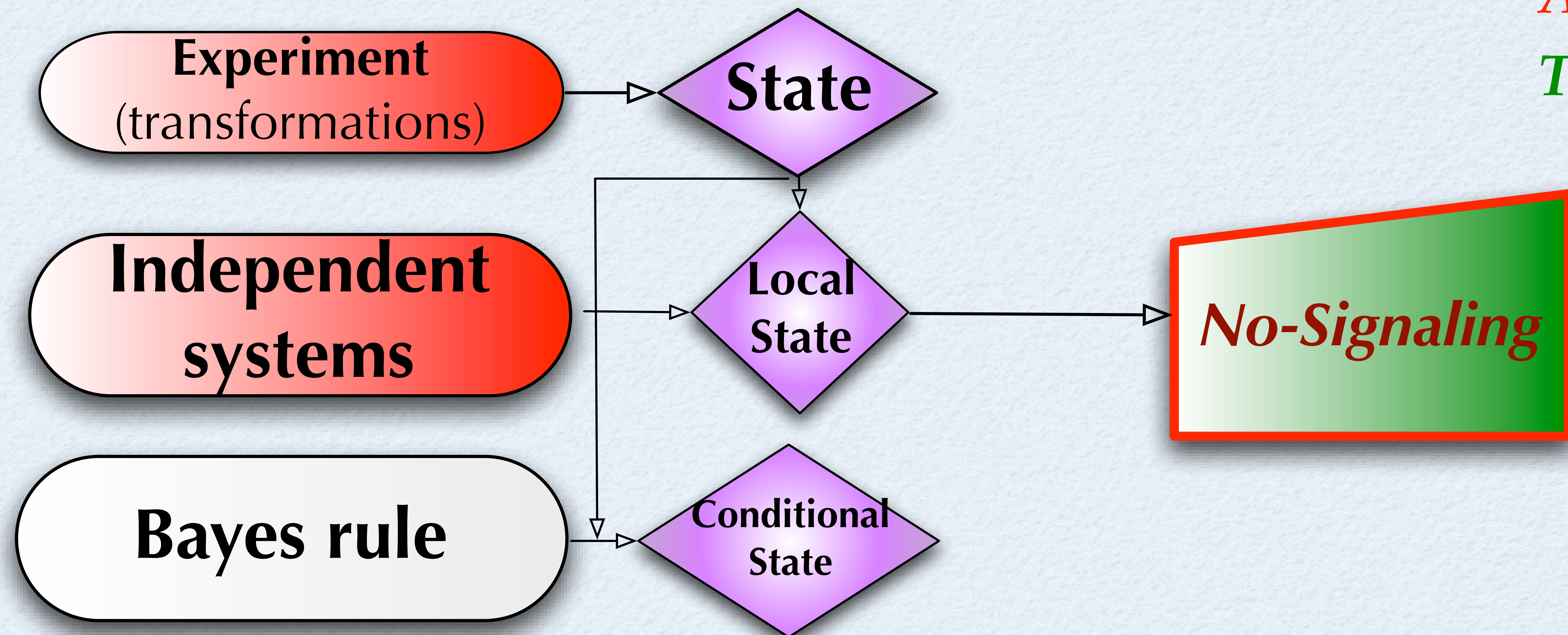

Informational compatibility

Multiplication by a scalar: for each transformation $\mathscr{A}$ the transformation $\lambda\mathscr{A}$ for $0 \leq \lambda \leq 1$ is defined as the transformation which is dynamically equivalent to $\mathscr{A}$ but occurs with probability $\omega(\lambda\mathscr{A}) = \lambda\omega(\mathscr{A})$

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Convex structure for transformations $\mathfrak{T}$
and for actions

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We call ***effect*** an informational equivalence class $\underline{\mathscr{A}}$ of transformations $\mathscr{A}$

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effects as positive linear l functionals over states:

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Convex structure for effects $\mathfrak{P}$

Generalized weights, transformations, and effects

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(real algebra)

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(real algebra)

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Real Banach spaces

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$\mathfrak{W}_{\mathbb{R}}$ $\mathfrak{P}_{\mathbb{R}}$ ***dual Banach pair*** under the pairing

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$\mathfrak{T}_{\mathbb{R}}$ ***Banach algebra***

Banach-space structures

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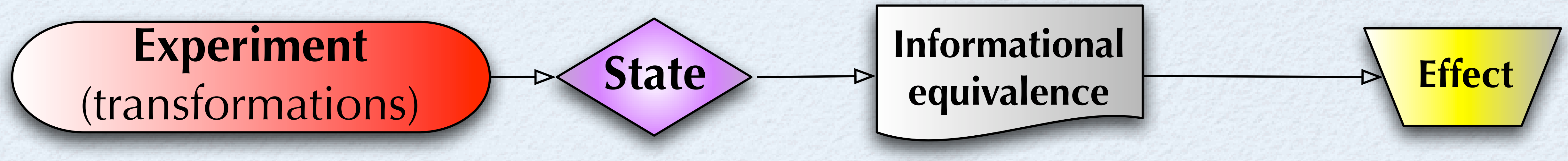

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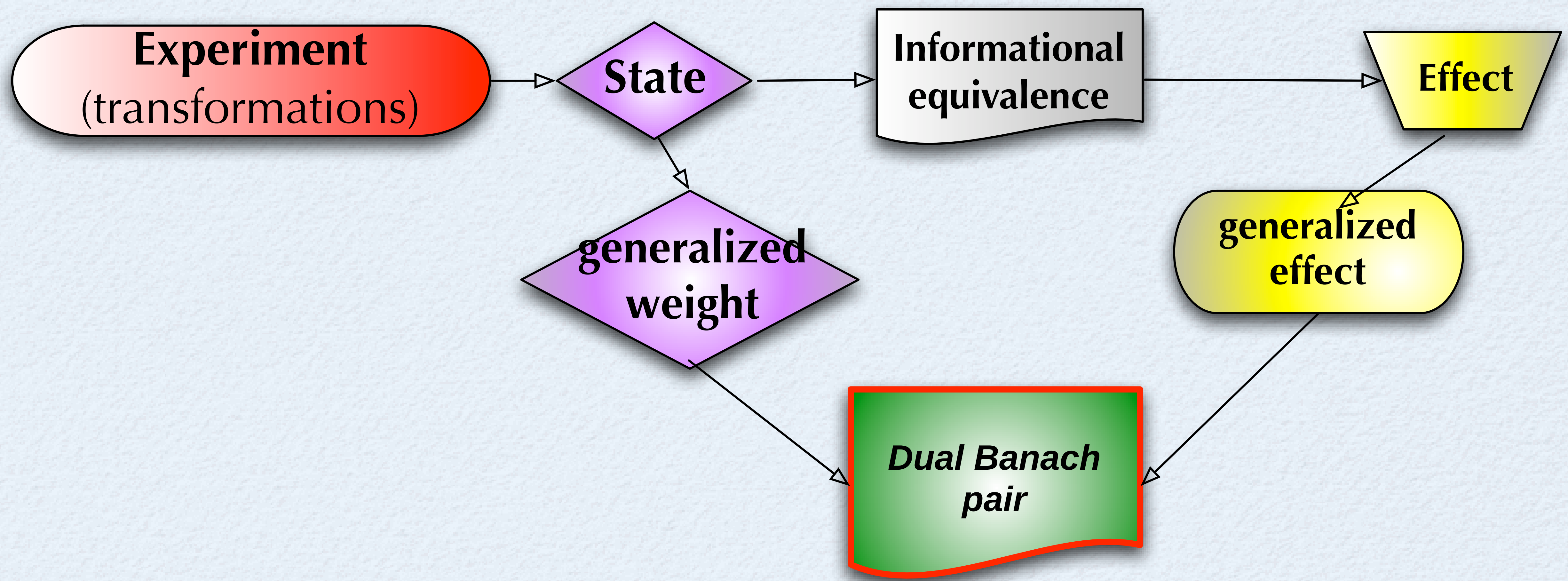

Observable

Observable: the complete set of effects $\{l_j\}$ of an experiment $\mathbb{A} = \{\mathscr{A}_j\}$, namely $l_j = \underline{\mathscr{A}_j} \quad \forall j$

Informationally complete observable

Informationally complete observable: an observable $\mathbb{L} = \{l_i\}$ is informationally complete if any effect l can be written as linear combination of elements of $\mathbb{L}$, namely there exist coefficients $c_i(l)$ such that

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affine dimension: $\dim(\mathfrak{S}) = |\mathbb{L}| - 1$, for $\mathbb{L}$ minimal informationally complete on $\mathfrak{S}$

Postulate 3: Local observability principle

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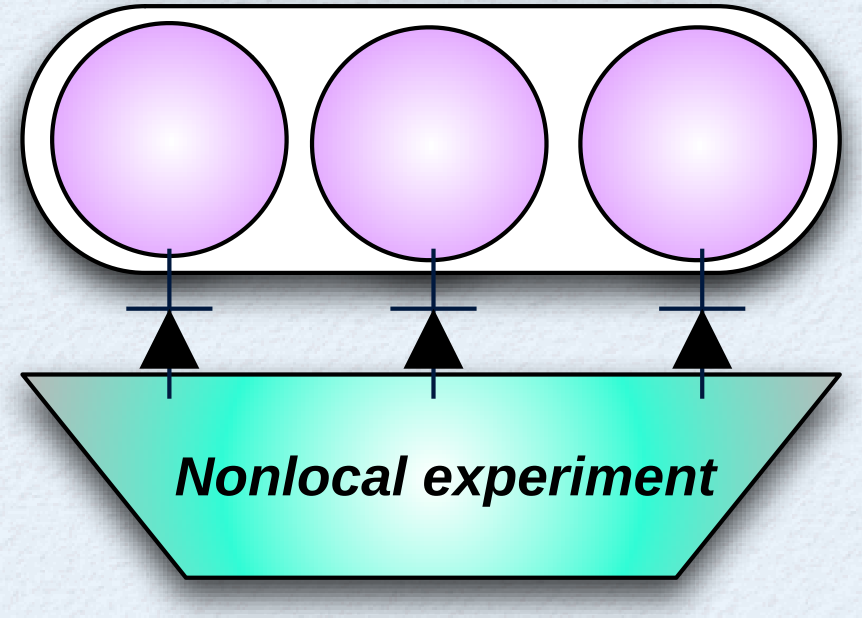

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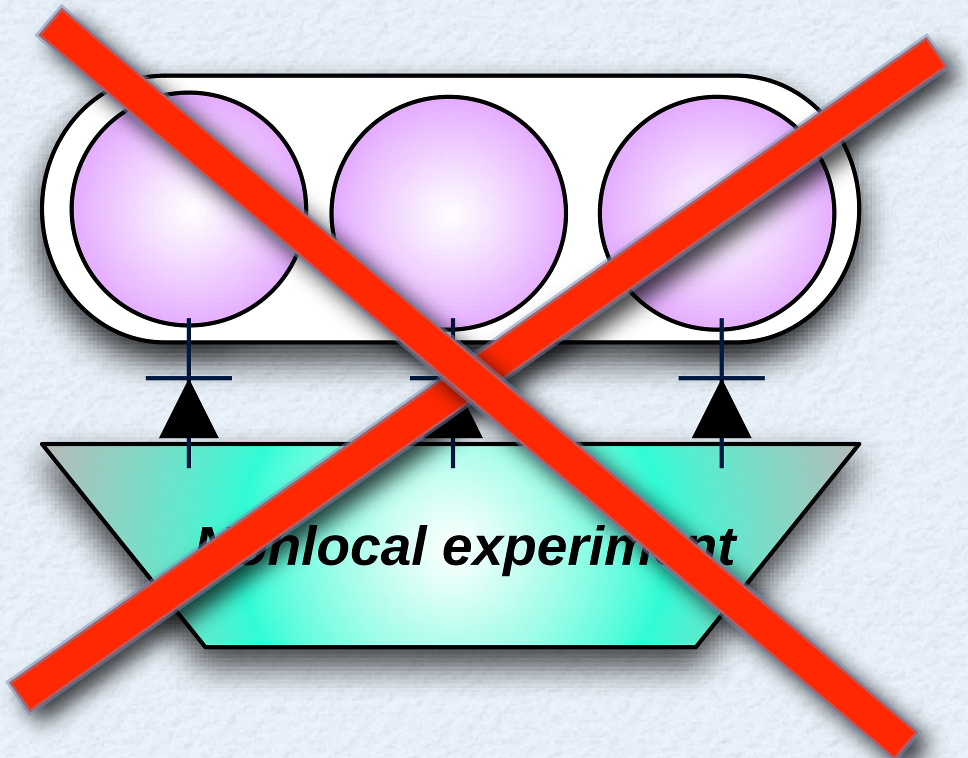

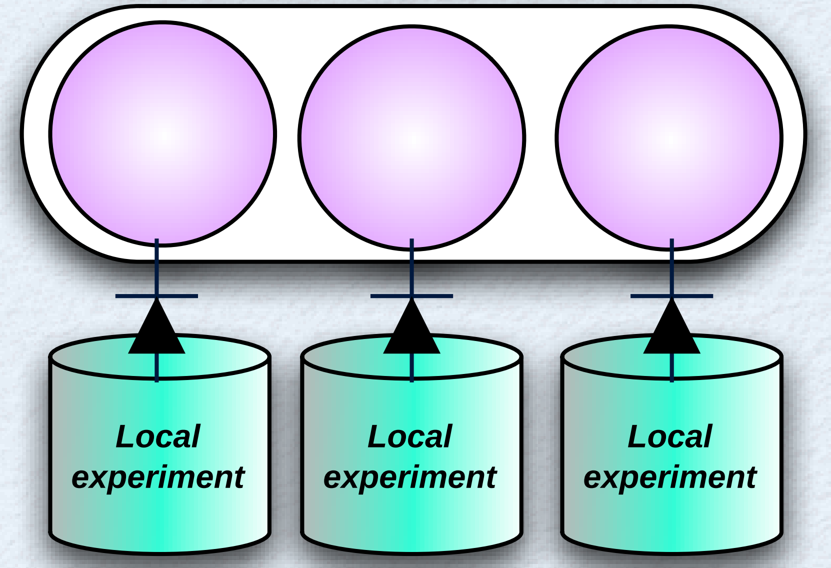

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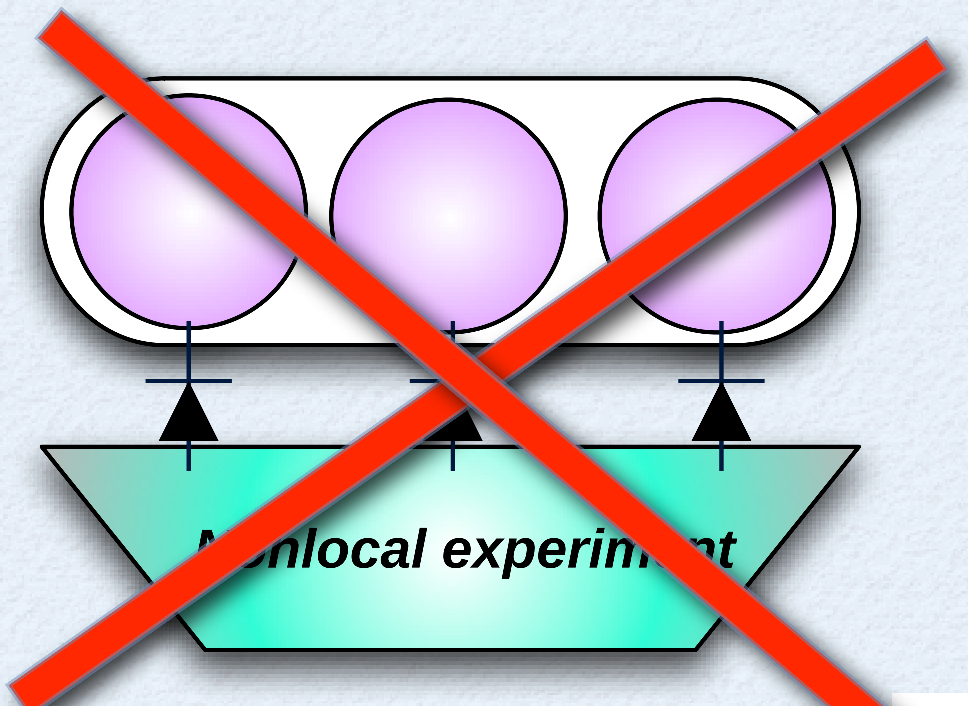

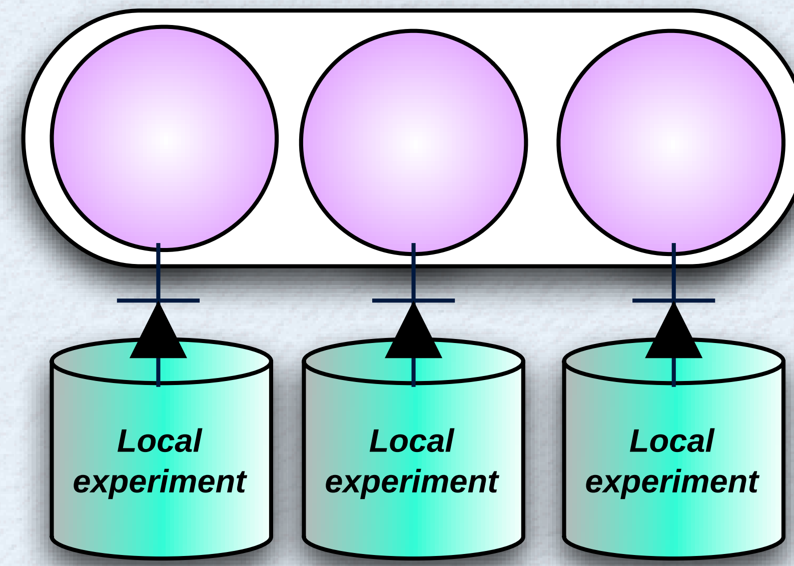

Holism  **Reductionism**

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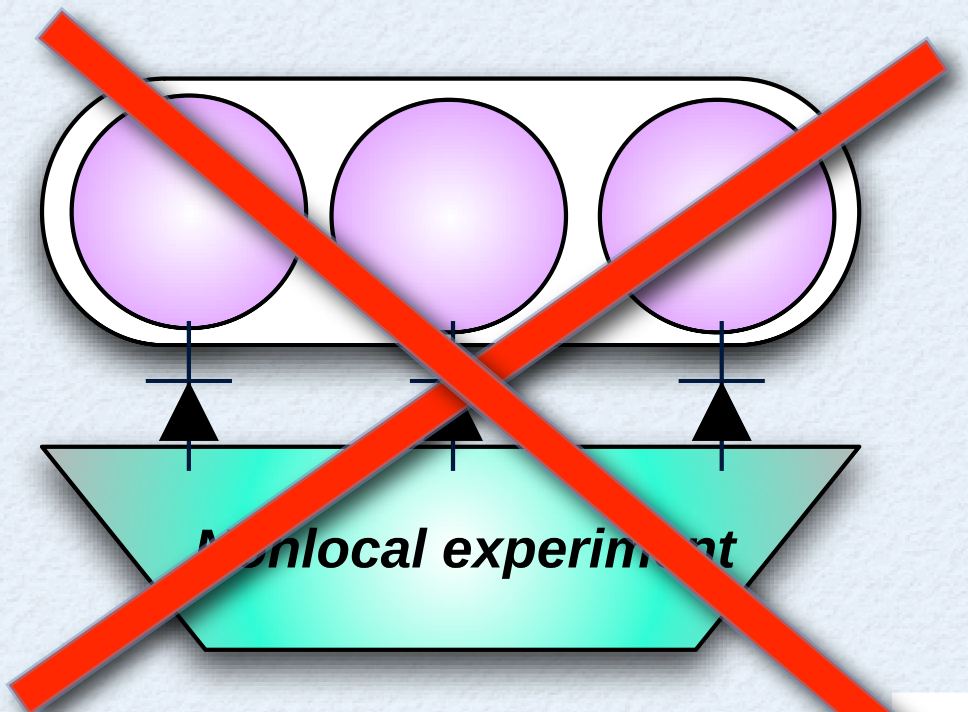
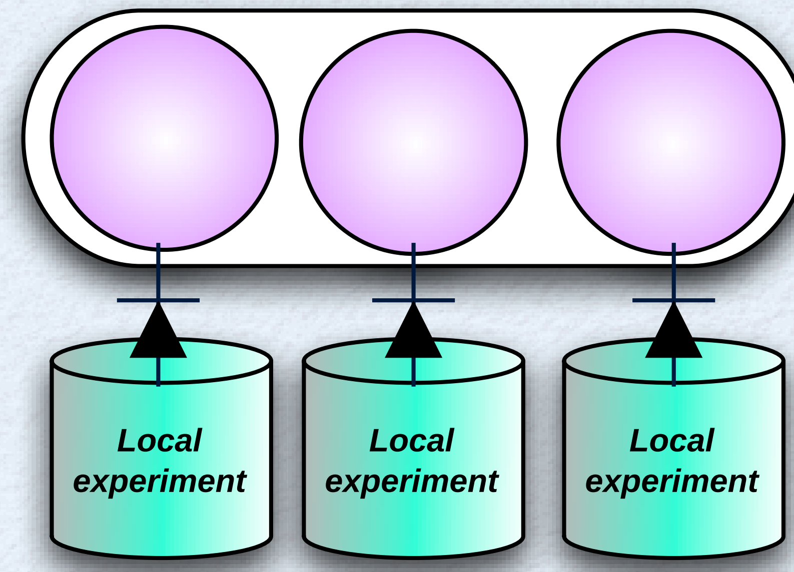




Holism  **Reductionism**

identity for the affine dimension of composite systems

$$\boxed{\dim(\mathfrak{S}_{12}) = \dim(\mathfrak{S}_1)\dim(\mathfrak{S}_2) + \dim(\mathfrak{S}_1) + \dim(\mathfrak{S}_2)}$$

Postulate 3: Local observability principle

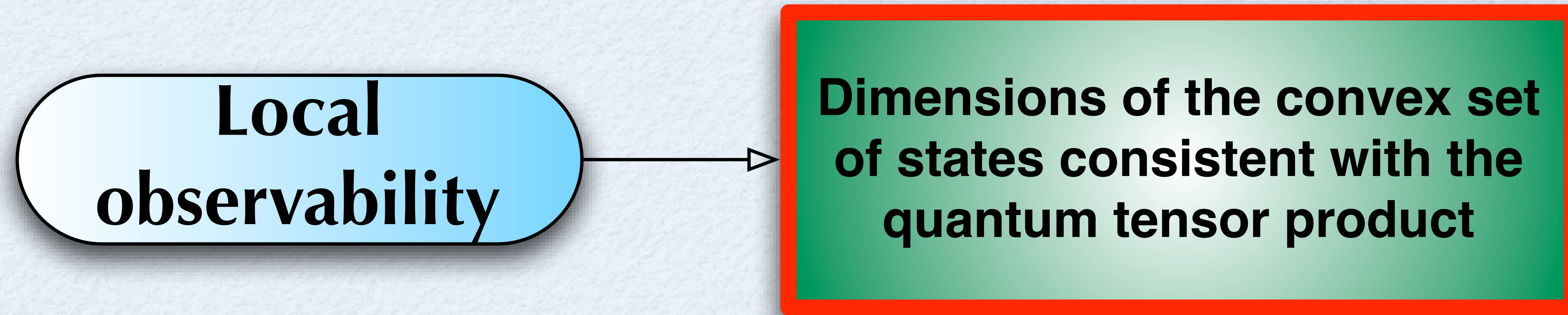
Postulates
Axioms
Theorems

Postulate 3: Local observability principle

Postulates

Axioms

Theorems



Block representation

$$l_{\underline{\mathscr{A}}} = \sum_j m_j(\underline{\mathscr{A}}) n_j \qquad l_{\underline{\mathscr{A}}}(\omega) = \boldsymbol{m}(\underline{\mathscr{A}}) \cdot \boldsymbol{n}(\omega) + q(\underline{\mathscr{A}})$$

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Conditioning: fractional affine transformation

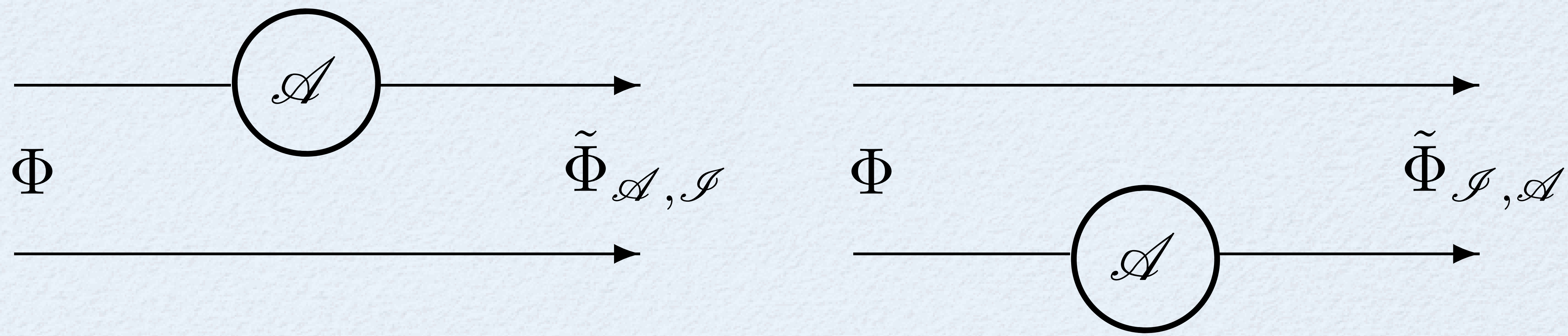
$$\boldsymbol{n}(\omega) \longrightarrow \boldsymbol{n}(\omega_{\mathscr{A}})$$

$$\boldsymbol{n}(\omega_{\mathscr{A}}) = \frac{\boldsymbol{M}(\mathscr{A})\boldsymbol{n}(\omega) + \boldsymbol{k}(\mathscr{A})}{\boldsymbol{m}(\underline{\mathscr{A}}) \cdot \boldsymbol{n}(\omega) + q(\underline{\mathscr{A}})}$$

$$M_{ij}(\mathscr{A}) = \begin{pmatrix} \boxed{q(\underline{\mathscr{A}})} & \boxed{\boldsymbol{m}(\underline{\mathscr{A}})} \\ \boxed{\boldsymbol{k}(\mathscr{A})} & \boxed{\boldsymbol{M}(\mathscr{A})} \end{pmatrix}$$

Faithful states

Dynamically faithful state: we say that a state Φ of a bipartite system is dynamically faithful if when acting on it with a local transformation $\mathscr{A}$ on one system the output conditioned weight $\tilde{\Phi}_{\mathscr{A},\mathscr{I}}$ is in 1-to-1 correspondence with the transformation $\mathscr{A}$



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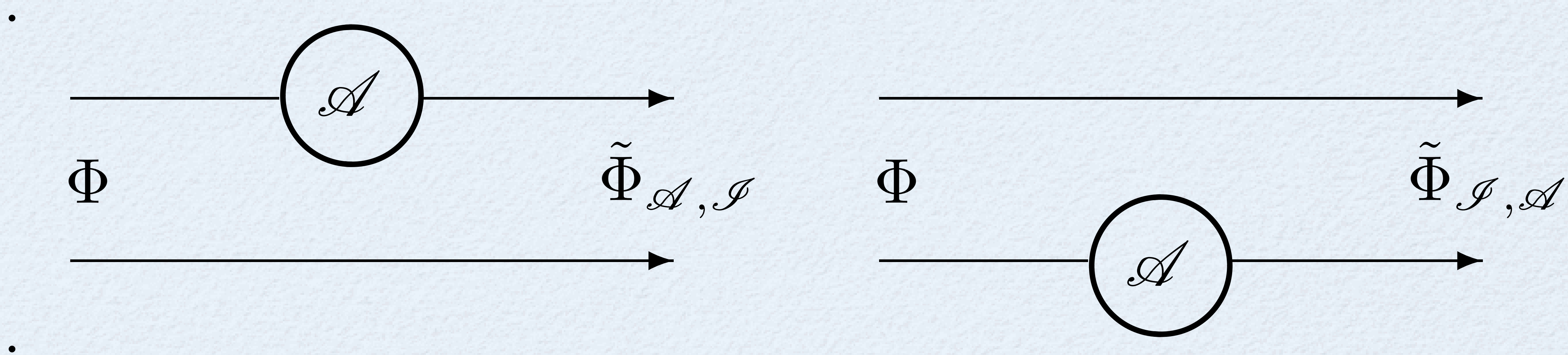
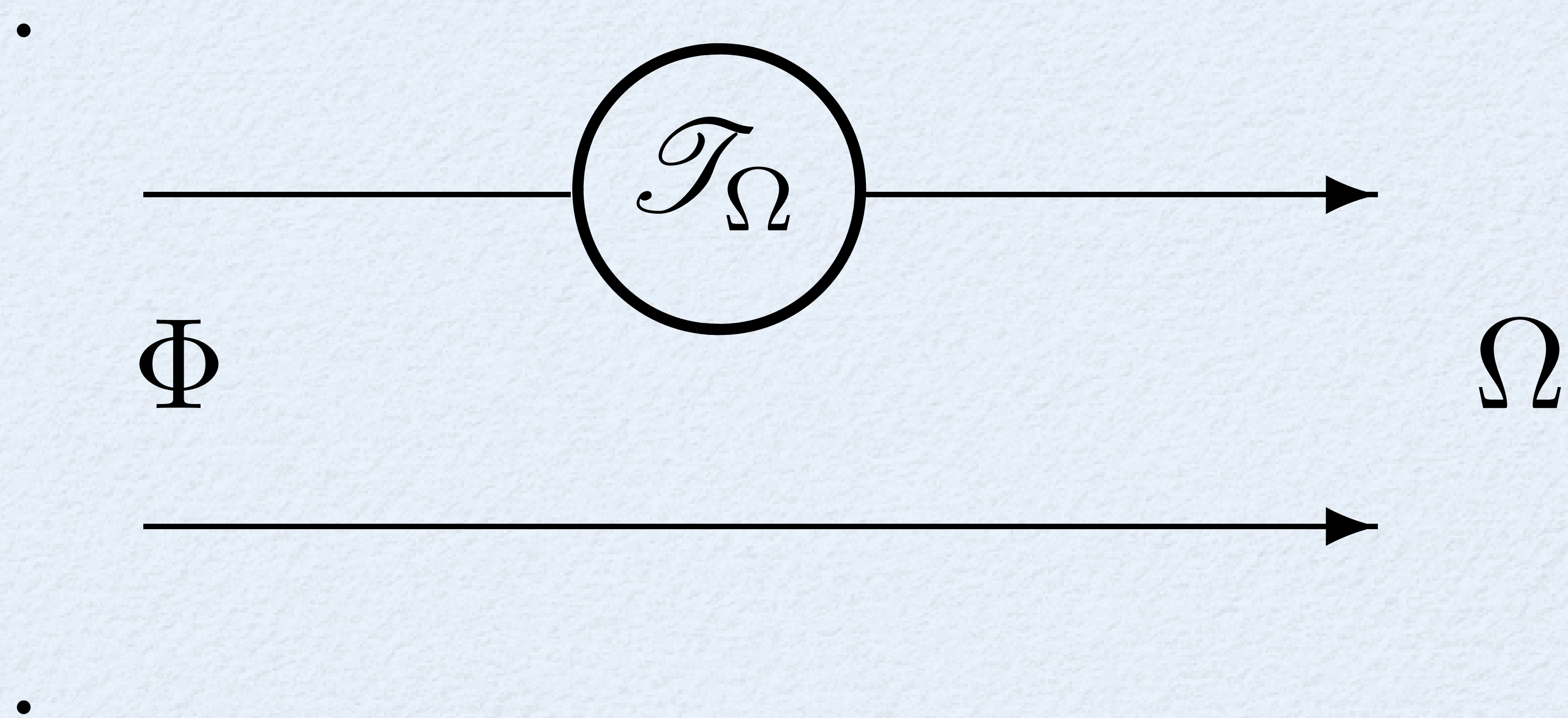


$$(\mathscr{A},\mathscr{I})\Phi = 0 \Longleftrightarrow \mathscr{A} = 0$$

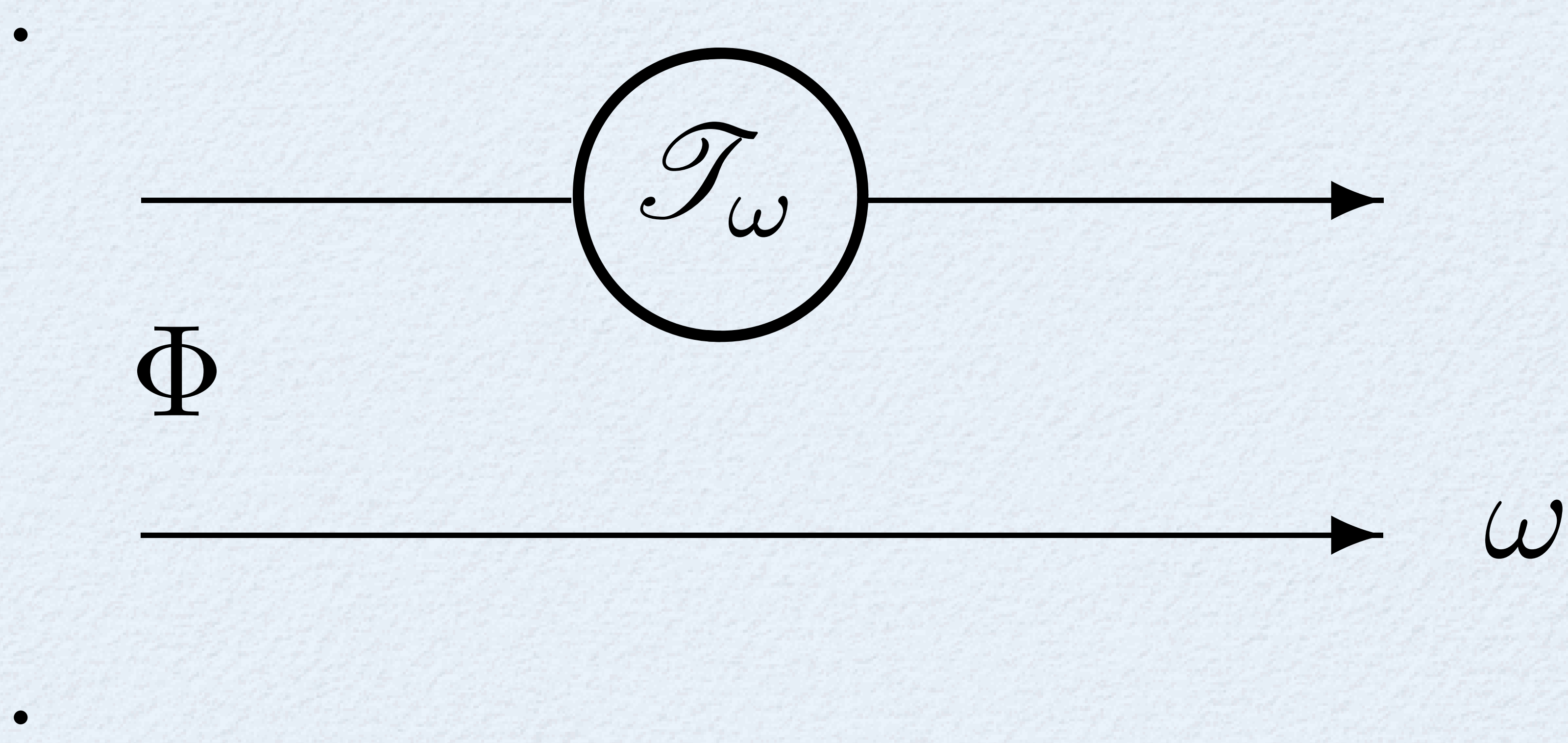
Faithful states

Preparationally faithful state: we say that a state Φ of a bipartite system is preparationally faithful if every joint states Ω can be achieved by a suitable local transformation $\mathscr{T}_\Omega$ on one system occurring with nonzero probability



Faithful states

Clearly a preparationally faithful state Φ of a bipartite system is also *locally* preparationally faithful, namely every local state ω of one component system can be achieved by a suitable local transformation $\mathscr{T}_\omega$ on the other component system



Faithful states

Symmetric bipartite state: we call a joint state Φ of a bipartite system symmetric if

$$\Phi(\mathscr{A},\mathscr{B}) = \Phi(\mathscr{B},\mathscr{A})$$

Perfectly discriminating observable

Perfectly discriminable states/observable $\{\omega_j\}$**:** there exists an observable $\mathbb{L} = \{l_i\}$ such that

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Informational dimension $\dim_{\#}(\mathfrak{S})$**:** maximal number of perfectly discriminable states

Postulate 4: Informationally complete discriminating observable

*For every system there exists a minimal informationally complete observable that can be achieved using a **joint discriminating observable** on the system + an "ancilla" (identical independent system).*

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$$\boxed{\dim(\mathfrak{S}) = \dim_{\#}(\mathfrak{S}^{\times 2}) - 1}$$

Dimensionality identities

$\Longrightarrow$		
P2 (infoc.)	$\dim(\mathfrak{P}_{\mathbb{R}}) = \dim(\mathfrak{S}) + 1$	(D2)
P3 (loc. obs.)	$\dim(\mathfrak{S}_{12}) = \dim(\mathfrak{S}_1)\dim(\mathfrak{S}_2) + \dim(\mathfrak{S}_1) + \dim(\mathfrak{S}_2)$	(D3)
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(D5+D35b)	$\dim_{\#}(\mathfrak{S}^{\times 2}) = \dim_{\#}(\mathfrak{S})^2$	($\otimes$)
P5 (faith.)	$\dim(\mathfrak{T}) = \dim(\mathfrak{S}^{\times 2}) + 1$	($\mathfrak{T}$)
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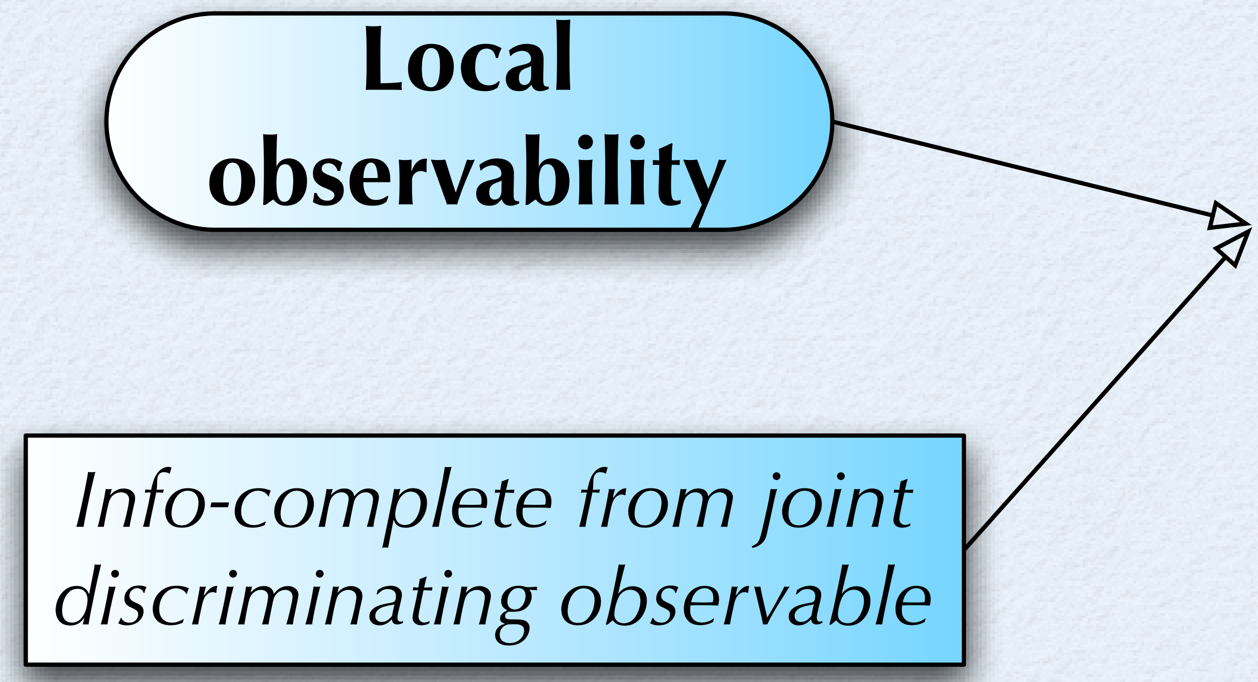

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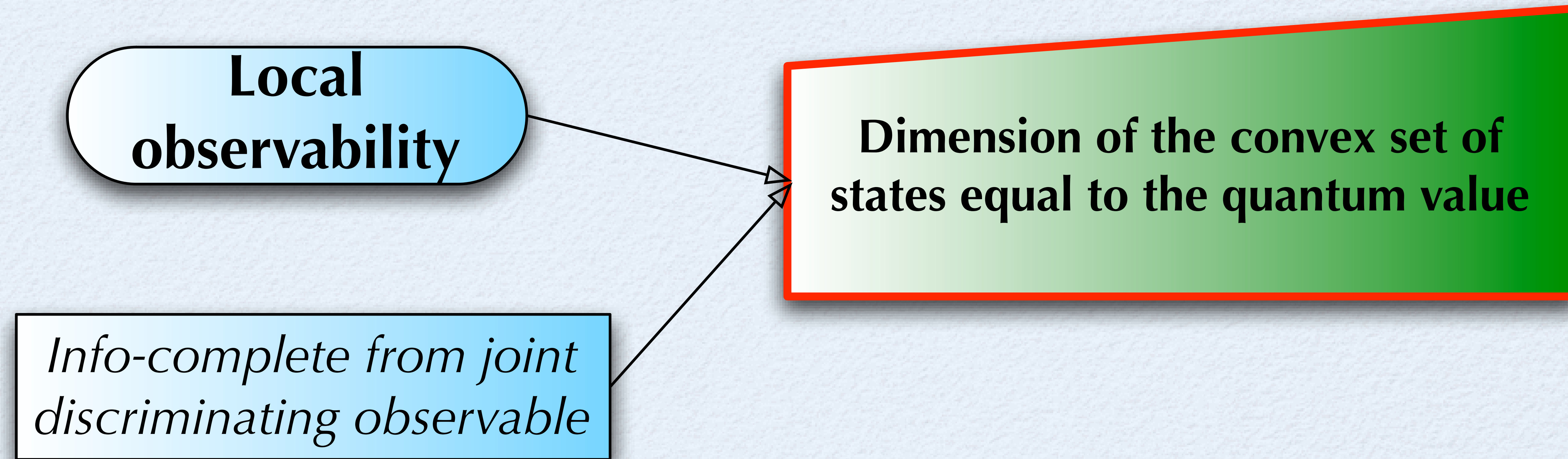

Scalar product over effects

Using the symmetric dynamically faithful state one introduces a strictly positive real scalar product over effects $\mathfrak{P}_{\mathbb{R}}$

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$\mathfrak{P}_{\mathbb{R}}$ real (pre)Hilbert space of dimension $\dim_{\#}(\mathfrak{S})^2$

Positive bilinear form

Positive form over generalized effects: from Φ real symmetric form over effects obtain the positive form (via informationally complete observable)

$$|\Phi| := \Phi_+ - \Phi_-$$

$$|\Phi|(\underline{\mathscr{A}}, \underline{\mathscr{B}}) = \Phi(\underline{\mathscr{A}}, \varsigma(\underline{\mathscr{B}})), \quad \varsigma(\underline{\mathscr{A}}) = (\mathscr{P}_+ - \mathscr{P}_-)(\underline{\mathscr{A}})$$

$$\varsigma^2 = \mathscr{I}$$

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$\Downarrow$

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The complex Hilbert space formulation

For finite dimensions the real Hilbert space $\mathfrak{P}_{\mathbb{R}}$ is isomorphic to the real Hilbert space of Hermitian complex matrices representing selfadjoint operators over a complex Hilbert space H of dimensions $\dim(\mathsf{H}) = \dim_{\#}(\mathfrak{S})$.

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This is the Hilbert space formulation of Quantum Mechanics

Positive bilinear form

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Then one can write the probability rule in terms of a real scalar product pairing between states and effects, with the convex cones of effects and states corresponding to the convex cone of positive matrices.

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Positive bilinear form

Since Φ is *preparationally* faithful, then for every state ω there exists a suitable transformation $\mathscr{T}_\omega$ such that $\omega = \Phi_{\mathscr{I},\mathscr{T}_\omega}|_1$ with probability $\Phi(\mathscr{I},\mathscr{T}_\omega) > 0$

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Then we can write the probability rule in terms of the pairing between states and effects:

$$\boxed{\begin{gathered}\omega(\underline{\mathscr{C}}) = \Phi_{\mathscr{I},\mathscr{T}_\omega}|_1(\underline{\mathscr{C}}) = |\Phi|(\underline{\mathscr{C}}, \underline{\widetilde{\mathscr{T}}}_\omega),\\[2em] \underline{\widetilde{\mathscr{T}}}_\omega = \frac{\varsigma(\underline{\mathscr{T}}_\omega)}{\Phi(\underline{\mathscr{I}}, \underline{\mathscr{T}}_\omega)}\end{gathered}}$$

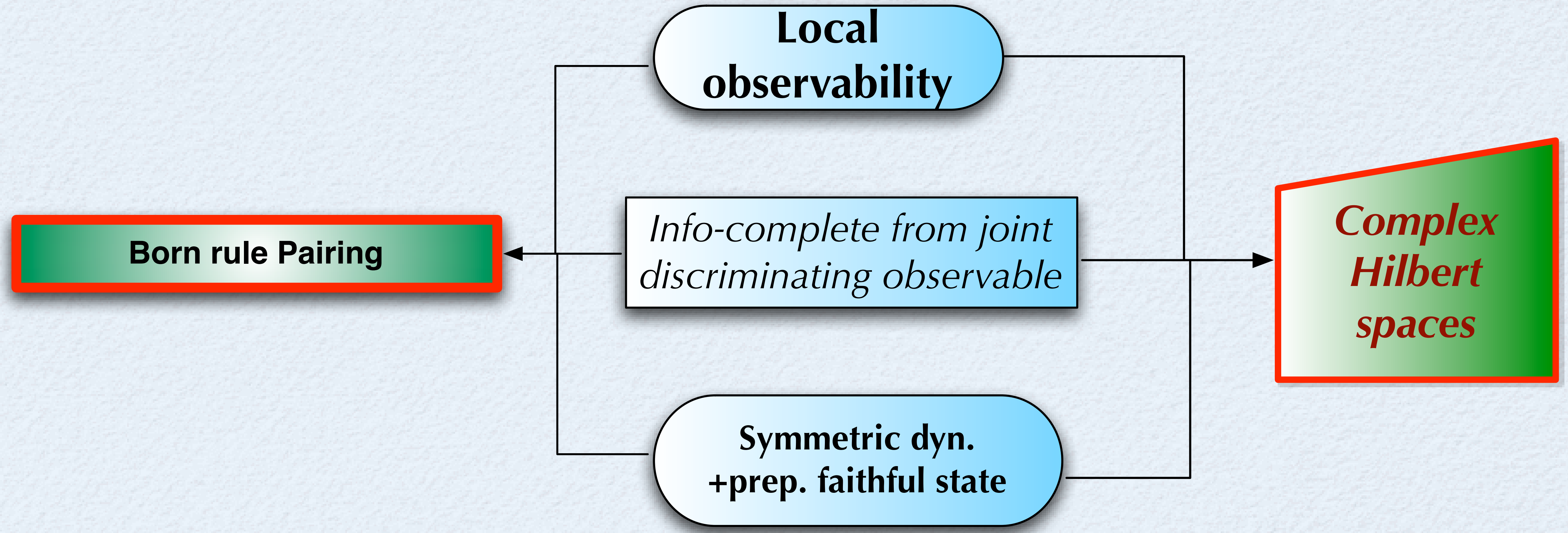
The complex Hilbert space formulation

Local observability

Info-complete from joint discriminating observable

Symmetric dyn. +prep. faithful state

The complex Hilbert space formulation



The complex Hilbert space formulation

End of story:

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- ***construct complex operators by complex linear combination of effects***

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- ***physical transformations are described by CP trace-decreasing maps***

The complex Hilbert space formulation

End of story:

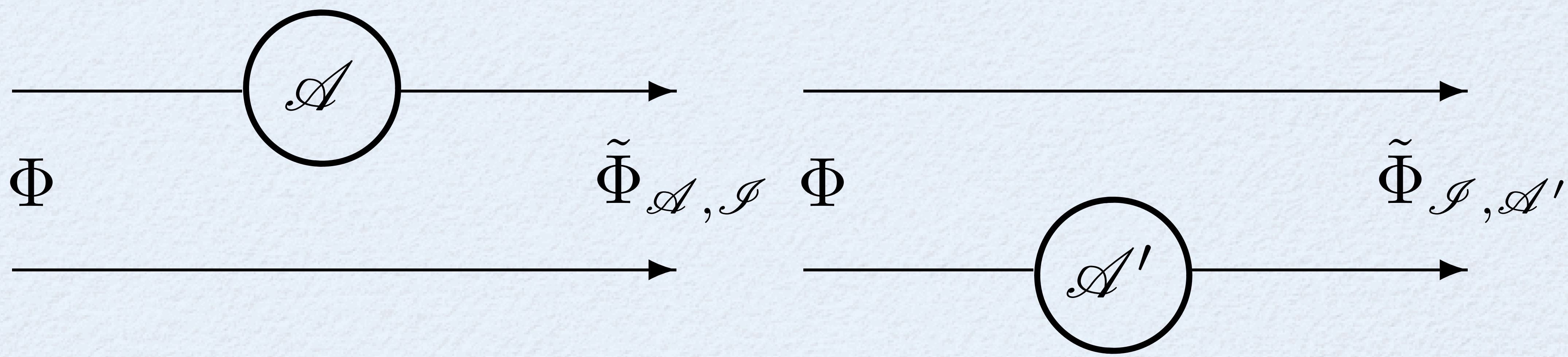
- ***construct complex operators by complex linear combination of effects***
- ***physical transformations are described by CP trace-decreasing maps***
- ***etc.***

Operational definition of *transposed*

Existence of symmetric faithful states

"transposition" over the real algebra $\mathcal{A}$ of (generalized) transformations

$$\mathscr{A} \Longleftrightarrow \mathscr{A}'$$



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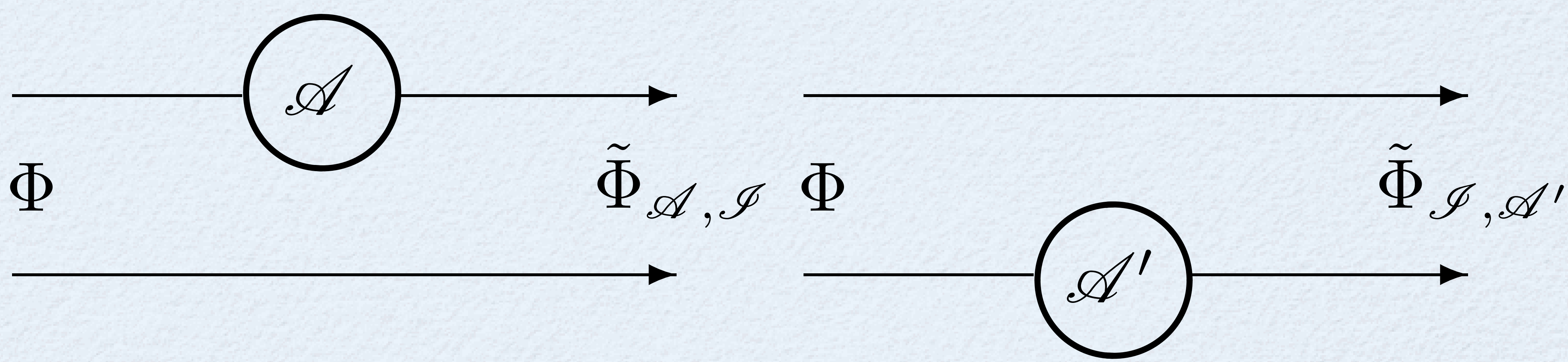

$$\boxed{\Phi(\mathscr{B} \circ \mathscr{A}, \mathscr{C}) = \Phi(\mathscr{B}, \mathscr{C} \circ \mathscr{A}')}$$

Operational definition of *transposed*

For *symmetric* faithful state it is easy to check that the involution $\mathscr{A} \iff \mathscr{A}'$ satisfies the properties of the transposed:

1. $(\mathscr{A} + \mathscr{B})' = \mathscr{A}' + \mathscr{B}'$
2. $(\mathscr{A}')' = \mathscr{A}\,,$
3. $(\mathscr{A} \circ \mathscr{B})' = \mathscr{B}' \circ \mathscr{A}'$

Operational definition of *transposed*

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Symmetric dyn. faithful state →

Operational definition of *transposed*

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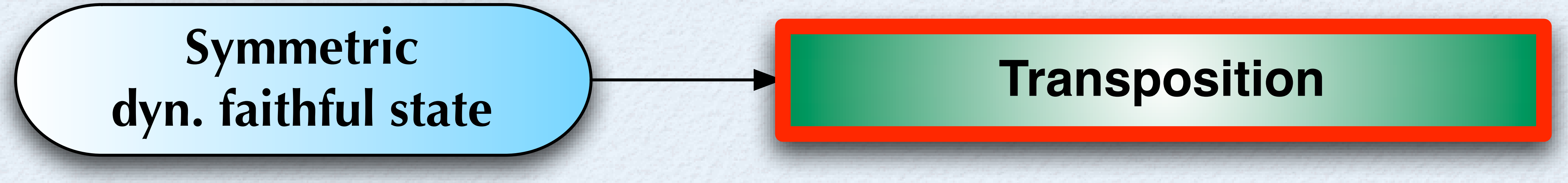

GNS construction for representing transformations

Extend ς to an involution over transformations

$$\varsigma(\mathscr{A}) =: \mathscr{A}^{\varsigma} \in \varsigma(\underline{\mathscr{A}})$$

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For ***composition-preserving*** ς, i.e. $\varsigma(\mathscr{B} \circ \mathscr{A}) = \mathscr{B}^{\varsigma} \circ \mathscr{A}^{\varsigma}$

ς works as a complex-conjugation in the sense that

$\mathscr{A}^{\dagger} := \varsigma(\mathscr{A}')$ works as an adjoint, namely

$$\boxed{{}_{\Phi}\langle \mathscr{C}^{\dagger} \circ \underline{\mathscr{A}} | \underline{\mathscr{B}} \rangle_{\Phi} = {}_{\Phi}\langle \underline{\mathscr{A}} | \mathscr{C} \circ \underline{\mathscr{B}} \rangle_{\Phi}}$$

GNS construction for representing transformations

Take complex linear combinations of generalized transformations and define $\varsigma(c\mathscr{A}) = c^* \varsigma(\mathscr{A})$ for $c \in \mathbb{C}$.

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c-generalized transformations: $\mathfrak{T}_{\mathbb{C}}$
c-generalized effects: $\mathfrak{P}_{\mathbb{C}}$

complex Banach spaces

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complex Banach spaces

complex *-algebra

GNS construction for representing transformations

Representations π_Φ of transformations $\mathscr{A} \in \mathcal{A}$ over effects $\mathcal{A}/\mathcal{I}$

$$\pi_\Phi(\mathscr{A})|\underline{\mathscr{B}}\rangle_\Phi \doteq |\underline{\mathscr{A} \circ \mathscr{B}}\rangle_\Phi$$

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The Born rule rewrites in the form of pairing:

$$\omega(\underline{\mathscr{A}}) = \Phi|_2(\pi_\Phi(\omega)^\dagger \pi_\Phi(\underline{\mathscr{A}})) \equiv {}_\Phi\langle \pi_\Phi(\underline{\mathscr{A}})|\pi_\Phi(\omega)\rangle_\Phi$$

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Experiment
(transformations)

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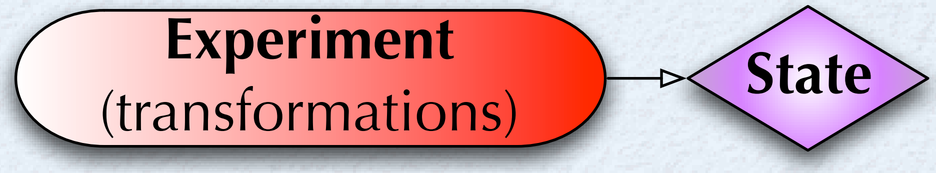

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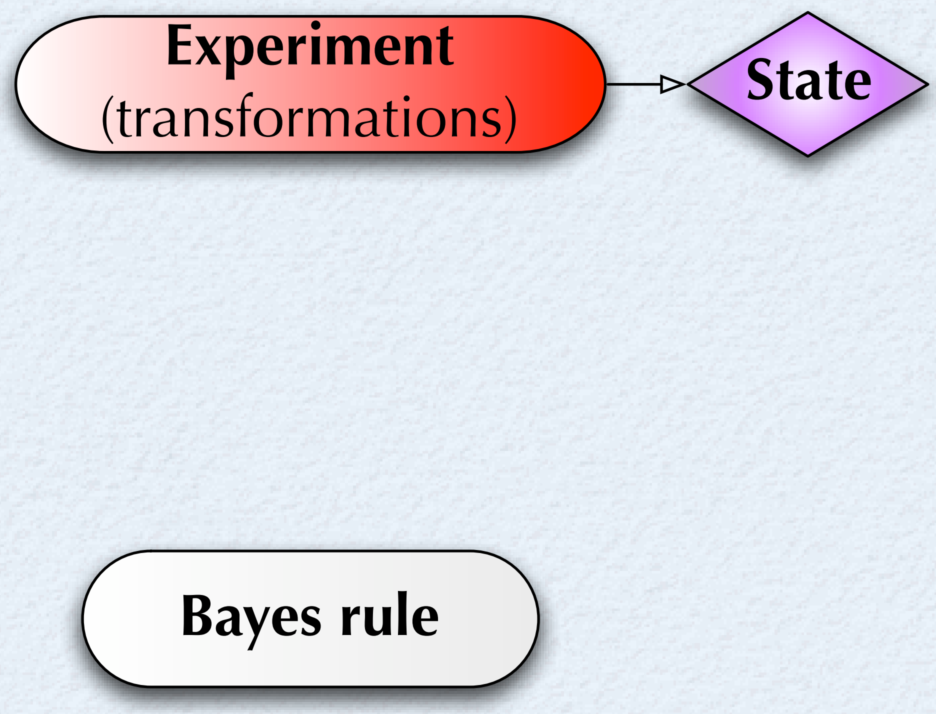

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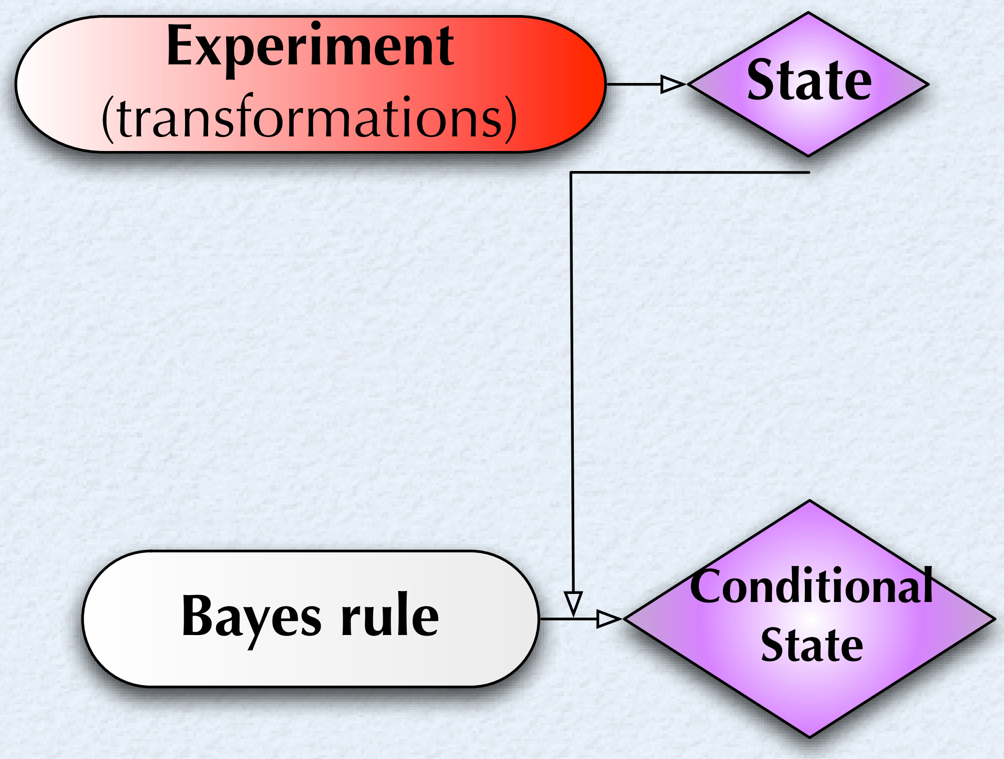

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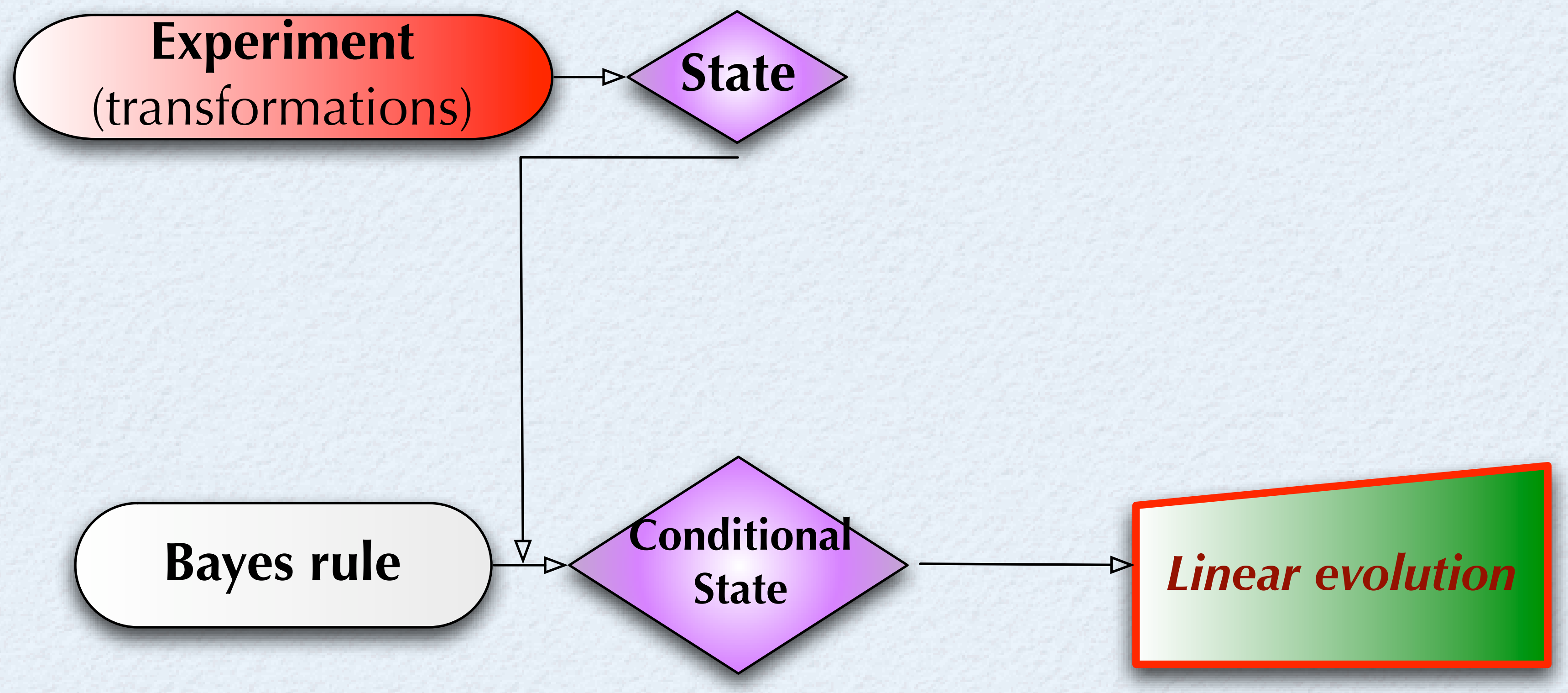

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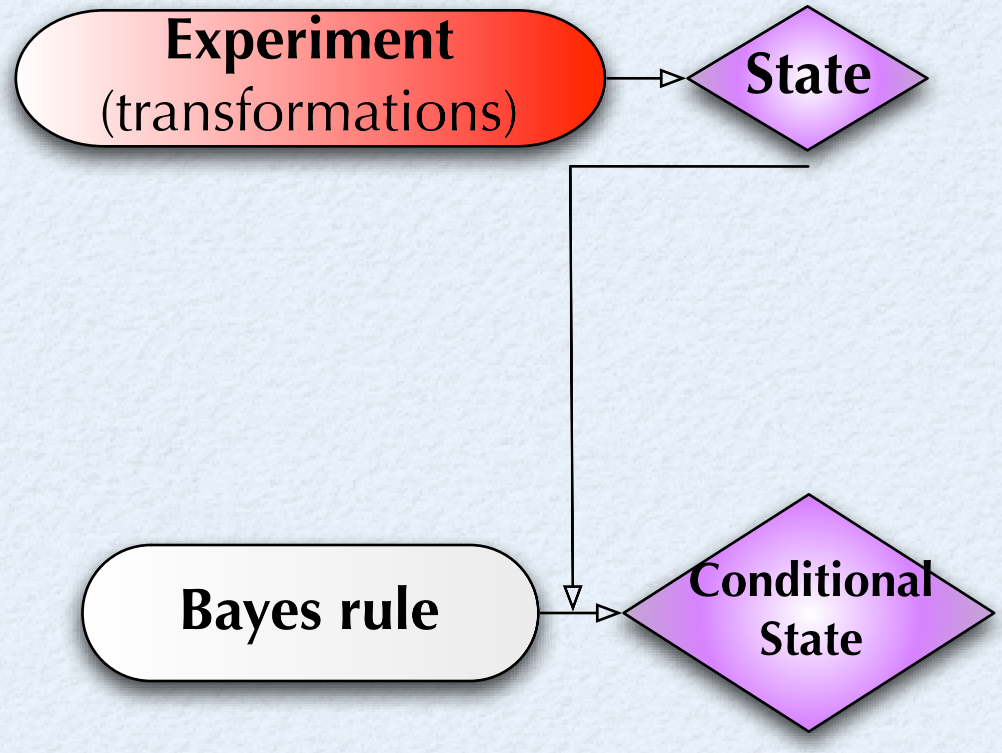

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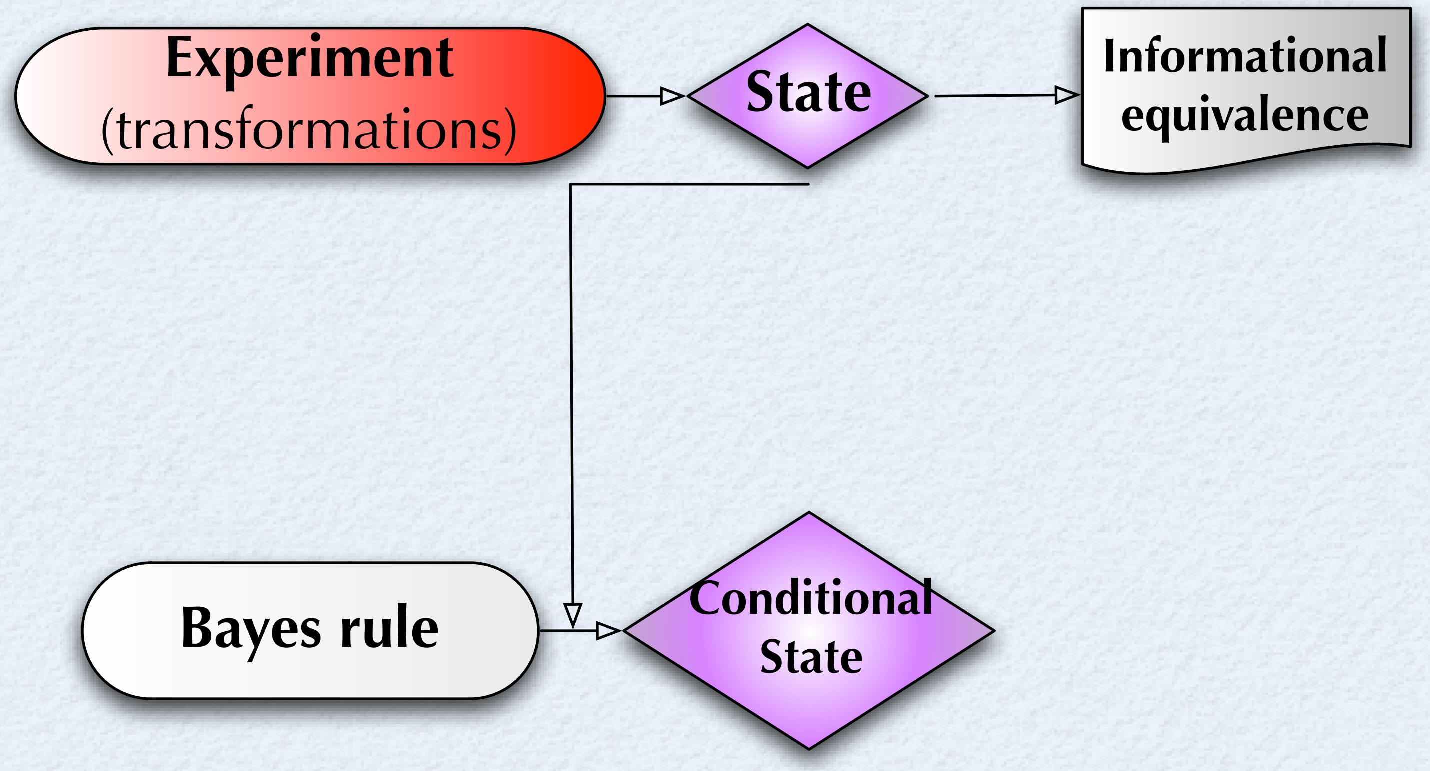

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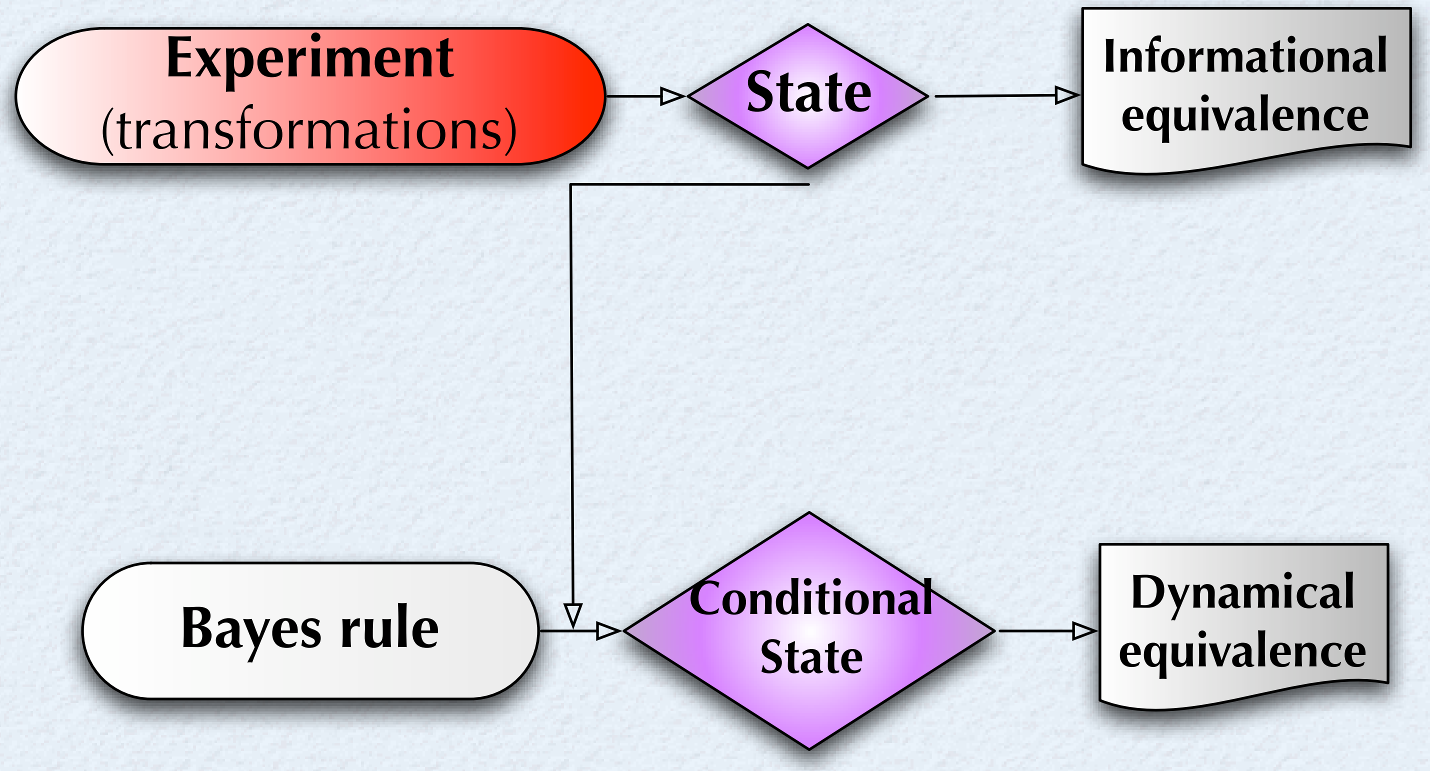

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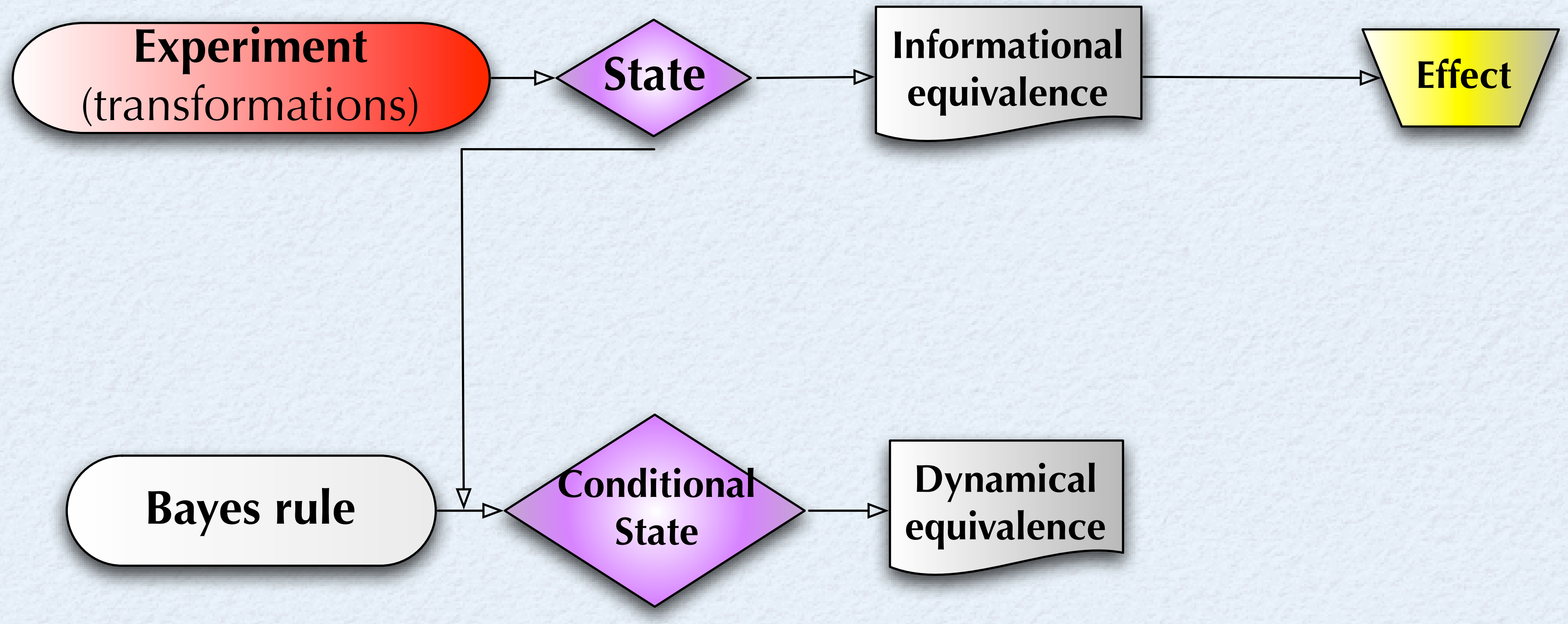

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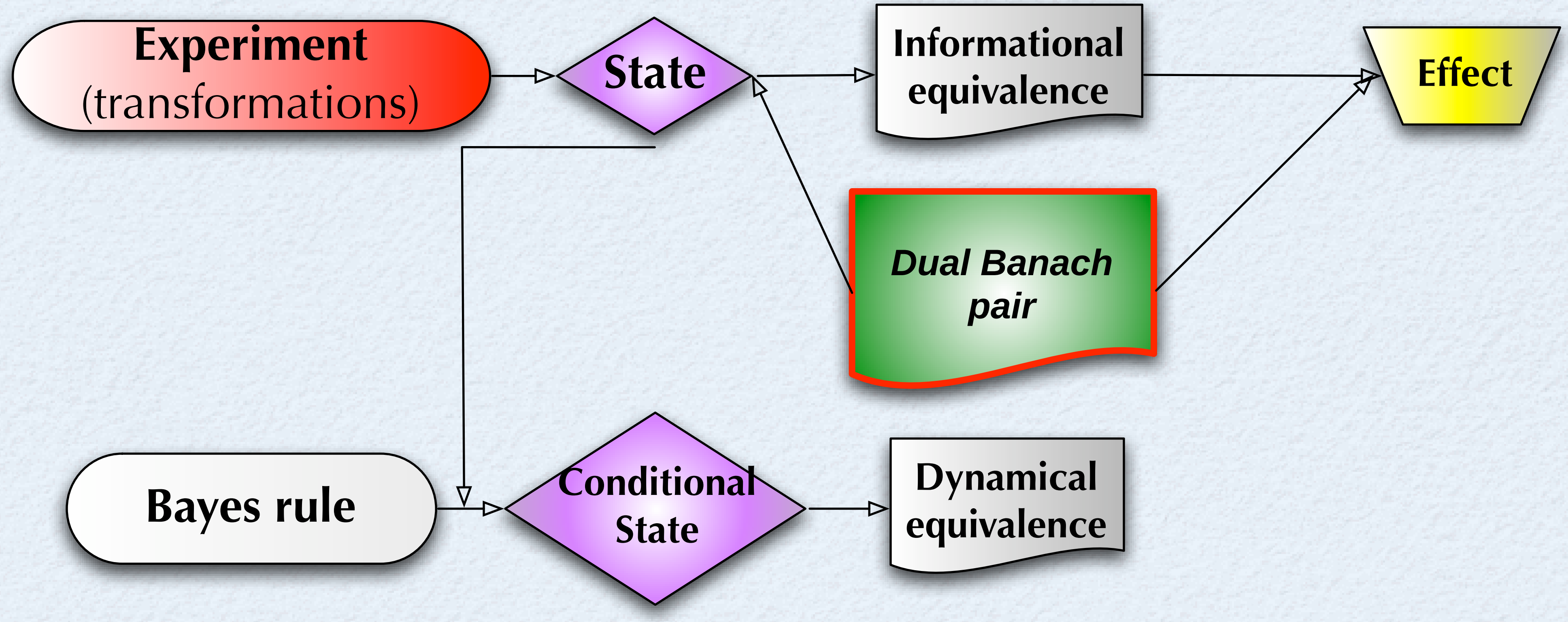

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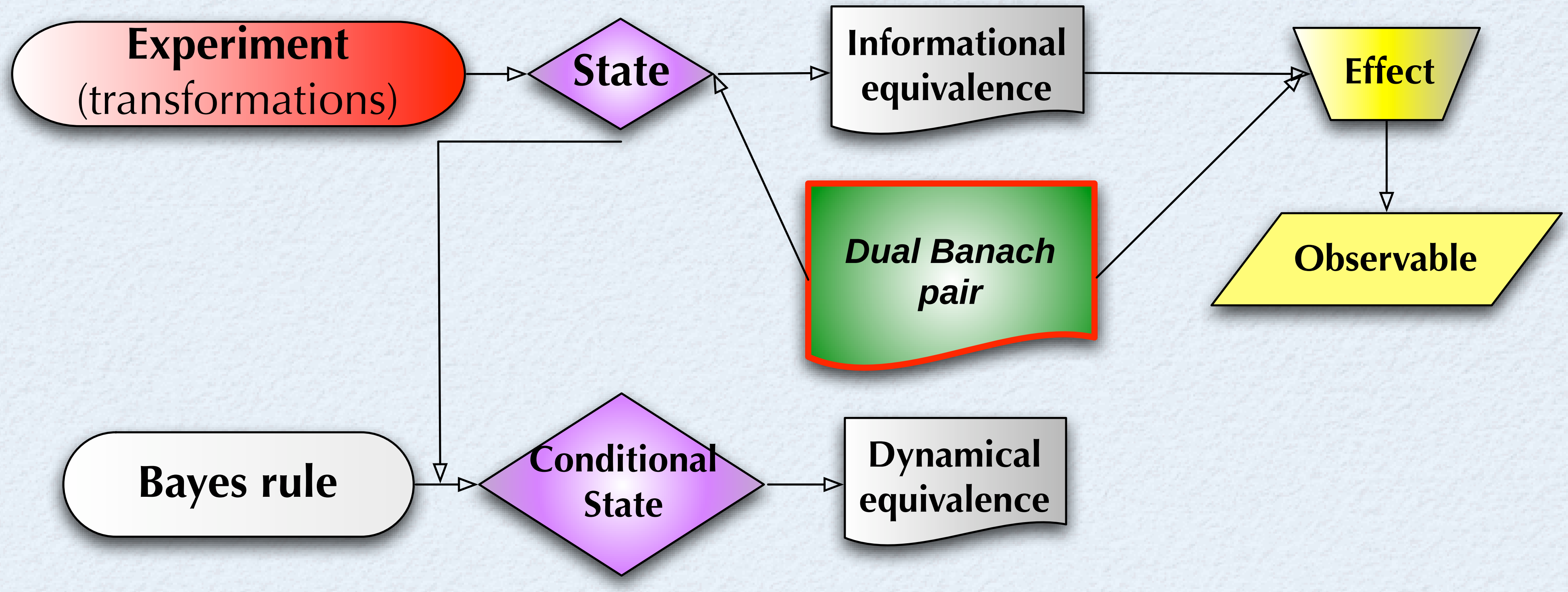

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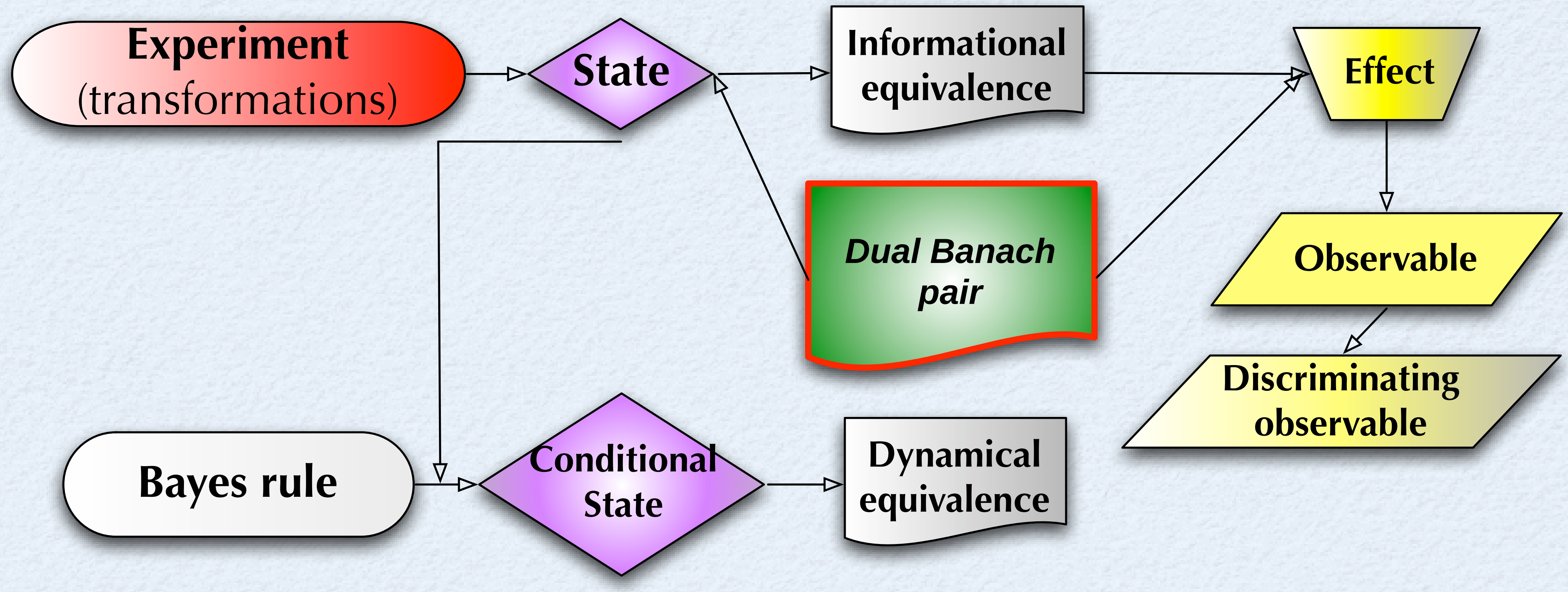

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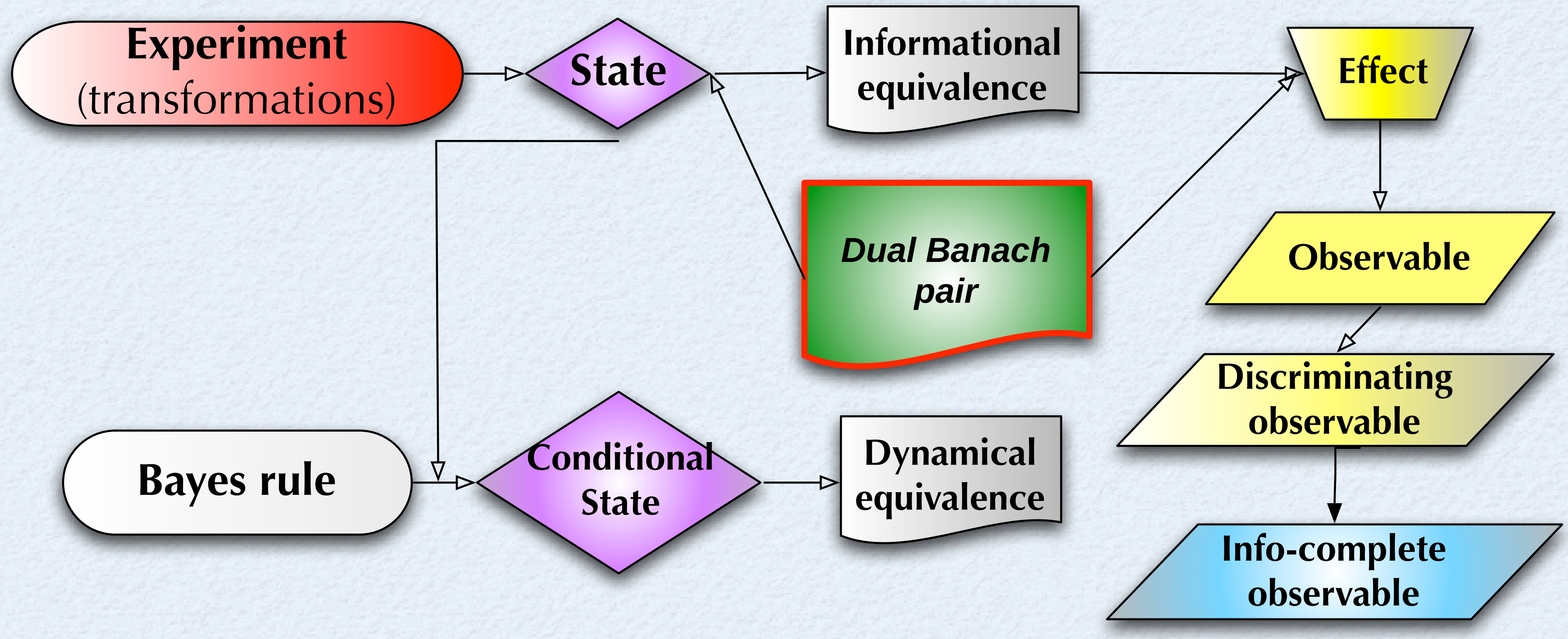

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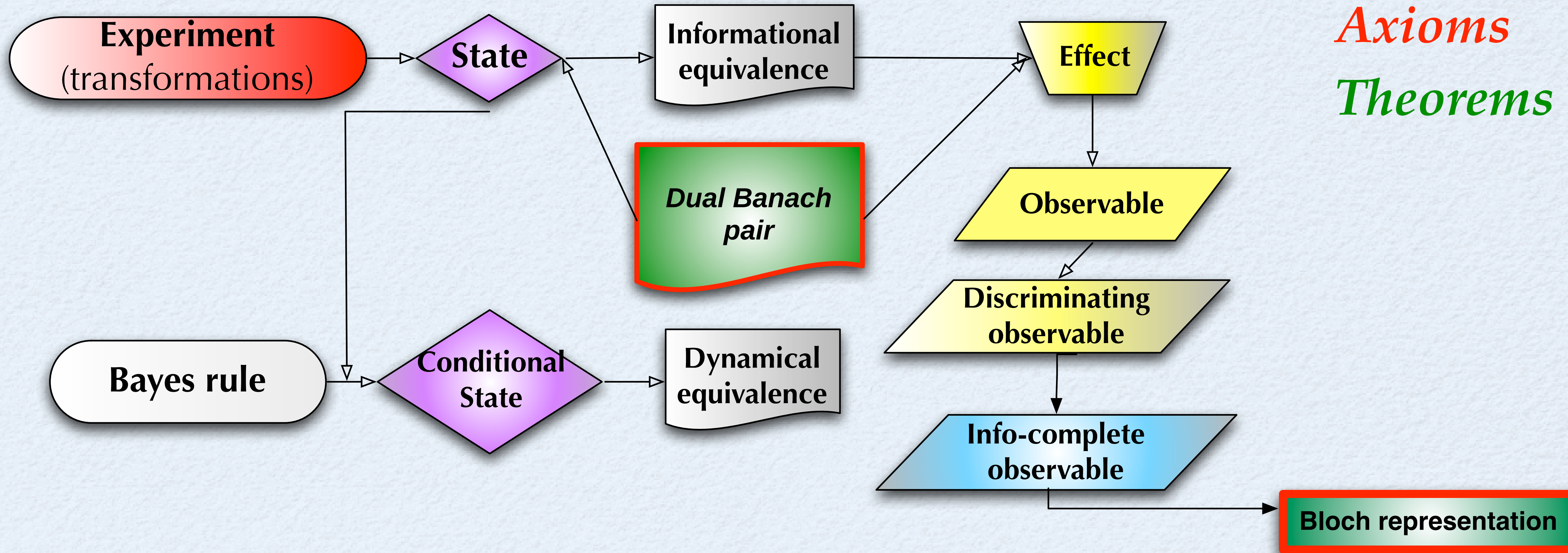

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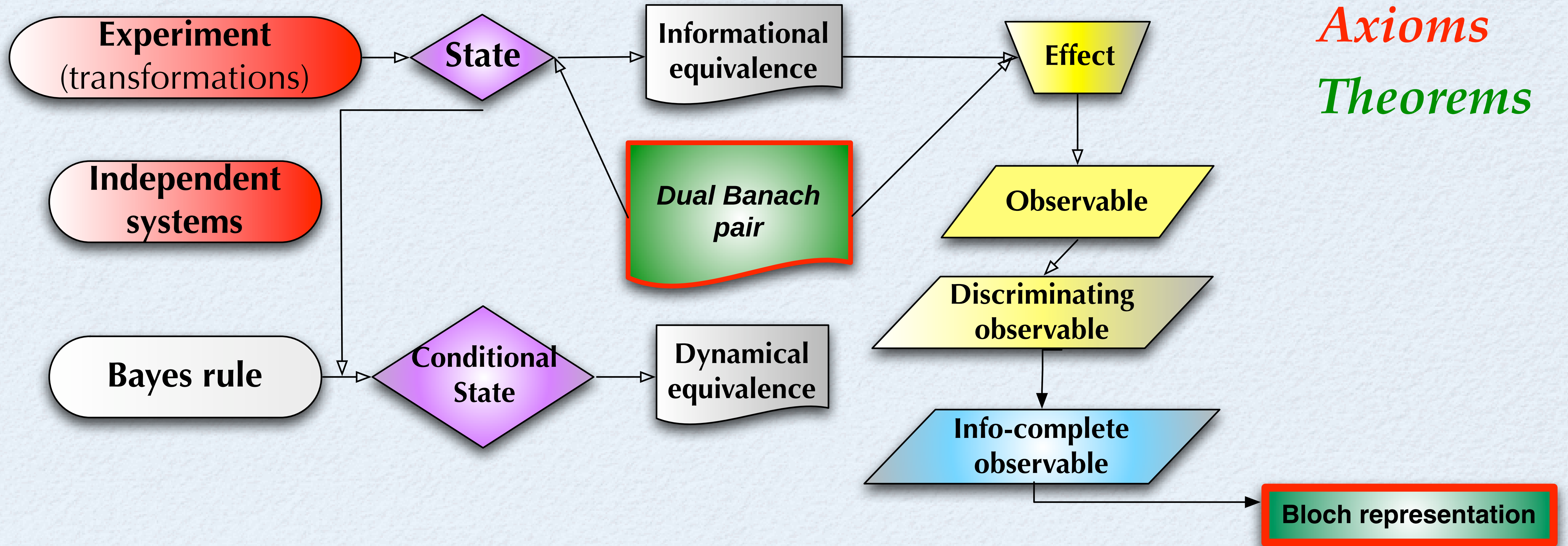

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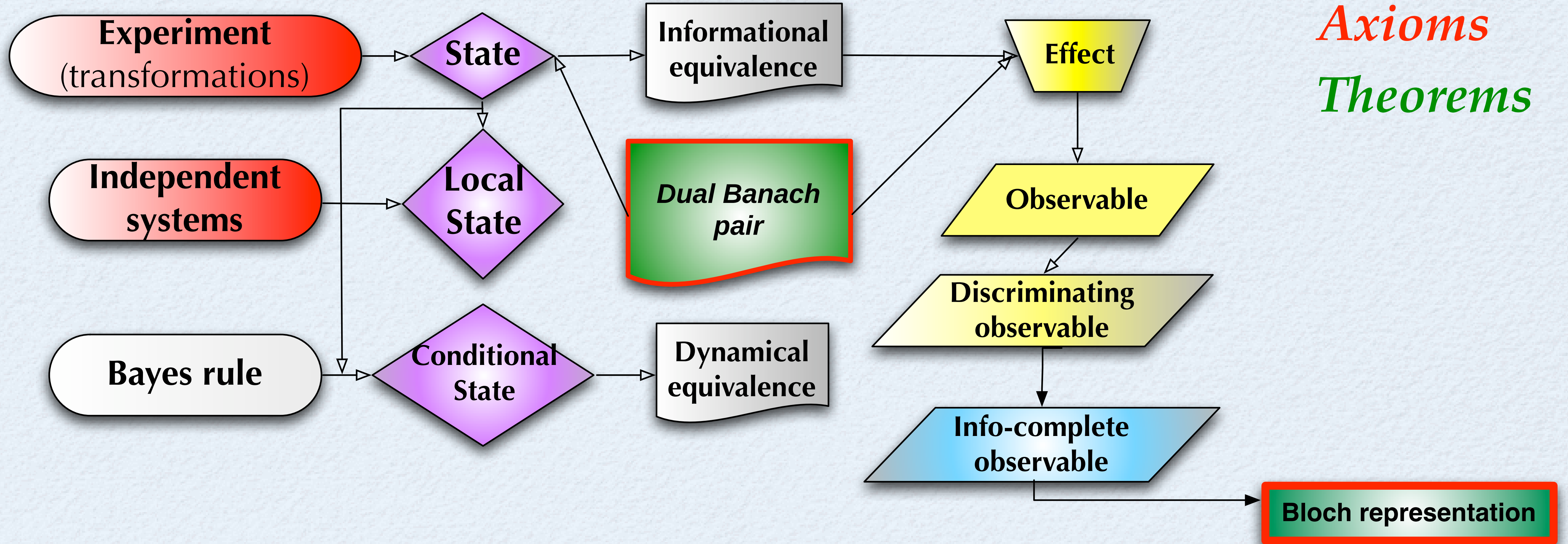

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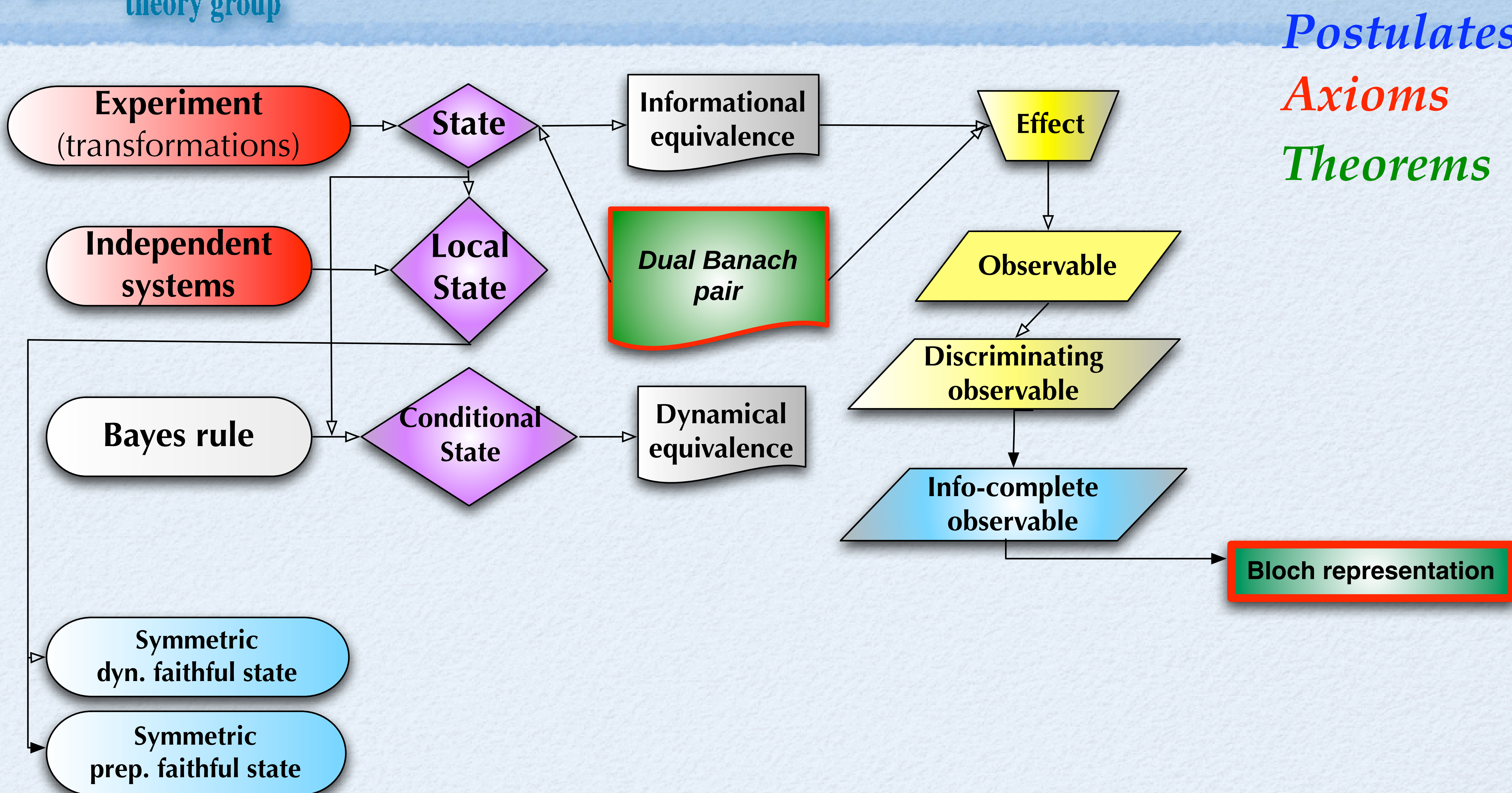

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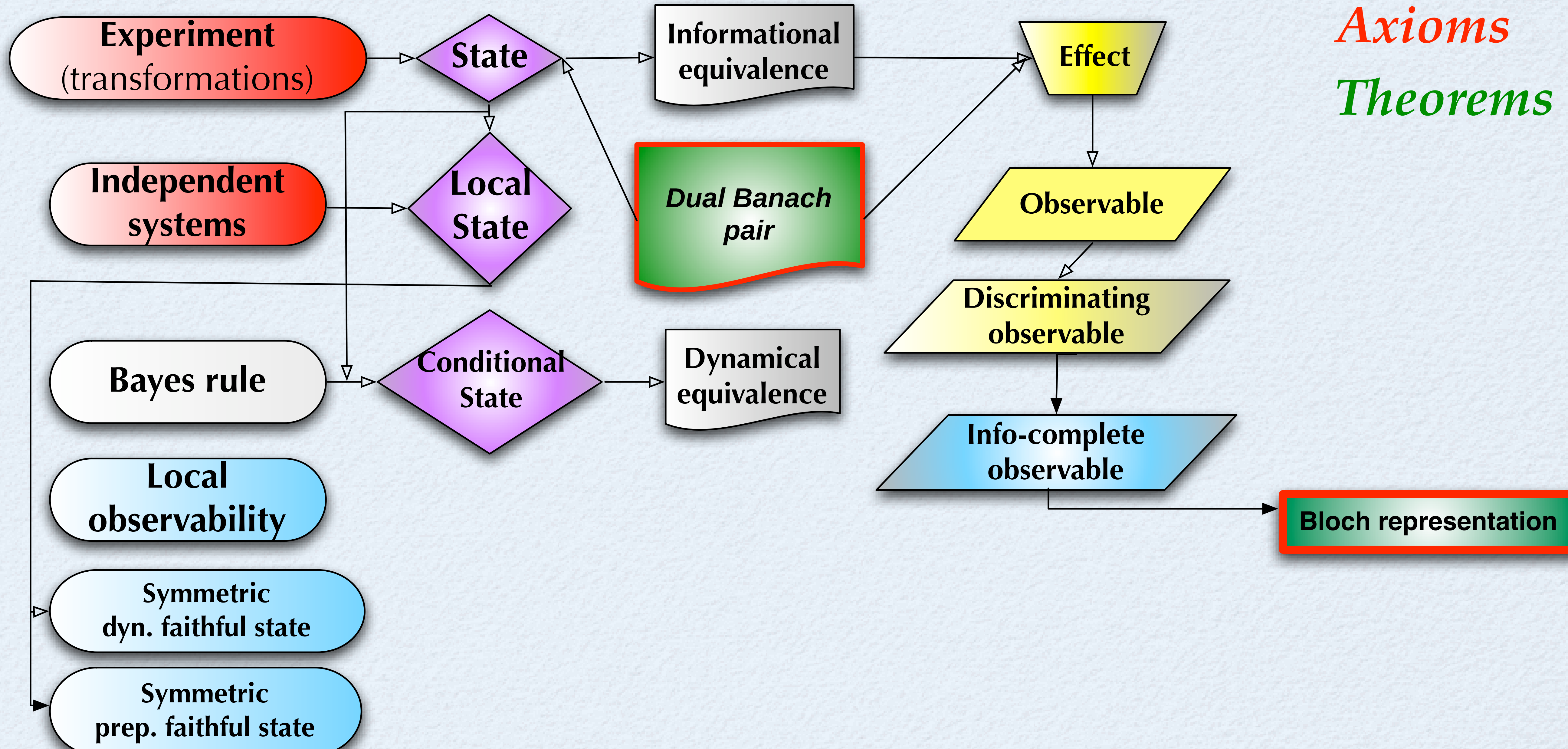

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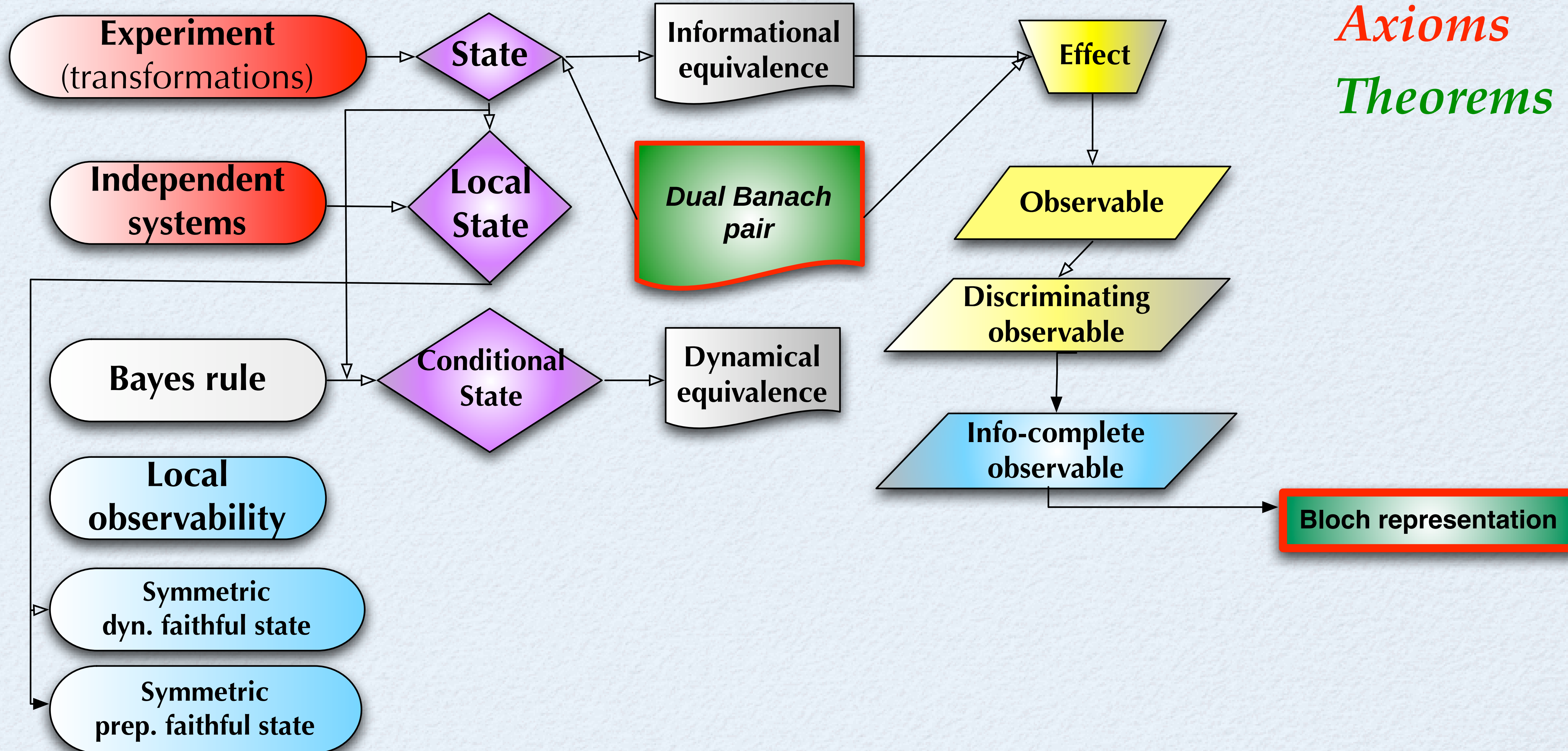

Info-complete from joint discriminating observable

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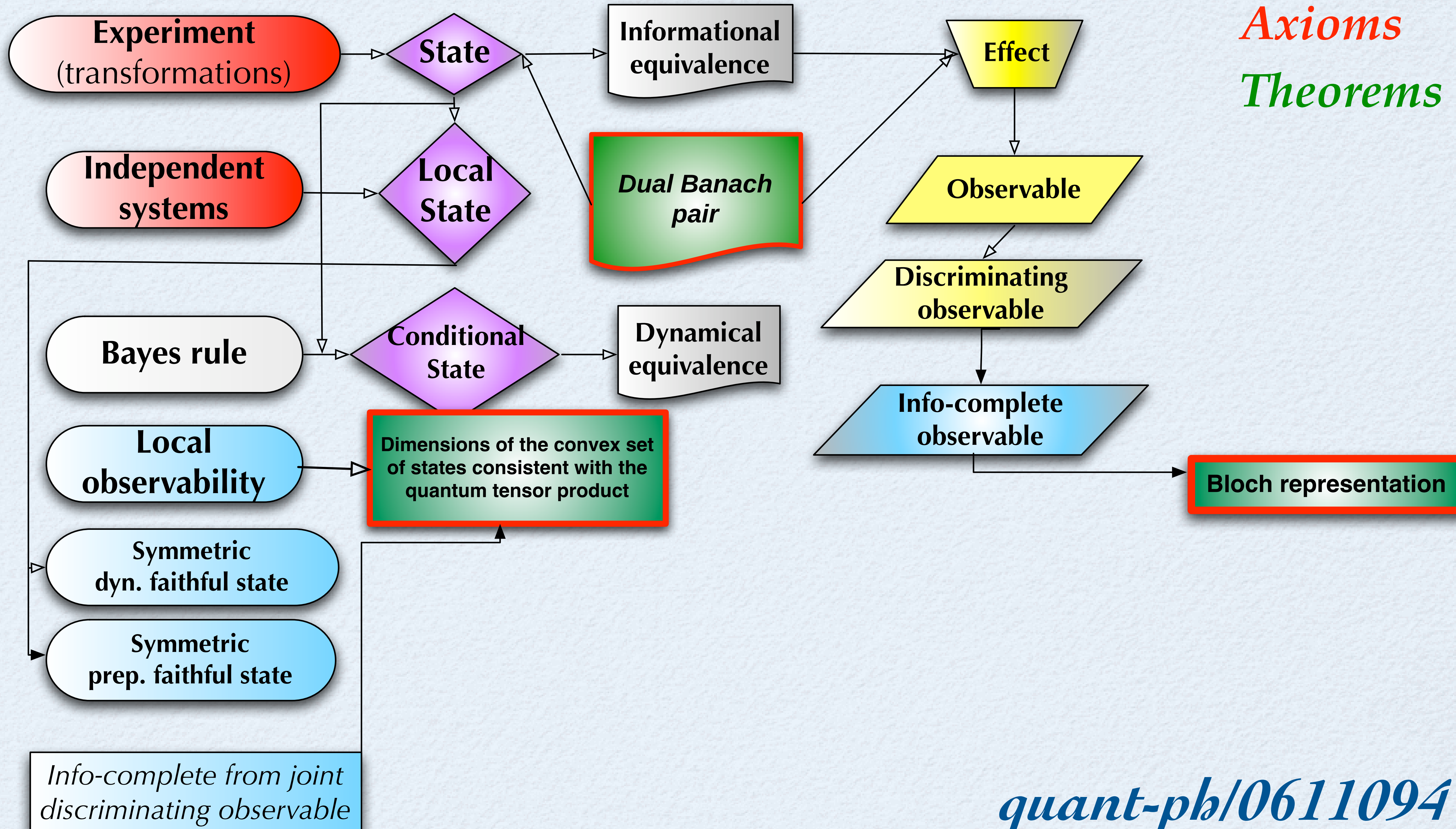

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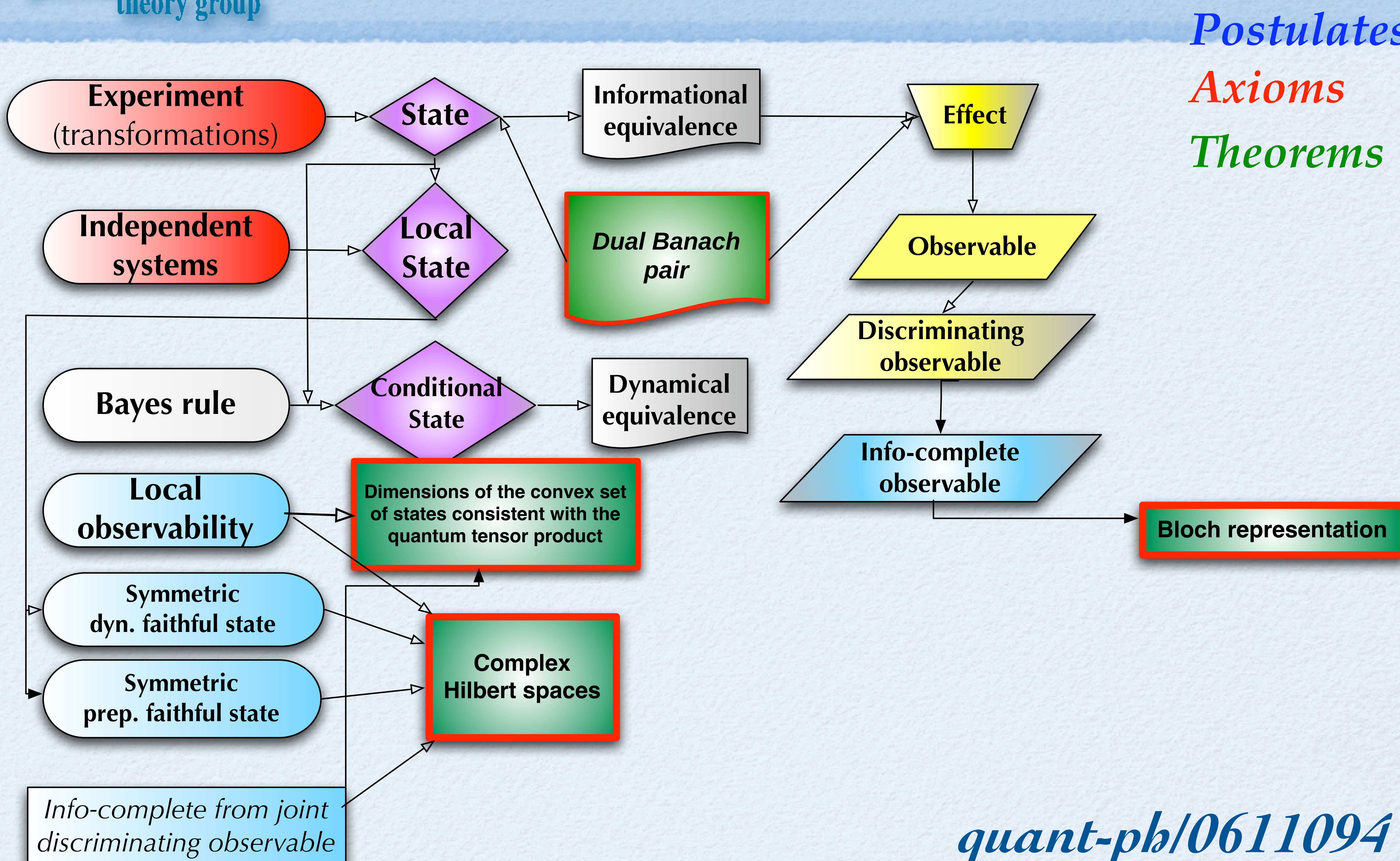

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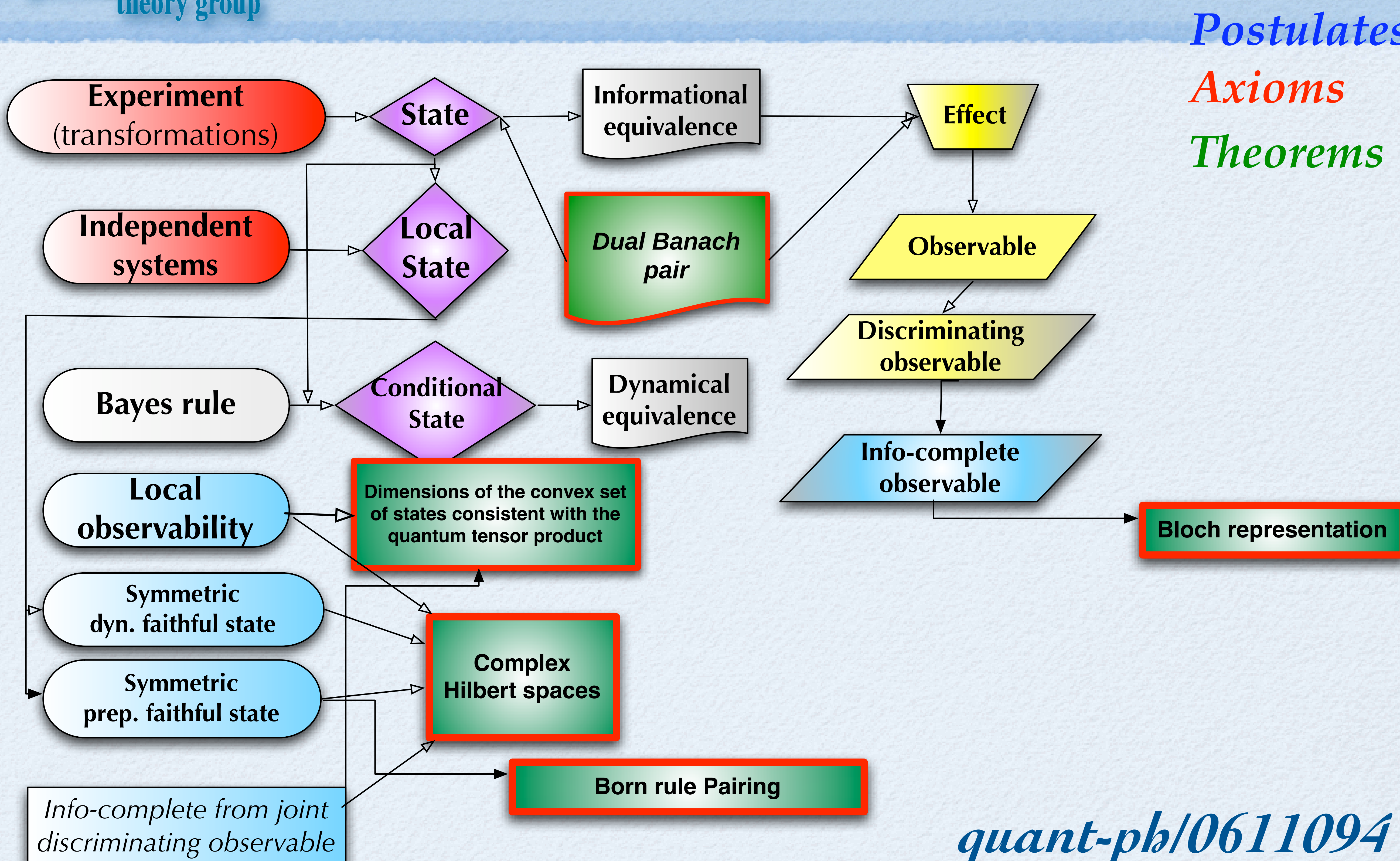

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