ISTITUTO LOMBARDO - ACCADEMIA DI SCIENZE E LETTERE

RENDICONTI

CLASSE DI SCIENZE MATEMATICHE E NATURALI

Vol. 140
(2006)

ESTRATTO

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QUANTUM CALIBRATION

 Istituto Lombardo (Rend. Scienze) 140, 015-036 (2006)

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GIACOMO MAURO D'ARIANO **

SUNTO. – Dopo una breve digressione su alcuni recenti progressi teorici in Meccanica Quantistica e nella teoria della misurazione, viene presentata una metodologia di recente scoperta che permette una caratterizzazione quantistica completa di un dispositivo, senza conoscerne il funzionamento interno. Il metodo è basato sul confronto con un apparato precedentemente calibrato – detto *tomographer* – e il confronto fra tomographer e apparato da calibrare è effettuato disponendo i due in un setup di correlazione con un input bipartito in uno stato cosiddetto *fedele*, ad esempio uno stato massimamente entangled. Il metodo è robusto a imperfezioni del tomographer, e funziona in pratica con la gran parte degli stati bipartiti di input.

The fun with 'Quantum Information' is that you can study the foundations of the enigmatic world of Quantum Mechanics, and, at the same time, you make something useful for practical applications.
(www.qubit.it homepage)

1. INTRODUCTION

'Information' is the paradigm of our times, as 'Energy' was the paradigm of the previous century. We are definitely in the era of computers

* Relazione presentata a: International Symposium - *Frontiere della Fisica*, tenuto all'Istituto Lombardo il 27 Novembre 2003.

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and of the Internet. On the other hand, we are also in the era of the Laser: Photonics is gradually taking place over Electronics, with the possibility of transmitting gargantuan amounts of informations with the optical wideband. We are approaching the Terabit era (with a Tbit/sec one could transmit the entire Wall Street Journal from its first issue in a handful of seconds!) The Laser, on the other hand, has also dramatically changed the face of the entire Physics. By the extremely intense radiation from a laser one can exert nonlinear optical effects – most of all frequency conversion – which would otherwise be negligible with conventional light. These effects on one side allow us to produce 'entanglement' – the most elusive quantum feature, and the main ingredient of the new science of Quantum Information [1], [2]. On the other side they help building up strong and controlled interactions with atoms, offering the unique opportunity of manipulating single atoms and photons, opening a new era for Quantum Optics and Atomic Physics. This facts along with the extreme miniaturization of nano-technology have renewed the interest in Quantum Mechanics. Finally, the marriage of Quantum Physics and Information Technology has recently opened the way to the realization of radically new information-processing devices, with the possibility of cryptographic communications which are guaranteed secure on the basis of physical laws [3], and with the perspective of the tremendous computational speedups which would be possible in principle with Quantum Computers [1], [2].

2. Recent results on Foundations of Quantum Mechanics

It is common opinion that there is little to be understood yet in Quantum Mechanics, and that the last relevant result was the work on nonlocality of John Bell [4]. This is definitely false. Indeed, the science of quantum 'Entanglement' is still at its early stages, and there is plenty of problems in Quantum Mechanics that are unsolved yet.

On the issue of nonlocality of Quantum Mechanics, Greenberger Horn an Zeilinger with their GHZ state [5] provided a kind of *Bell theorem without inequalities* as a very strong counter-argument to the Einstein Podolski Rosen incompleteness of Quantum Mechanics, clarifying how nonlocality and contextuality are intrinsic quantum features, independently on the probabilistic framework. Another small revolution

has been the *no-cloning theorem* [1] [6]-[12] which is considered a cornerstone of the theory of Quantum Information. From the foundational point of view, this theorem has also allowed to assess rigorously the impossibility of determining the wave function of a single system [13]. This may seem now a trivial issue: however, we should not forget the numerous attempts to determine the wave-function of a single system, that continued to appear up to the middle of the nineties, including papers by authoritative scientists [14]-[18] exploring concrete measurement schemes for such an impossible goal. Moreover, the no-cloning theorem has also lead to new insights on non-causality of entanglement, and on the impossibility of superluminal communications [21].

Another small revolution of the last years has certainly been the quantum teleportation of Bennett, Brassard, Crèpau, Jozsa, Peres, and Wotters [22], which showed that even though the state of a single system is unknowable and unclonable, the nonlocality of entanglement can be used to teleport the quantum state, using only two bits per spin. In order to achieve teleportation one performs a new kind of quantum measurement which gives no information on the state, nevertheless plays a crucial role in the accomplishment of teleportation. Francesco De Martini [23] in Roma La Sapienza and Anton Zeilinger in Innsbruck [24] have pioneered the first teleportation experiments. To somebody

[1] The *no cloning theorem* is usually attributed to W. K. Wootters, W. H. Zurek [7], and sometimes also to D. Dieks [8]. The paper by Wootters and Zurek originated as a comment against a paper by Nick Herbert [6], who devised a method for superluminal communication based on the assumption of perfect cloning of polarized photons by means of stimulated emission in a laser. (Ironically Nick Herbert even patent his method! Notice that the first impossibility proof for superluminal communications [19] appeared four years before his paper). However, without the general no-cloning theorem in those years it was not easy to understand how the quantum noise from spontaneous emission could nullify Herbert's method. The first proof of the no-cloning theorem was actually provided by Giancarlo Ghirardi in an unpublished referee report on the manuscript of Herbert's paper [6] (he actually recommended rejection). Ghirardi underestimated the value of his proof, and published it only later [9] (for a full story of the birth of the no-cloning theorem see Ref. [11]). One should notice that these early proofs of the no-cloning theorem were still not stringent, since they assessed that the cloning transformation violates the superposition principle, which applies to a set of at least *three* states. Actually, one cannot clone even *two* states, if they are not orthogonal, since this would violate unitarity (this more stringent version of the theorem is due to H, Yuen [10]). Finally, my favored way of stating the theorem is that the cloning transformation is non *isometric*, whence it cannot be achieved deterministically [12].

now quantum teleportation may seem a little thing, but it has dramatically changed our understanding of Quantum Mechanics.

In these last twenty years also the field of Quantum Measurements got a renewed interest. The paradigm has shifted from the negative viewpoint of 'uncontrollable disturbance of measurement' of the Messiah book [25] to the positive attitude of 'control of coherence' in engineering new quantum measurements. A relevant example is that of the Standard Quantum Limit for measuring the position of a free mass, which is involved in the problem of achieving the astonishing sensitivities needed for gravitational wave detection. The issue became the subject of a big debate in the '80. The Standard Quantum Limit was posed by Carlton Caves, Kip Thorne and other authors [26], following a faulty derivation by Braginski [27] – the disciple of Weber, the constructor of gravitational antennas – based on a misuse of the Heisenberg principle [28]. Then Yuen [29] showed that such limit can be breached, and Ozawa provided a feasible measurement scheme [30] for it.

One of the results of the debate on the Standard quantum Limit was to attract attention on the faulty interpretation of the of the Heisenberg principle itself, in the form that was originally formulated by Heisenberg in his 'γ-ray microscope gedanken experiment'. According to its popular version (which was then elevated to 'principle' by Ruark [31], and became the paradigm itself of Quantum Mechanics) it is impossible to measure one variable, say the momentum, of a conjugated pair (e.g. position q and momentum p) without 'disturbing' the value of the conjugated variable q of an amount Δq no less than the order of $\hbar/\Delta p$, where Δp is the accuracy of the measurement. In the correct interpretation both Δp and Δq are *a priori* uncertainties, and neither will result as a consequence of the disturbance due to the measurement[2]. As a matter

[2] Let me clarify the common confusion between 'uncertainty relations' and 'uncertainty principle', the former concerning the statistics of repeated measurements on an ensemble of equally prepared identical quantum systems, the latter, on the contrary, concerning a sequence of measurements on the same quantum system (this difference is well emphasized in the Jammer book [32]). The 'uncertainty relations' do not have any bearing on the issue of the measurement disturbance, since it can be experimentally tested by measuring each of the observables separately: at most one of the two rms deviations, say Δp can be considered as the precision of the *preparation*, e.g. by a collimator of particle momentum, and then Δq will results from the statistics of measuring only q. In other words, both Δp and Δq are *a priori* uncertainties according to the Born rule, and neither will result as a consequence of the disturbance due to the measurement.

of fact, since both Δp and Δq are intrinsical to the wave function before the measurement, they cannot be logically connected to the interaction with the apparatus. And in fact, a measurement model was provocatively proposed by Ozawa [33] in which the position of the particle is measured leaving the particle in a eigenstate of the momentum!

That the Heisenberg principle is just a folklore is witnessed by the fact that 'classics' of quantum mechanics do not even mention it: see, for example, the Landau and Lifshitz book [34], or the the book of Asher Peres [35][3]. Clearly, the original spirit of the 'γ-ray microscope gedanken experiment' is still valid in the sense of trade-off between information retrieved by a measurement and disturbance on the measured system: however, the general tradeoff is certainly not represented by the Heisenberg principle, and it is very hard to quantify (I personally derived a tradeoff of this kind in Ref. [36]).

Another commonplace in the theory of Quantum Measurements which has been recently reconsidered is the paradigm of repeatable and objective measurement, i.e. the von Neumann orthogonal measurement [37]. In Ref. [38] we have shown that, contrarily to the widespread belief, repeatable measurements are not necessarily described by orthogonal projectors – the customary *observable*. These are only few examples to show how research in Quantum Mechanics is still very active at the foundational level.

All this renewed energy has been brought back to foundations mainly from the new possible applications in Quantum Optics and Atomic Physics, and, above all, by the promises of Quantum Cryptography and Quantum Computation. On the application side, what makes Quantum Information so interesting is the fact that the goal of designing radically new physical devices involves all types of approaches, from the level of pure mathematics, to theory, computer simulation, and experiment. Indeed, as in any engineering process one has an optimization stage based on a complete classification of all possibilities, a feasibility study based on theory and simulations, and finally the experimental verification. And it is this variety of approaches which makes researchers fond of this field.

In the field of Quantum Measurements our group has also focused a coordinated effort to engineer a radically new generation of quan-

[3] In the Asher Peres Book, if you look for 'uncertainty principle' in the subject index, you'll discover that the page number is just the page of the index itself – a deliberate error, since there is no mention to the principle in the whole book.

tum devices useful for the forthcoming Quantum Information technology, most of all programmable quantum detectors [39] and universal detectors and tomographers [40]. The motivation is that in Quantum Mechanics one is faced with the problem that there are infinitely many observables, and there is no simple rule to measure any desired one by combining a small set of apparatuses. For example, even though one has an apparatus that measures the observable A and another one that measures the observable B, the two apparatuses are of no use for measuring the observable $A+B$ or the commutator $-i[A,B]$, and often one needs to rebuild a completely new device which is generally functioning in a completely different way from those used for the measurement of A and B. A *programmable detector* would achieve such task: this is a device that can be tuned to measure any desired observable. On the other hand, a *universal detector* is a device that performs a special kind of measure – named *informationally complete* – from which by only changing the data-processing of the measurement outcomes one can obtain the expected value of any desired observable.

3. Quantum Calibration

It is the theory which decides what we can observe!
Albert Einstein to Werner Heisenberg

The calibration of measuring apparatuses is at the basis of any actual experiment. What does it mean calibration? For example, calibrating a 'scale' means to put different known weights on it and annotate a corresponding notch for each pointer position. The same procedure would be not so easy for calibrating, for example, a photocounter, since we don't have standard sources with precise numbers of photons, and, moreover, we cannot be sure that the photon has been actually absorbed by the detector. This leads us to pose the following problem: if we have an apparatus which performs a quantum measurement, how can we know *what* and *how much* it measures? A kind of answer was given by Einstein to Heisenberg in the quotation reported above[4]. The

[4] Assessing the need of theory for observation seems to contrast the firm realistic beliefs of Albert Einstein! Indeed, it is well known that in regards of the objectiv-

experimentalist may not be happy of needing a theoretician in the lab to asses what it is being measured! But Quantum Mechanics is so: in order to establish which observable is measured by an apparatus, one needs a complete theoretical description of the interaction of the apparatus with the measured system. A paradigmatic case is that of the *photocounter* [41], where the number of photons claimed to be detected – usually very uncertain – is typically inferred from the cascading mechanism of the amplification process. The calibration is given essentially in terms of quantum efficiency and dark-current, and mostly saturation effects categorize detectors into the major classes of 'linear' and 'single-photon'. Even in a very simplified model, a theoretical description accounting for the above features is very involved [42], [43], and the resulting theoretical calibration is exceedingly indirect. Would it be nice to have a way to 'calibrate' a photocounter – and more generally any quantum measuring apparatus – without using a complete theoretical statistical mechanics description of its inner functioning?

To accomplish such task we need just a little theory about the phenomenological description of a measuring device, independently on its inner functioning. In a quantum mechanical description the complete characterization of a measuring apparatus corresponds to the knowledge of its POVM (positive operator-valued measure [44]), which gives the probability $p(n)$ of any measurement outcome n for arbitrary input state, via the Born rule

$$p(n) = Tr[\rho P_n]. \tag{1}$$

In Eq. (1) ρ is the density operator of the state of the system, and the POVM is given by the set of operators $\{P_n\}$. To ensure that $p(n)$ is a probability, the POVM must satisfy the positivity and normalization constraints $P_n \geq 0$, $\sum_n P_n = I$.

The concept of POVM generalizes the familiar von Neumann observable describing perfect measurements, according to which the probability of obtaining outcome n is given by $p(n) = |\langle\psi|o_n\rangle|^2$, $\{|o_n\rangle\}$ denoting a complete orthonormal basis of states, i.e. in this case the POVM is made of the one-dimensional projectors $P_n = |o_n\rangle\langle o_n|$. The physical interpretation of the measurement is given via a quantization

ity issue of quantum measurements, Einstein loved to mention that the Moon exists in its place even though we don't look at it. But in the present quotation actually Einstein is assessing the need of theory to know if the Moon is still there!

rule that associates a self-adjoint operator O to a classical observable, $|o_n\rangle$ being the eigenvector of O corresponding to its nth eigenvalue o_n. Such concept of observable, however, does not cover many practical situations – e.g. phase-estimation [45], [46], joint measurements of incompatible observables [47], [48], discrimination among non-orthogonal states [49], [50], informationally complete measurements [51], transmission of reference frames [52] – and here the POVM description is needed. But then, in absence of a direct physical interpretation of the measurement, the problem of assessing the correct functioning of the measuring apparatus becomes even more compelling.

The problem now is if it possible to measure experimentally the POVM of an apparatus without knowing its inner functioning, e.g. by comparing the apparatus with other (previously calibrated) apparatuses.

Another similar problem is the following. Is it possible to determine experimentally the transformation affected by an apparatus, namely the evolution of the state within the apparatus? In Quantum Mechanics the most general evolution of a state ρ – including the evolution due to a quantum measurement – is described by means of a *quantum operation* $\mathcal{E}$ as follows

$$\rho \rightarrow \frac{\mathcal{E}(\rho)}{Tr[\mathcal{E}(\rho)]}, \tag{2}$$

where the normalization factor $Tr[\mathcal{E}(\rho)] \leq 1$ also gives the probability that the transformation occurs, and the map $\mathcal{E}$ is linear and sends density operators to (unnormalized) density operators, i.e. it preserves positivity (technically, the map is completely positive, i.e. it preserves positivity of any bipartite state when applied locally). The simplest case of quantum operation is just the usual unitary evolution of isolated systems. More generally the quantum operation is a sum of the form (called Kraus form)

$$\mathcal{E}(\rho) = \sum_n K_n \rho K_n^\dagger \tag{3}$$

3.1. *Quantum tomography*

Now, in order to determine experimentally the operators P_n describing a measuring apparatus or the quantum operation $\mathcal{E}$ of any device, we need a method called *quantum tomography* [53], [54], [55]. This is simply a method to estimate the ensemble average $\langle H\rangle$ of any arbitrary operator H by measuring a *quorum* of observables $\{O_l\}$, namely simply a set of Hermitian operators O_l on which one can expand any operator H as

$$H = \sum_l \langle Q_l, H\rangle\, O_l. \tag{4}$$

Then the tomographic estimation of the ensemble average $\langle H\rangle$ is simply obtained by averaging over both the ensemble and the quorum (in infinite dimensions the derivation of the expansion is more involved, and, as we will see, becomes conveniently nonlinear). The estimation of the density matrix element ρ_{ij} corresponds to the operator given by the outer product of $H = |i\rangle\langle j|$. The method is very powerful, because is very robust to any kind of experimental imperfections, and a general approach is available to unbias instrumental noise [53]. Moreover, there are techniques (such as adaptive methods [56], maximum-likelihood strategies [57], etc) by which one can increase the precision of the estimation dramatically.

3.1.1. *Homodyne tomography*

The most popular example of quantum tomography belongs again to the field of Quantum Optics, and is called homodyne tomography, since it is based on homodyne detection (a schematic of an homodyne detector is depicted in *Fig. 1*). The quantum system is the harmonic oscillator associated to a given mode of the electromagnetic field, described by the annihilation and creation operators a and $a^\dagger$, respectively, with commutator $[a, a^\dagger] = 1$. The *quorum* is the set of *quadratures* $X_\varphi = \frac{1}{2}(a^\dagger e^{i\varphi} + a e^{-i\varphi})$, i.e. the real e.m. field at tunable phase φ relative to the local oscillator of the homodyne detector (see *Figure 1*). The actual form of the expansion depends on boundedness properties of the operator to be expanded. For example, for Hilbert Schmidt operators H one has

$$H = \int_0^\pi \frac{d\varphi}{\pi} E_H(X_\varphi;\varphi),\ E_H(x;\varphi) = \frac{1}{4}\int_{-\infty}^{+\infty} dk|k|\mathrm{Tr}[He^{ikX_\varphi}]e^{-ikx}. \tag{5}$$

As one can see that the expansion in X_φ is actually nonlinear. We introduced the original theoretical method in Pavia [58] in the beginning of 1993. The name 'tomography' came from an analogy with the Radon transform algorithm used for imaging in computerized axial tomography: here the probability distributions of the quadratures for varying φ are the marginals (or projections) of the Wigner function of the state, similarly to the set of radial radiographies for the density of mass to be reconstructed in the imaging process (see *Fig. 1*).

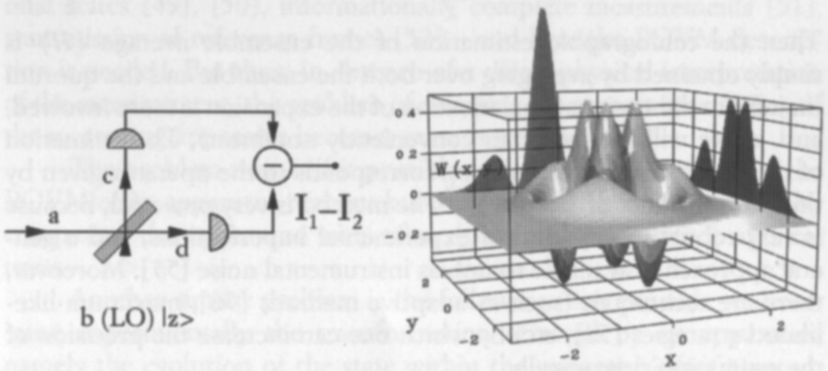

Fig. 1.

LEFT: Scheme of a balanced homodyne detector. The 'signal' mode a is combined by means of a 50-50 beam splitter with a 'local oscillator' (LO) mode b operating at the same frequency of a, and prepared in an 'intense' coherent state $|z\rangle$. At the output of the beam splitter one has the sum and difference modes $c=(a+b)/\sqrt{2}$ and $d=(a-b)/\sqrt{2}$. These output modes are detected by two identical photodetectors, and finally the difference of photocurrents I_1-I_2 (at zero frequency) is rescaled by $2|z|$, giving the quadrature X_φ where $\varphi=\arg(z)$. RIGHT: The probability distributions of the quadratures for varying φ are the marginals (or projections) of the Wigner function of the state, similarly to the set of radial radiographies for the density of mass to be reconstructed in the imaging process. In the figure the marginal distributions are the dark functions plotted on the axes, and the Wigner function is the two-variables function in the center. The marginals are obtained by integrating the Wigner function in the orthogonal direction (i.e. along the x axis for the marginal plotted on the y axis and along the y axis for the marginal plotted on the x axis). [In particular, this Wigner function refers to the state $\frac{1}{\sqrt{2}}(|0\rangle+|3\rangle)$]. [Figure kindly provided by L. Maccone.]

3.1.2. *Pauli tomography*

The situation for finite dimensions is much easier. For example, for dimensions two (namely a spin, or 'qubit') the tomographic expansion is just the Bloch form of an operator, namely the linear expansion over the Pauli matrices

$$H=\frac{1}{2}[\vec{\sigma}\cdot Tr(\vec{\sigma}H)+I\,Tr(H)]. \qquad (6)$$

In Quantum Optics the two-state system is a single photon with vertical or horizontal polarization, and the analogous of the homodyne detector is simply obtained using a polarizing beam splitter, two single-photon detectors, and a wave-plate as in *Fig. 2*.

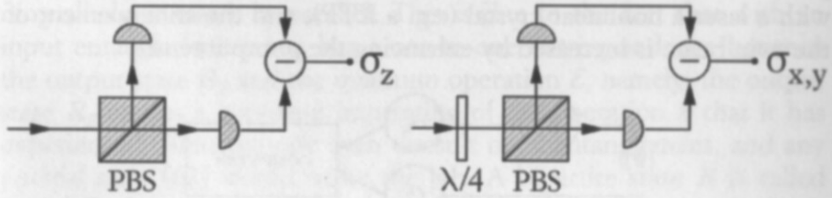

Fig. 2.

In Quantum Optics the qubits are encoded on polarization of single photons $|\uparrow\rangle\equiv|1\rangle_h|0_v|\downarrow\equiv|0\rangle_h 1\rangle_v$, and the Pauli matrices are given in terms of the two modes, e.g. $\sigma_z=h^\dagger h-v^\dagger v$. In this representation the Pauli-matrix detectors are realized as in figure, using polarizing beam splitters [(PBS), single-photon detectors, and $\lambda/4$ wave-plates].

3.2. *The quantum calibration method*

Now, let's come back to the original problem, namely how to determine experimentally the POVM or the quantum operation of a device.

3.2.1. *Calibration of a Quantum Operation of an apparatus*

In order to determine the transformation carried out by a device, a brute force method consists in performing an exhaustive scanning of all possible inputs, making a tomographic reconstruction of the corresponding outputs. This method, however, requires many input states (not just an orthonormal basis, but also some special linear combinations) [5], or a sufficiently over-complete set of states, such as coherent states [59]. This stringent requirements makes the method actually infeasible. As a matter of fact, in Quantum Optics producing states with a definite number of photons is and will remain an impossible task for many years – even for few photons. On the other side, 'coherent' over-complete sets of states are not easily available for atomic systems. Then, if we

[5] According to the polarization identity we need also all linear combinations $|n\rangle+i^k|m\rangle$ of elements of an orthonormal basis $\{|n\rangle\}$ with $k=0,1,2,3$, i being the imaginary unit.

cannot use the brute force method, what can we do? The solution was proposed by us in Ref. [60], [61], and uses just a single entangled state at the input. And, indeed in Quantum Optics it is very easy to produce entangled states: these are the well known twin-beams produced by parametric downconversion of vacuum, which are achieved by pumping with a laser a nonlinear crystal (e.g. a KTP), and the entanglement of the twin beam is increased by enhancing the pump intensity.

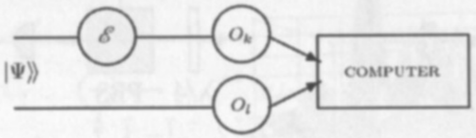

Fig. 3 – General experimental scheme of the method for the tomographic estimation of a quantum operation.

Two identical quantum systems are prepared in an entangled state $|\psi\rangle\rangle$. One of the two systems undergoes the quantum operation $\mathcal{E}$, whereas the other is left untouched. At the output one makes a quantum tomographic estimation, by measuring jointly two random observables from a quorum $\{O_l\}$ (see the text).

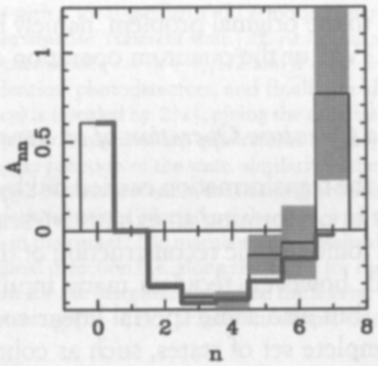

Fig. 4 – Homodyne tomography of the quantum operation A corresponding to the unitary displacement of one mode of the radiation field.

Diagonal elements A_{nn} (shown by thin solid line on an extended abscissa range,) with their respective error bars in gray shade, compared to the theoretical probability (thick solid line). Similar results are obtained for all upper and lower diagonals of the quantum operation matrix A. The reconstruction has been achieved using an entangled state $|\psi\rangle\rangle$ at the input corresponding to parametric downconversion of vacuum with mean thermal photon $\bar{n}$ and quantum efficiency at homodyne detectors η. Here $z=1$, $\bar{n}=3$, $\eta=0.7$ and 300 blocks of $2{\cdot}10^5$ data have been used. These are the same parameters of the experiment in Ref.[67]. (From Ref. [60]).

The main idea of the method which uses a single entangled state in place of many input states is sketched in *Fig. 1*. Here only one of two entangled quantum systems (e.g. the twin beams) experiences the quantum operation, whereas the other system is left untouched. In this way a single entangled input state $|\psi\rangle\rangle$ performs the same work of scanning all input states in parallel. This is due to the fact that with an input entangled state one has a one-to-one correspondence between the output state $R_{\mathcal{E}}$ and the quantum operation $\mathcal{E}$, namely, the output state $R_{\mathcal{E}}$ carries a complete imprinting of the operation $\mathcal{E}$ that it has experienced. Actually, one even doesn't need entanglement, and any *faithful state* [62] would make the job. A bipartite state R is called 'faithful' when acting with a device on R in a 'fork' scheme as in *Fig. 3* the output $R_{\mathcal{E}}$ will carry a complete information about the operation $\mathcal{E}$ of the device. One can easily see that a state R is faithful when its associated map $\mathscr{R}$ defined as $\mathscr{R}(\rho) = Tr_1[(\rho^{\mathsf{T}} \otimes I) R]$ is invertible (X^{T} denotes the transposition with respect to a reference basis), namely R can be obtained from a maximally entangled state via an invertible map $\mathscr{R}$. Then, since invertibility is a 'dense' condition, essentially any state is faithful: however, the knowledge of the map $\mathcal{E}$ from the measured output state $\mathscr{R}_{\mathcal{E}}$ will be affected by increasingly large statistical errors when the input state R approaches a non-faithful one [6]. A computer simulation of the homodyne tomography of a quantum operation is reported in *Fig. 4*.

De Martini in his labs in Rome performed an experiment as a demonstration of the method for characterizing a single qubit device, based on Pauli tomography [63]. In *Fig. 5* the experimental results are reported, in very good agreement with the theoretical prediction. Actually, the De Martini team invented a method for preparing in a very controlled way all sorts of bipartite states (mixed, entangled, etc.) [64] (see a sketch of the setup in *Fig. 6*), whereas in Pavia we developed a method [65] to characterize the bipartite state more efficiently than with quantum tomography, based on some prior knowledge of the state (based on the technique called *entanglement witness*): some experimental results are reported in *Fig. 6*.

[6] In infinite dimensions there are also convergence issues which give thresholds for mixing noise on the input state.

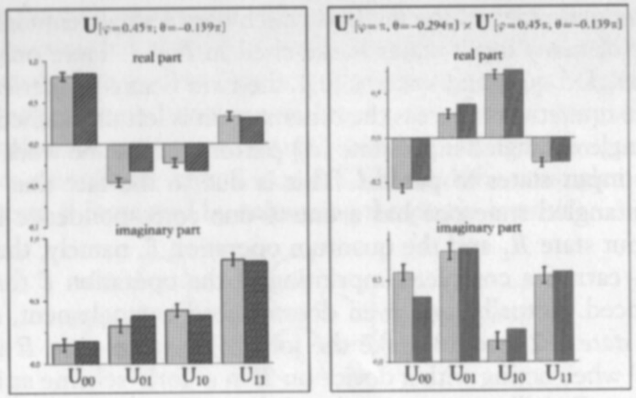

Fig. 5 – The first experiment of quantum calibration performed in Rome [63].

LEFT: Experimental characterization of a single optical wave-plate with retardation phase $\varphi = 0.45\pi$ and orientation angle of the optical axis respect to the laboratory horizontal direction $\theta = -0.138\pi$. The experimental matrix elements U_{nm} of the wave-plate are superimposed to statistical errors for 8000 events, and compared with the theoretical values.
RIGHT: The same experimental characterization as on the left figure, here for a device made of a series of two optical wave-plates: a wave-plate with $\varphi = 0.45\pi$ and $\theta = -0.138\pi$ followed by another wave-plate with $\varphi = \pi$ and $\theta = +0.29\pi$ [figures from Ref. [63].

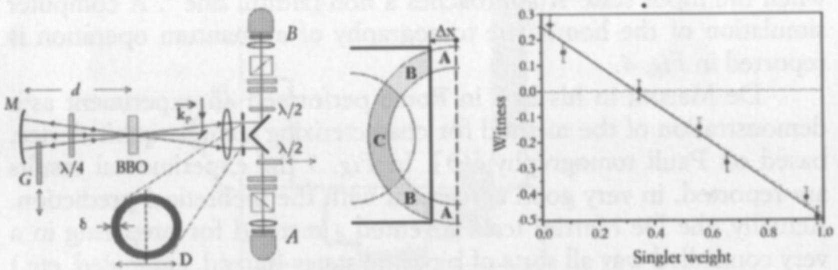

Fig. 6 – Experiment of entanglement witness performed in Rome [66].

LEFT: Scheme of the experimental apparatus. The polarization entangled photons are generated by spontaneous parametric downconversion in a nonlinear BBO crystal, and are detected by two silicon-avalanche photodiodes preceded by polarization analyzers. In this experiment a novel geometry is exploited for collecting down-converted radiation, which results in a very bright source of entangled photons, with also the advantage of complete freedom in choosing the two-photon state.
RIGHT: Experimental results of entanglement detection for Werner states. These states range from the pure singlet at weight $p = 1$ to the totally chaotic state at $p = 0$, with the transition between entangled and separable states at $p = 1/3$ The straight line corresponds to the theoretical prediction. The experimental results verify the transition between separable and entangled Werner states, occurring at zero-witness at $p = 1/3$.

3.2.2. *Calibration of the POVM of a measuring apparatus*

How we can calibrate a POVM? We need to use a faithful state R as the input in the calibration scheme sketched in *Fig. 7*, and use the simple formula

$$P_n = p(n)\,[R^{-1}(\rho_n)]^{\mathsf{T}}. \tag{7}$$

where $p(n)$ is the probability of the outcome n, $\mathscr{R}$ is the associated map of the faithful state R, and ρ_n is the conditioned state to be determined by quantum tomography. The probability $p(n)$ is determined easily as the experimental frequency of the outcome n. The knowledge of the state R (whence the associated map), clearly requires a pre-calibration stage, in which we determine the state R of the apparatus by performing a joint tomography with two tomographers (e.g. homodyne detectors). Then, we substitute one of the two tomographers by our measuring apparatus, and perform the true calibration stage. In this way in principle we need only two tomographers and a single faithful state to calibrate any measuring apparatus. In Quantum Optics the homodyne detector can be calibrated in a relatively easy way using just the vacuum state (in the homodyne setup photodetectors work in the linear regime). The special instance in which the measuring apparatus actually measures a customary 'observable' can be easily recognized directly from the homodyne data, which give a commutative POVM. In such case one only needs to reconstructs the conditioned probability distribution $p(n|k) = \langle k|P_n|k\rangle$. Then, when using the 'calibrated observable' the measurement will be unbiased, however, generally at expense of some increasing statistical error.

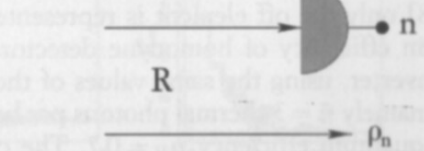

Fig. 7 – Experimental setup to determine the POVM of the unknown measurement apparatus.

The apparatus is used jointly with a 'tomographer' on a bipartite system prepared in a predetermined state R. The tomographer determines the state ρ_n conditioned by the measurement outcome n at the measurement apparatus. The results are then numerically processed, for different outcomes n to finally obtain the POVM $\{P_n\}$ of the measurement apparatus.

In principle the method can be used to calibrate a real photocounter, using a twin beam from a pumped nonlinear crystal and two homodyne detectors. For the real experiment there are, however, technical issues that I'll not discuss here, mainly related to the mode matching between the two different kinds of detectors – the homodyne and the photocounter – and to the control of the phase of the LO at homodyne detectors with respect to the pump of the doenconverter. Alex Lvovsky of the university of Konstanz and the group of Marco Genovese at Galileo Ferraris in Torino are interested in performing the experiment. In principle one can even calibrate the full shape of the photocurrent, and each feature (e.g. peak area, location, height, shape, etc.) will correspond to a different observable compatible with the number of photons (i.e. commuting with it). Moreover, it must be mentioned that the pre-calibration stage has been already performed in an old experiment that we did with Prem Kumar at Northwestern in 2000 [67], where all diagonal matrix elements of a twin-beam have been tomographically characterized, using two homodyne detectors and a twin-beam from parametric down-conversion by a nonlinear optical amplifier (see *Fig. 8*).

Actually, if one wants also the off-diagonal matrix elements, then there is the technical issue of the control of the phases of the LO's at the homodyne detectors relative to the pump.

In *Fig. 9* a computer simulation is reported of a homodyne calibration of an On/Off photodetector [68]. You can see qualitatively how the On element of the POVM (on the right) responds more to photons, where the Off element on the left responds more to the vacuum. The simulation program, written by Paoloplacido Lo Presti, is very sophisticated, and uses the best state-of-the-art maximum-likelihood algorithms precisely designed for this purpose. In *Fig. 10* from Ref. [68] only the off element is represented for different values of quantum efficiency of homodyne detectors and of the gain of the down-converter, using the same values of the Northwestern experiment [67], namely $\bar{n} = 3$ thermal photons per beam, $N = 10^6$ data and homodyne quantum efficiency $\eta_H = 0.7$. The calibration of the photocounter has been considered mostly for the sake of demonstration. Indeed a simple photocounter maybe more easily calibrated by other means. Our technique, however, is general, and can be used in principle for any measuring apparatus, on any physical quantum system.

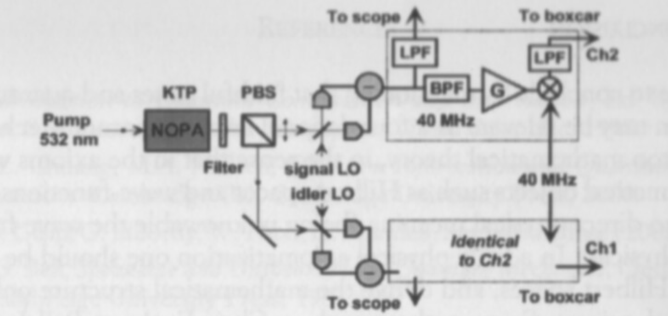

Fig. 8 – The experiment performed at Northwestern by Prem Kumar and coworkers [67] in order to measure the joint photon-number probability distribution of a twin-beam.
Schematic of the experimental setup: NOPA, non-degenerate optical parametric amplifier; LOs, local oscillators; PBS, polarizing beam splitter; LPFs, low-pass filters; BPF, band-pass filter; G, electronic amplifier. Electronics in the two channels are identical. [From Ref. [67]].

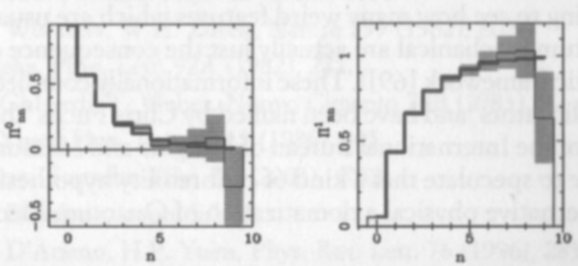

Fig. 9 – Homodyne tomography of an On/Off photodetector with quantum efficiency h = 0.4 and thermal noise photon number n = 0.1.
The reconstruction is obtained by pattern-function averaging of $1.5 \cdot 10^6$ data, for homodyne quantum efficiency $\eta = 0.9$ and twin beam thermal photon $\bar{n} = 3$. [From Ref. [68]].

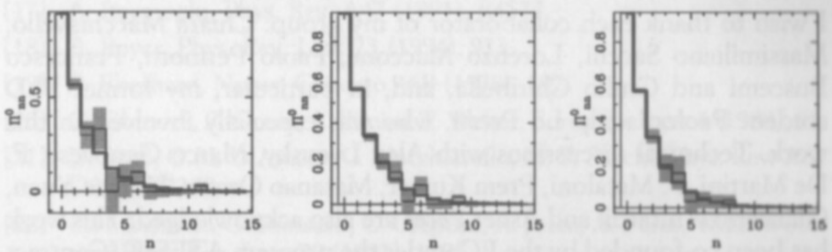

Fig. 10 – Homodyne tomography of an On/Off photodetector with quantum efficiency h = 0.4 and thermal noise photon number n = 0.1, with $\bar{n} = 3$ photons in the twin-beam.
The ML estimation of the diagonal of the only Off POVM element are reported for different values of sample size N and homodyne quantum efficiency η_H. Left: $N = 10^5$, $\eta_H = 0.7$; Middle: $N = 10^4$, $\eta_H = 0.9$; Right: $N = 10^6$, $\eta_H = 0.7$ [From Ref. [68]].

4. Conclusion

I'd like to conclude by mentioning that faithful states and quantum calibration may be relevant at a foundational level. Quantum Mechanics is still a too mathematical theory, in the sense that in the axioms we have mathematical objects such as Hilbert spaces and wave-functions, which have no direct physical meaning (being unknowable the wave-function is unphysical). In a truly physical axiomatization one should be able of avoid Hilbert spaces, and derive the mathematical structure only from physical axioms. Some authors such as Chris Fuchs at Bell Labs and Lucien Hardy at Oxford have shown that the so-called 'informationally complete' POVM's – which are essentially a kind of 'calibrators' – play an important role at the foundational level, since they allow to derive the full quantum mechanical description in a purely probabilistic settings (it is amazing to see how many weird features which are usually considered quantum mechanical are actually just the consequence of a purely probabilistic framework [69]). These informationally complete POVM's are our 'calibrators' and have been named by Chris Fuchs 'the quantum standards of the International Bureau of Weights and Measures à Paris'. So, I'd love to speculate that a kind of calibrability hypothesis may support an alternative physical axiomatization of Quantum Mechanics.

Acknowledgements

I wish to thank each collaborator of my group: Chiara Macchiavello, Massimiliano Sacchi, Lorenzo Maccone, Paolo Perinotti, Francesco Buscemi and Giulio Chiribella, and, in particular, my former PhD student Paoloplacido Lo Presti, who was especially involved in this work. Technical discussions with Alex Lvovsky, Marco Genovese, F. De Martini, P. Mataloni, Prem Kumar, Masanao Ozawa, Horace Yuen, Giancarlo Ghirardi, and Asher Peres are also acknowledged. This work has been co-founded by the EC under the program ATESIT (Contract No. IST-2000-29681), the MIUR cofinanziamento 2002, and MIUR cofinanziamento 2003. Partial support also comes from the MURI program administered by the U.S. Army Research Office under Grant No. DAAD19-00-1-0177.

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