

## Isotropic Phase Number Squeezing and Macroscopic Quantum Coherence.

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**Summary.** — A new master equation performing isotropic phase number squeezing is suggested. The phase properties of coherent superpositions are analysed when the state evolves in the presence of a bath with fluctuations squeezed in this isotropic way. We find that such a reservoir greatly improves the persistence of coherence with respect to either a customary thermal bath, or to an anisotropically squeezed phase-sensitive bath.

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### 1. - Introduction.

Recently, much attention has been focused on quantum interference effects for superpositions of macroscopically distinguishable states [1-10], with attempts at observing non-classical features at a macroscopic level [11]. Considerable effort has been devoted to the investigation of the influence on macroscopic superpositions due to dissipation, which rapidly destroys quantum coherence, and makes quantum effects non-detectable in practice [12].

With the aim of reducing the effect of dissipation on a macroscopic superposition, Kennedy and Walls [3] have studied the time evolution of an initial superposition of

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coherent states for a single mode of the field interacting with a bath having squeezed fluctuations [13]. They have shown that when the fluctuations are squeezed in the right quadrature a squeezed bath becomes more efficient than a thermal one in preserving interference fringes in the output photocurrent of a homodyne detector [14]. More recently, Bužek, Kim and Gantsog [8] have investigated the phase properties of quantum superpositions of two coherent states under squeezed amplification, showing that a suitable phase-sensitive amplifier is able to preserve the phase distribution of the input state (Schrödinger-cat state [15]).

In ref. [3] and [8] a superposition of two coherent components is considered as a test quantum superposition. In this case the quantum state itself determines a privileged direction—the line joining the two peaks in the complex plane—which selects the quadrature suited to squeezing. Nevertheless, superpositions of macroscopically distinguishable states can be formed by more than two states. For example, during the time evolution of an initial coherent state in a Kerr medium [2], one can have superpositions of three, four, and even more coherent states. For this reason, in this paper we suggest a new master equation for «isotropic» phase number squeezing, which is much more efficient in preserving coherence of any general superposition compared to an anisotropic squeezed bath. We analyse coherence by observing the phase (quasi) probability that is marginal to the Wigner function [16],

$$(1) \quad P(\phi) = \int_0^\infty r W(r \exp[i\phi], r \exp[-i\phi]) dr,$$

where the Wigner function [17] is defined by

$$(2) \quad W(\alpha, \alpha^*) = \int \frac{d^2\lambda}{\pi^2} \exp[-\lambda\alpha^* + \lambda^*\alpha] \text{Tr} \{ \hat{\rho} \exp[\lambda a^\dagger - \lambda^* a] \},$$

and  $\hat{\rho}$  is the density matrix of the system. We will compare numerical results for the time evolution of an initial superposition state in the cases of isotropic, directional, and vanishing squeezing, concluding that the isotropic squeezing is much more effective in preserving the peak structure of the phase distribution than the other two cases.

This paper is organized as follows: in sect. 2 we introduce the model and the master equation. In sect. 3 we show the results of numerical integration in terms of  $P(\phi)$  and of the Wigner function. Section 4 concludes the article with a short discussion and some remarks.

## 2. - The master equation.

The master equation for the reduced density operator of a single mode of the electromagnetic field in a squeezed bath can be derived from the knowledge of the correlation functions of the bath operators [18]. In the interaction picture and in the rotating-wave approximation one obtains [3]

$$(3) \quad \frac{d\hat{\rho}}{dt} = \gamma(N+1)(2a\hat{\rho}a^\dagger - a^\dagger a\hat{\rho} - \hat{\rho}a^\dagger a) + \gamma N(2a^\dagger\hat{\rho}a - aa^\dagger\hat{\rho} - \hat{\rho}aa^\dagger) - \\ - \gamma M(2a^\dagger\hat{\rho}a^\dagger - a^\dagger a^\dagger\hat{\rho} - \hat{\rho}a^\dagger a^\dagger) - \gamma M^*(2a\hat{\rho}a - aa\hat{\rho} - \hat{\rho}aa),$$

where  $a, a^\dagger$  are the boson annihilation and creation operators of the mode,  $\gamma$  is the damping constant, and  $M = |M| \exp[i\psi]$  is the squeezing complex parameter satisfying  $|M|^2 \leq N(N+1)$ . For  $M=0$  the reservoir reduces to the usual thermal bath and  $N$  becomes the mean number of thermal photons; for  $|M|^2 = N(N+1)$  the squeezing is maximum. The master equation (3) describes a situation in which the noise transferred to the system increases quadrature fluctuations in one direction more rapidly than in the other ones. The choice of the squeezing direction is determined *a priori* by the phase  $\psi$  of the squeezing parameter  $M$ . In this sense the master equation (3) represents a «unidirectionally» squeezed bath.

In order to obtain an «isotropic» squeezing we modify eq. (3) in such a way that the phase of  $M$  is dynamically shifted as a function of the phase of the field. For highly excited states  $\hat{\rho}$  the phase factor of the state can be approximately given by the following expectation value:

$$(4) \quad \exp[\pm i\phi] \approx \text{Tr}[\hat{e}^\mp \hat{\rho}],$$

where  $\hat{e}^-$  and  $\hat{e}^+ \equiv (\hat{e}^-)^\dagger$  are the shift operators

$$(5) \quad \hat{e}^- = (aa^\dagger)^{-1/2} a, \quad \hat{e}^+ = a^\dagger (aa^\dagger)^{-1/2},$$

acting on the Fock space as follows:

$$(6) \quad \hat{e}^+ |n\rangle = |n+1\rangle, \quad \hat{e}^- |n\rangle = |n-1\rangle.$$

This suggests using  $\hat{e}^\mp$  as dynamical phase factors in the master equation. As the phase of squeezing rotates at double frequency with respect to the average field, the shift operators should appear in pairs in the squeezing part of the Liouvillian (3). This last observation, along with the requirement of an isometrical master equation (which must preserve normalization of  $\hat{\rho}$ ) leads us to consider the following substitutions in the squeezing term of eq. (3):

$$(7) \quad a^\dagger \rightarrow \hat{e}^- a^\dagger, \quad a \rightarrow a \hat{e}^+.$$

Taking into account the following operator identities:

$$(8) \quad \hat{e}^- a^\dagger = a \hat{e}^+ = \sqrt{\hat{n}+1},$$

the substitutions (7) suggest changing the master equation (3) into the form

$$(9) \quad \frac{d\hat{\rho}}{dt} = \gamma(N+1)(2a\hat{\rho}a^\dagger - a^\dagger a\hat{\rho} - \hat{\rho}a^\dagger a) + \gamma N(2a^\dagger \hat{\rho} a - a a^\dagger \hat{\rho} - \hat{\rho} a a^\dagger) - \\ - 2\gamma \Re(M)[2\sqrt{\hat{n}+1}\hat{\rho}\sqrt{\hat{n}+1} - (\hat{n}+1)\hat{\rho} - \hat{\rho}(\hat{n}+1)].$$

The master equation (9) here derived in a heuristic way could represent a feedback-driven thermal bath which detects the phase of the state dynamically. As we will see, the relaxation (9) is very effective in preserving coherence, and thus a more fundamental derivation of (9) is motivated (work is in progress along this line). Notice that the stationary solution of eq. (9) is still the thermal distribution, but the decay of the off-diagonal terms is achieved for longer times.

The time evolution of an initial coherent state  $|\alpha_0\rangle = |4.0\rangle$  according to the master

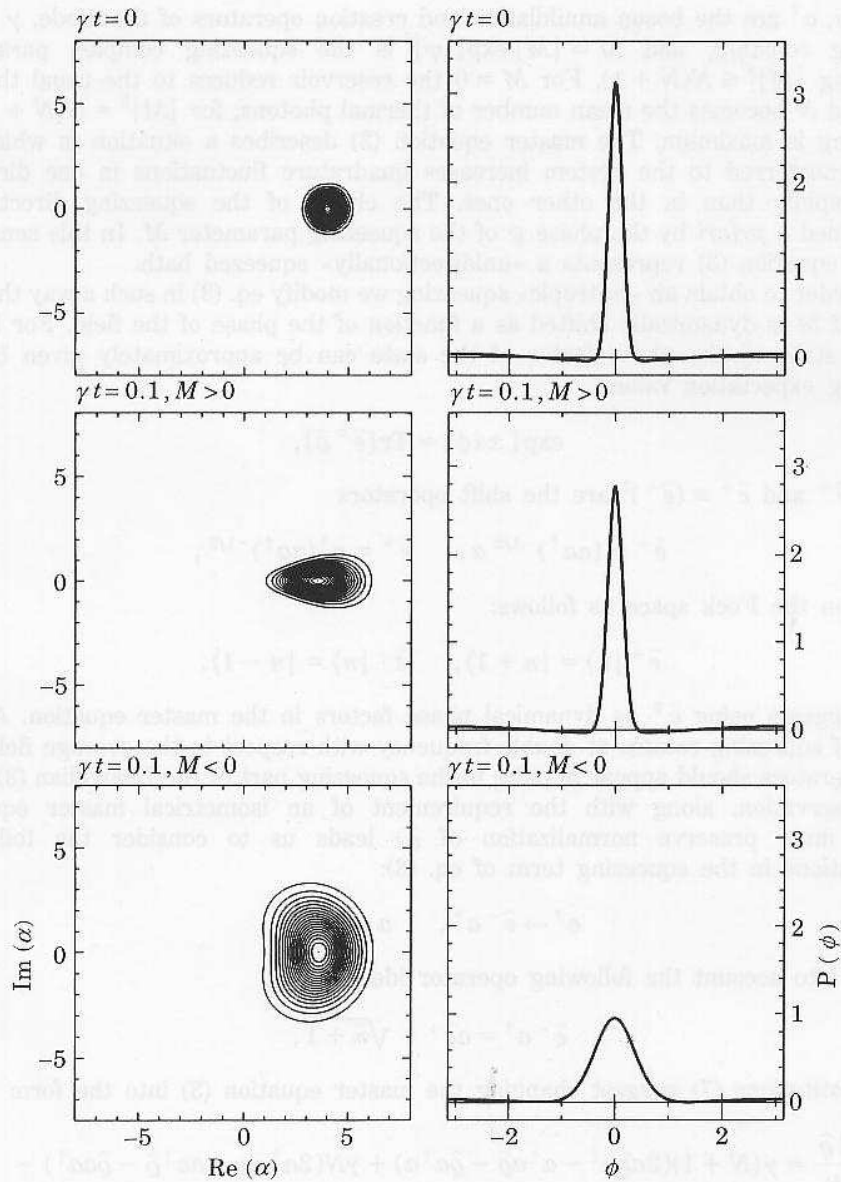


Fig. 1. — The effect of isotropic squeezing (master equation (9)) for positive and negative values of  $M$  on a coherent state with  $\alpha_0 = 4$ . Contour plots of the Wigner function and  $P(\phi)$  distribution evolved in time for  $N = 10$  and  $|M| = \sqrt{N(N+1)}$ .

equation (9) is given in fig. 1 for two different values of  $M$ . Here the Wigner function and the phase distribution  $P(\phi)$  are plotted at  $t = 0.1 \gamma^{-1}$  for  $N = 10$  and  $M = \pm \sqrt{N(N+1)}$  (actually,  $M$  now becomes a real parameter, because its imaginary part does not contribute in (9)). It is evident that the effect of the isotropic squeezing

corresponds to phase-squeezing for  $M > 0$  and to number-squeezing for  $M < 0$ . Due to dissipation, in addition to squeezing the Wigner function also moves slightly towards the origin of the phase space. In sect. 3 we will study the time evolution of a superposition of three coherent states (generated via Kerr effect), and we will analyse the persistence of phase coherence of the state.

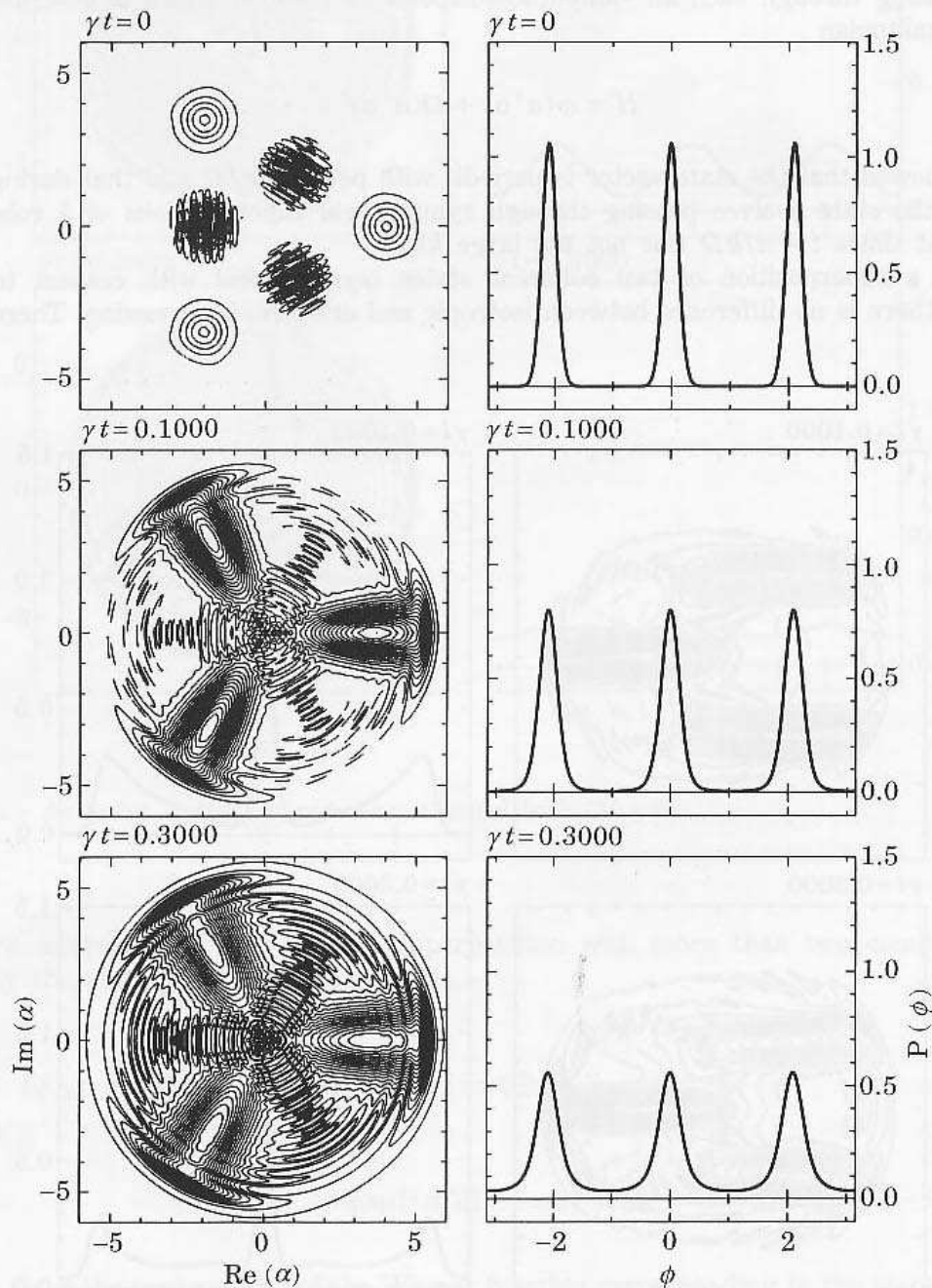


Fig. 2. - Contour plots of the Wigner function and  $P(\phi)$  distribution evolved in time for initial state (11), and for isotropic squeezing (master equation (9)). Here  $N = 30$ ,  $M = \sqrt{N(N+1)}$  and  $\alpha_0 = -4$ .

### 3. - Persistence of phase coherence.

The Kerr non-linearity in quantum optics is probably the best candidate to produce quantum superpositions of macroscopically distinguishable states. In ref. [2], Yurke and Stoler investigated the time evolution of an initial coherent state propagating through such an «amplitude-dispersive» medium which is described by the Hamiltonian

$$(10) \quad H = \omega(a^\dagger a) + \Omega(a^\dagger a)^2.$$

They showed that the state vector is periodic with period  $2\pi/\Omega$  and that during one period the state evolves passing through symmetrical superpositions of  $k$  coherent states at times  $t = \pi/k\Omega$  (for not too large  $k$ ).

For a superposition of two coherent states (symmetrical with respect to the origin) there is no difference between isotropic and anisotropic squeezing. Therefore,

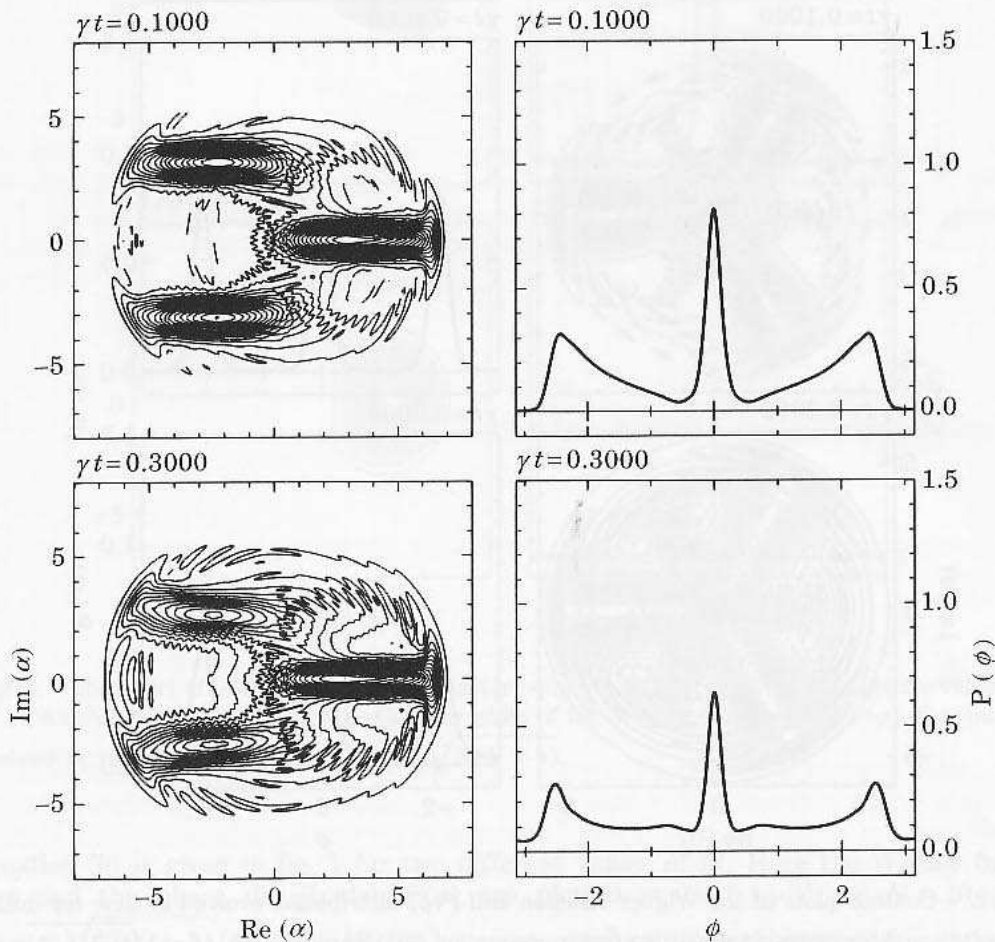


Fig. 3. - As in fig. 2, but for anisotropic squeezing (master equation (3)).

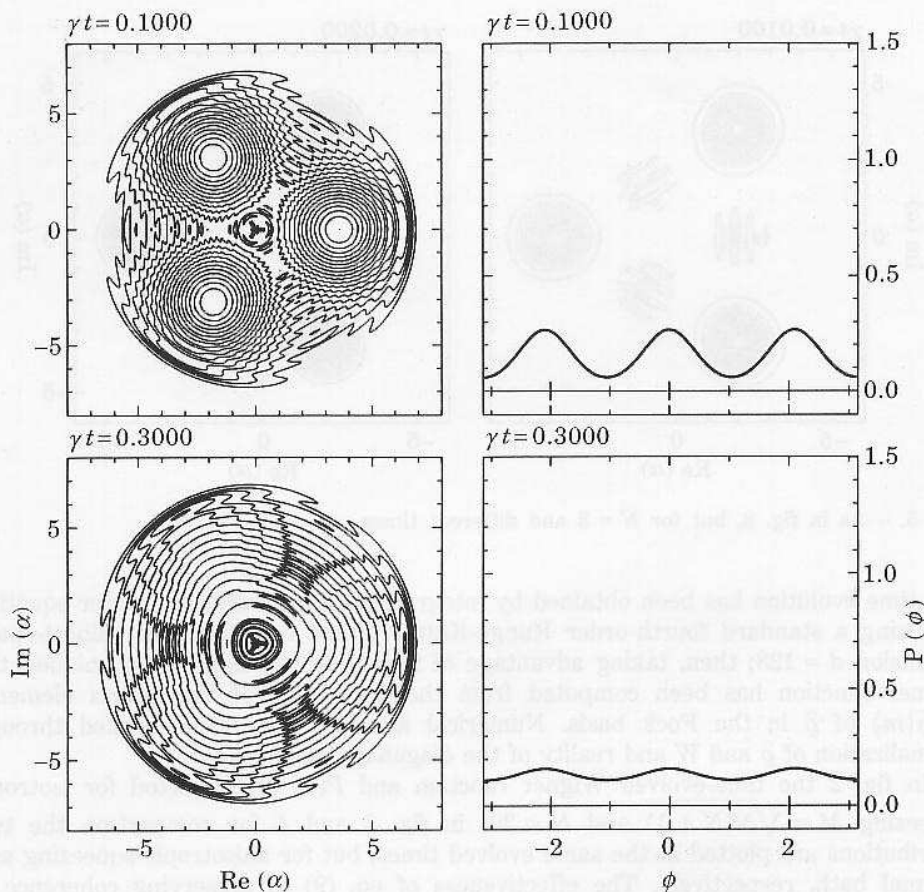


Fig. 4. — As in fig. 2, but for customary thermal bath ( $M = 0$ ).

we are interested in the simplest superposition with more than two components, namely the state obtained for  $t = \pi/3\Omega$ ,

$$(11) \quad |\phi\rangle_3 = \frac{1}{\sqrt{3}} [\exp[-i\pi/6] |\alpha_0 \exp[i\pi/3]\rangle + \exp[i\pi/2] |-\alpha_0\rangle + \exp[-i\pi/6] |\alpha_0 \exp[-i\pi/3]\rangle].$$

In fig. 2 the contour plot of the Wigner function corresponding to the state (11) is given for  $\alpha_0 = -4.0$ . In the following the phase properties of the time evolution of the state  $|\phi\rangle_3$  according to the master equation (9) are investigated and the results are compared with those obtained with the same initial state for the thermal bath—eq. (3) with  $M = 0$ —and for the anisotropically squeezed bath—eq. (3) with  $M^2 = N(N + 1)$ .

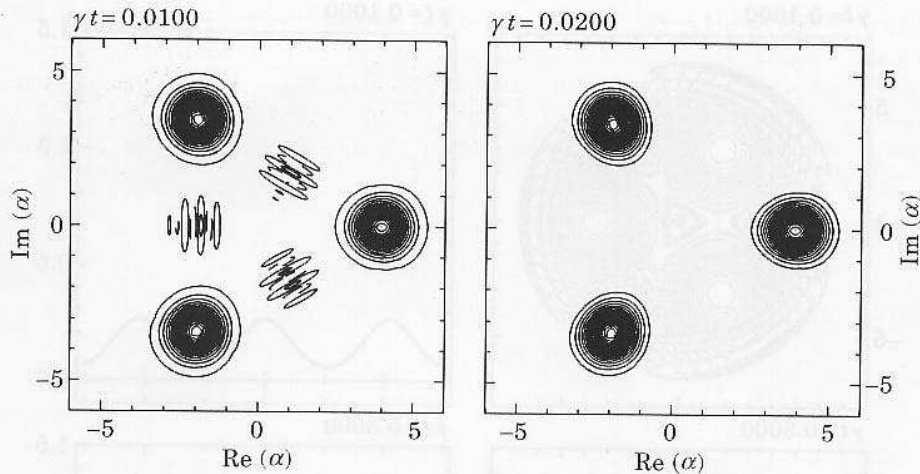


Fig. 5. - As in fig. 2, but for  $N = 3$  and different times.

The time evolution has been obtained by integrating numerically the master equation (9), using a standard fourth-order Runge-Kutta routine for truncated Hilbert-space dimension  $d = 128$ ; then, taking advantage of fast-Fourier-transform techniques, the Wigner function has been computed from the knowledge of the matrix elements  $\langle n|\hat{\rho}|m\rangle$  of  $\hat{\rho}$  in the Fock basis. Numerical accuracy has been checked through normalization of  $\hat{\rho}$  and  $W$  and reality of the diagonal elements of  $\hat{\rho}$ .

In fig. 2 the time-evolved Wigner function and  $P(\phi)$  are depicted for isotropic squeezing  $M = \sqrt{N(N+1)}$  and  $N = 30$ ; in fig. 3 and 4 for comparison the two distributions are plotted at the same evolved times, but for anisotropic squeezing and thermal bath, respectively. The effectiveness of eq. (9) in preserving coherence is evident. For isotropic squeezing the phase distribution  $P(\phi)$  survives for much longer times, and the three peaks remain identical to each other for all times. On the contrary, for anisotropic squeezing the central peak is the only one surviving at long times, whereas the other two are smeared out. Notice that the height of the central peak for anisotropic squeezing is slightly greater than that of the three peaks for isotropic squeezing at the same times: such survival of coherence in the directional case, however, is fortuitous, being strongly dependent on the direction of squeezing with respect to the angular shape of the Wigner function in the complex plane.

In concluding this section, we notice that the effect of the vacuum component of the squeezed bath in washing out quantum interference is stronger for lower values of  $N$  and  $M = \sqrt{N(N+1)}$ . In fig. 5 this is shown for  $N = 3$ , where at  $\gamma t = 0.01$  the interference patterns are weakly visible, whereas at  $\gamma t = 0.02$  they are already totally absent. These plots should be compared with those in fig. 2, which correspond to much longer times.

#### 4. - Summary and conclusions.

In this paper we have suggested a new master equation which squeezes states isotropically in the complex plane. This novel kind of squeezing turns out to be very

absent. These plots should be compared with those in fig. 2, which correspond to much longer times.



effective in increasing the persistence time of coherence, independently of the quantum state. This suggests an improved scheme of detection and generation of Schrödinger-cat states, based on isotropically squeezed reservoirs. The master equation of the isotropically squeezed bath has been derived heuristically: a suitable feedback mechanism should now be envisaged, which supports this new type of dissipative dynamics.

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