Tomographic Measurement of Joint Photon Statistics of the Twin-Beam Quantum State

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We report the first measurement of the joint photon-number probability distribution for a two-mode quantum state created by a nondegenerate optical parametric amplifier. The measured distributions exhibit up to 1.9 dB of quantum correlation between the signal and idler photon numbers, whereas the marginal distributions are thermal as expected for parametric fluorescence.

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The quantum correlation between the modes of electromagnetic radiation has served as a testbed for modern physics by providing an elegant means for verifying the foundations of the quantum theory. Experimentally generated in the process of parametric down-conversion, wherein one pump photon produces a pair of correlated signal and idler photons, such correlation has been used to show the EPR paradox, to test violation of Bell's inequalities, to demonstrate quantum-state teleportation, etc. [1] The realization of three-mode quantum correlation, as exhibited by the Greenberger-Horne-Zeilinger state, emphasizes the difference between the quantum and the classical worlds even more strongly [2]. Therefore, the development of theoretical and experimental tools for studying, measuring, and, ultimately, utilizing the quantum state of a multimode field is an important task in quantum physics. Although the state of a single quantum system cannot be measured, it can be inferred from a set of repeated measurements performed on an ensemble of identically prepared systems [3].

The simplest of the measurement tools in quantum optics, direct detection, allows one to obtain the degree of photon-number correlation between the modes by registering either the coincidence count rate [1] or the difference-photocurrent noise produced by photodetectors measuring the various light beams [4]. In most cases, however, this method does not allow one to measure the quantum statistics of the light modes beyond the knowledge of the first few moments. Since the quantum states of various modes become more sensitive to losses with their increasing mean photon numbers, the observation of inherently quantum features, such as the even-odd oscillations of the photon-number distribution [5], calls for single-photon detection capability. Direct detectors with such capability have serious drawbacks: they either have low quantum efficiencies (as is the case with photomultipliers) or cannot distinguish between registrations of one and more photons (as in avalanche photodiodes). Even more importantly, all direct detectors have poor mode selectivity. By accepting photons from many spatial, temporal, and polarization modes, they provide no means for measuring the photon statistics of a single radiation mode.

A powerful alternative to direct detection that allows a complete measurement of the quantum state of any radiation mode is provided by the method of optical homodyne tomography (OHT), developed over the past decade [6–10]. This method is based on the fact that the knowledge of the field-quadrature probability distributions $\hat{X}^{\phi} = \hat{X}\cos\phi + \hat{Y}\sin\phi$ for all ϕ allows one to reconstruct the Wigner function (joint quasiprobability distribution) $W(X,Y)$ that completely describes the quantum state, and is equivalent to knowledge of the full density matrix. Here, $\hat{X}^{\phi} = [\hat{a}\exp(-i\phi) + \hat{a}^{\dagger}\exp(i\phi)]/2$ is the quadrature operator at phase ϕ, $\hat{X} = [\hat{a} + \hat{a}^{\dagger}]/2$, $\hat{Y} = [\hat{a} - \hat{a}^{\dagger}]/2i$, and $\hat{a}$ and $\hat{a}^{\dagger}$ are the creation and annihilation operators of the mode of interest. The reconstruction procedure is given by the inverse Radon transform [6,7]. The issue of numerical instability of this method has recently been resolved by the advent of an improved reconstruction algorithm called "direct sampling." In this procedure, measurement of the density-matrix elements ρ_{nm} in photon-number representation is reduced to averaging certain "pattern functions" $F_{nm}(X^{\phi},\phi) = f_{nm}(X^{\phi})\exp[i(n-m)\phi]$ over the experimental quadrature outcomes X^{ϕ}, obtained by balanced-homodyne detection, and over the local-oscillator (LO) phases ϕ [8]. The direct sampling procedure is greatly simplified if one is interested only in the diagonal elements of the density matrix, which give the photon-number distribution of the quantum state [9]. This is because the pattern functions for the diagonal elements ρ_{nn} are independent of ϕ; they can be averaged over quadrature outcomes taken at random LO phases, which simplifies the experiment by eliminating the need for phase locking.

As noted above, the quadrature measurement is practically implemented in OHT by means of balanced homodyne detection, wherein a strong coherent-state LO selects a single mode of the field corresponding to its spatial, temporal, and polarization profiles. Moreover, because the strong LO power overcomes both Johnson and dark-current noise, fast *p-i-n* photodiodes having high quantum efficiency can be employed. The issue of high quantum efficiency is extremely important in studies of the states of light that possess inherently quantum features. These features wash out very rapidly with degradation of the

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quantum efficiency from unity. Therefore, single-mode selectivity, single-photon resolution, and high quantum efficiency provided by the OHT method have become an indispensable asset for researchers in the quest for quantum features in photon-number distributions.

Successes in measurements of single-mode states have prompted the application of OHT to studies of multimode quantum states. The theoretical basis for multimode OHT has recently been developed [11,12]. In the case of several modes that cannot be easily separated, direct sampling can be done by employing a single LO scanning across all possible superpositions of the modes [13]. On the other hand, for the modes that can be separated and detected by use of independent random-phase LOs, the multimode direct sampling of the joint photon-number distribution is as simple as averaging a product of the pattern functions for each mode over the simultaneously taken quadrature data: $P(n_1, n_2, \ldots) = \langle f_{n_1 n_1}(X_1^{\phi_1}) f_{n_2 n_2}(X_2^{\phi_2}) \ldots \rangle_{X_1^{\phi_1}, \phi_1, X_2^{\phi_2}, \phi_2, \ldots}$. An experimental application of OHT to the measurement of the two-time correlation function of the classical field emitted by a semiconductor laser has recently been demonstrated [14]. In this paper, we report the first, to the best of our knowledge, application of OHT to the measurement of the joint photon-number distribution of a two-mode quantum state, viz., the twin-beam state emerging from a nondegenerate optical parametric amplifier (NOPA). Our measurement clearly demonstrates the presence of inherently *quantum* features in such a two-mode state.

The quantum correlation is imposed onto the twin (signal and idler) beams by the nature of the parametric scattering process in the NOPA, wherein one pump photon down-converts into a pair of photons that belong to two different modes. This corresponds to the creation of the following two-mode state: $|\Psi\rangle = (\bar{n} + 1)^{-1/2} \times \sum_{n=0}^{\infty} [\bar{n}/(\bar{n} + 1)]^{n/2} |n, n\rangle$, where $\bar{n} = g - 1$ is the average number of photons in each mode with g being the gain of the parametric amplifier. The total photon number in the two modes is always even because of the pairwise nature of the photon creation process. Hence, the probability distribution for the total photon number exhibits even-odd oscillations similar to those for a squeezed-vacuum state [5,10]. Although the photons in the signal and idler modes are perfectly correlated, their statistics in each mode alone are thermal, yielding the following joint probability distribution:

$$P(n, m) \equiv |\langle n, m \mid \Psi \rangle|^2 = \frac{\delta_{nm}}{\bar{n} + 1} \left(\frac{\bar{n}}{\bar{n} + 1} \right)^n, \quad (1)$$

which has zero probabilities everywhere, except along the main diagonal. We have previously demonstrated the thermal character of the photon statistics of either of the twin beams alone [15]. In order to overcome the mode mismatch between the LO and the amplified field, whose spatiotemporal profile is modified during the pulsed traveling-wave amplification process, we employed a self-generated matched LO (self-homodyne tomography). This approach, however, is not suitable for a joint measurement of the twin beams because it renders the NOPA phase sensitive, which distorts the self-generated LO, making the matching of the LO with the mode of the quantum state of interest inefficient. Hence, our observation of quantum features in the joint distribution relies on matching, to the best possible extent, an external LO to the NOPA output.

A schematic of our experimental setup is shown in Fig. 1. The NOPA, consisting of a 5-mm-long $KTiOPO_4$ (KTP) crystal, is pumped by the second harmonic of a Q-switched and mode-locked Nd:YAG laser. The laser produces a 100-MHz train of 120-ps duration pulses at 1064 nm (85 ps for the second harmonic at 532 nm) with a 205-ns wide Q-switch envelope (145 ns for 532 nm) having a 1-kHz repetition rate. The 1064-nm orthogonally polarized twin beams emitted by the KTP crystal NOPA are detected separately by two balanced-homodyne detection setups using two independent LOs derived from the same laser [16]. Low- and high-frequency parts of the resulting photocurrents are separated. The peak amplitudes of the 5-MHz low-pass-filtered photocurrents from the photodiodes in the signal and the idler arms are monitored by an oscilloscope. A 10-MHz-wide band of radio frequencies near $\Omega/2\pi = 40$ MHz is selected in each arm by means of a bandpass filter and amplified with a low-noise amplifier (the level of electronic noise in our setup is about 8 dB below the shot-noise level). The amplified noise photocurrent is then down-converted to the near-dc region by use of an rf mixer and sampled by a boxcar integrator (signal arm, by channel 2; idler arm, by channel 1). The outputs of the boxcar channels are a measure of the quadrature amplitudes X_S^ϕ and X_I^ψ of the signal and idler modes $\hat{A}_S^{(\xi_S)} = [\hat{a}_S(\omega + \Omega)e^{-i\xi_S} + \hat{a}_S(\omega - \Omega)e^{i\xi_S}]/\sqrt{2}$ and $\hat{A}_I^{(\xi_I)} = [\hat{a}_I(\omega + \Omega)e^{-i\xi_I} + \hat{a}_I(\omega - \Omega)e^{i\xi_I}]/\sqrt{2}$, respectively. Here $\omega/2\pi$ is the optical frequency, ϕ (ψ) is the phase of the signal (idler) LO, and ξ_S (ξ_I) is the corresponding phase of the rf LO driving the mixer. The joint photon-number probability distribution $P(n, m)$ of the twin beams is then obtained by averaging the two-mode pattern function $f_{nn}(X_S^\phi) f_{mm}(X_I^\psi)$ over the quadrature

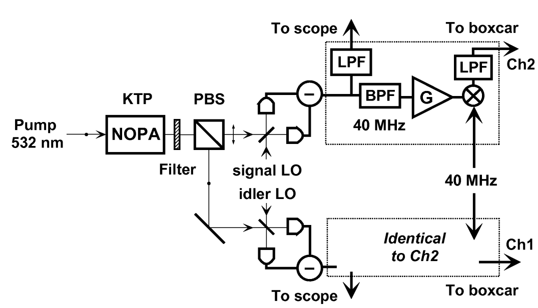

FIG. 1. A schematic of the experimental setup. NOPA, nondegenerate optical parametric amplifier; LOs, local oscillators; PBS, polarizing beam splitter; LPFs, low-pass filters; BPF, bandpass filter; G, electronic amplifier. Electronics in the two channels are identical.

samples X_S^{ϕ} and X_I^{ψ}, and over the independently randomly varying LO phases ϕ and ψ. In our experiment, we vary $(\phi + \psi)$ by moving a piezoelectric transducer in the pump path, and vary $(\phi - \psi)$ by use of an electro-optic phase modulator in the LO path.

The quantum efficiency in our experiment is determined by two factors: (i) the detection and propagation losses and (ii) the efficiency of the homodyne overlap. The former factor is estimated to be about 0.7, which includes the detection and propagation efficiency of 0.85 as well as the effect of nonideal splitting by the polarizing beam-splitting cubes. Estimation of the factor in (ii) above is more difficult because of the evolution of the spatiotemporal beam profile in the process of pulsed traveling-wave parametric amplification [15,17]. In contrast to the case of an OPA in a cavity, where the generated parametric beams have a single-mode Gaussian structure, the beams generated in our OPA are inherently multimode. This brings up the issue of matched LOs for optimizing the correlation between the measured signal and idler photocurrents. In the simplest approximation, our OPA amplifies the input fields at every spatiotemporal point independently, with the gain determined by the pump intensity at that point. Therefore, the twin beams exhibit 100% point-to-point correlation, and the matched LOs are two orthogonally polarized delta-function modes corresponding to the same spatiotemporal point. However, the effects of beam walk-off owing to critical phase matching in the KTP crystal, gain-induced diffraction [17], and limited spatial bandwidth of the NOPA [18] complicate the amplification mechanism, leading to spreading of the correlation within some finite radius. While the theoretical problem of finding a matched LO in this complex case has not yet been solved, we chose the focusing arrangement of the pump and LO beams in our experiment in such a way as to optimize the amount of observed correlation for a given crystal length and pump power. An IR seed signal beam at the input of the NOPA was used for alignment purposes. In this case, the homodyne-overlap efficiency between the LO and the amplified signal beam was measured to be 0.73, and that between the LO and the generated idler varied between 0.55 and 0.60 for the data presented in this paper. The efficiency between the idler and the LO approximates that for homodyning of the quantum noise of twin beams from the unseeded NOPA, at least at low gains [19].

The measured joint photon-number distributions are shown in Fig. 2 (left) for three different values of the pump power (or parametric gain). The less-than-unity quantum efficiency results in the spreading of the distribution around the main diagonal $n = m$, whereas a deltalike correlation is expected for 100% efficiency. The typical marginal distributions for the signal or the idler beam alone are shown in Fig. 3. They indicate good agreement with the theoretically predicted thermal distributions for the same mean photon numbers.

To show the quantum character of the measured distribution, we used it to find the photon-number correlation $d(\Delta n)$ between the two modes [12]:

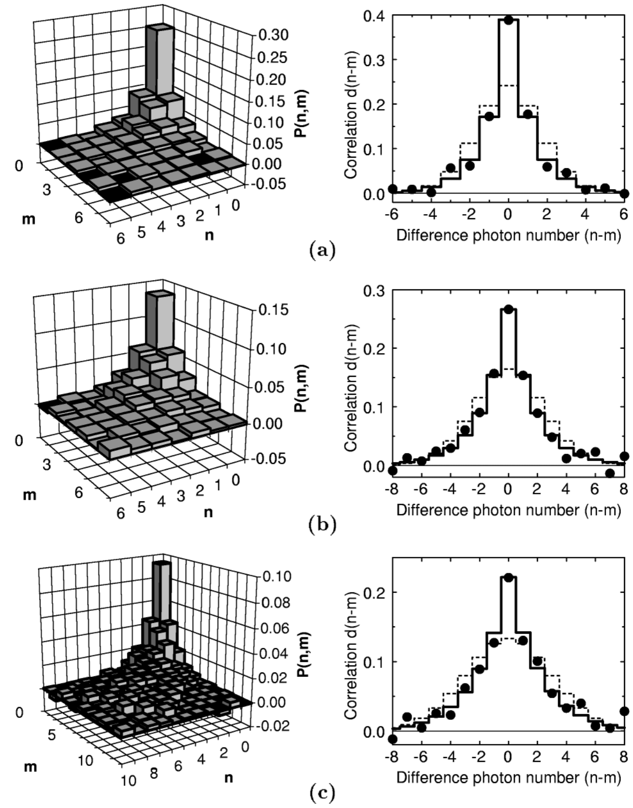

FIG. 2. Left: Measured joint photon-number probability distributions for the twin-beam state. Right: Difference photon-number distributions corresponding to the left graphs (filled circles, experimental data; solid lines, theoretical predictions; dashed lines, difference photon-number distributions for two independent coherent states with the same total mean number of photons and $\bar{n} = \bar{m}$). (a) 400 000 samples, $\bar{n} = \bar{m} = 1.5$, $N = 10$; (b) 240 000 samples, $\bar{n} = 3.2$, $\bar{m} = 3.0$, $N = 18$; (c) 640 000 samples, $\bar{n} = 4.7$, $\bar{m} = 4.6$, $N = 16$.

$$d(\Delta n) = \sum_{k=\max(-\Delta n,0)}^{N} P(k + \Delta n, k), \qquad (2)$$

which is shown in Fig. 2 (right). The number N is determined by the size of our reconstructed distribution. In the limit of $N \to \infty$, $d(\Delta n)$ is the probability of finding the difference between the signal and the idler photon numbers to be Δn. In the case of ideal homodyne detection, $d(\Delta n)$ is expected to be the Kronecker $\delta_{\Delta n 0}$. For a less-than-unity quantum efficiency, however, the correlation $d(\Delta n)$ is no longer a delta function; it spreads around $\Delta n = 0$. Nevertheless, it can be narrower than the correlation function for two independent coherent-state beams having the same total mean photon number as the twin beams. The latter represents the standard quantum limit for photon-number correlation between classical states. In Fig. 2 (right) we compare the photon-number correlation observed in our measurements (filled circles) with the standard quantum limit (dashed lines). The measured twin-beam correlation function is narrower than the coherent-state correlation function, indicating the

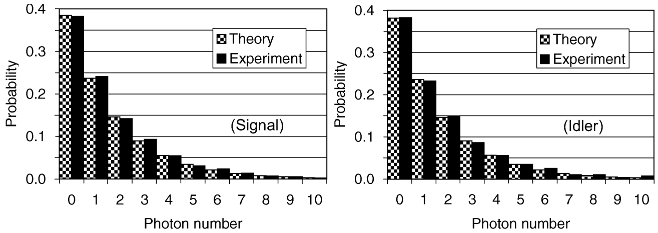

FIG. 3. Marginal distributions for the signal and idler beams reconstructed from the same data as the distribution in Fig. 2(a). Theoretical distributions for the same mean photon numbers are also shown.

inherently quantum character of the twin-beam state. For substantial deviations $|\Delta n|$ from the main diagonal, $d(\Delta n)$ becomes randomly oscillating due to the increasingly large contribution of the statistical errors in the measurement of the joint distribution. We also show the correlation functions (solid lines) reconstructed from the theoretical joint quadrature distributions [12], where the overall quantum efficiency η is used as a fitting parameter, and the contribution of the electronic noise is taken into account. We can see that our experimental data agree well with the theory for (a) $\eta = 0.35$, (b) $\eta = 0.16$, and (c) $\eta = 0.15$. The value $\eta = 0.35$ is also close to our previous estimate of 0.38–0.42 for the overall quantum efficiency. In the case of higher parametric gains [as in (b) and (c)], the discrepancy between the best-fitting η and the estimate based on homodyne overlap between the LO and the idler mean field increases. This is mainly because, at the NOPA output, an imperfect overlap can mix thermal-state rather than vacuum-state noise into the photocurrent, which enhances the detrimental effect of the mismatch, especially at high gains, and leads to an effectively higher degradation of the quantum efficiency from unity. We note here that the degradation of quantum efficiency at high gains can be avoided by employing a matched LO generated by the use of a second identical OPA [19].

To quantify the amount of measured nonclassical correlation between the twin beams, we find the difference photon-number noise normalized to the shot-noise level $F = \mathrm{var}[n - m]/(\bar{n} + \bar{m})$ by averaging appropriate pattern functions over the two-mode quadrature data [20], while subtracting the contribution of the electronic noise. We obtain $F = 0.65$ (-1.9 dB), $F = 0.85$ (-0.71 dB), and $F = 0.86$ (-0.66 dB) for the data used in reconstructions of Figs. 2(a), 2(b), and 2(c), respectively. These clearly satisfy the sufficient condition for nonclassicality of the two-mode correlation, i.e., $F < 1$ [20]. Note that the measured nonclassicality factors F are in excellent agreement with the theoretical limits of $1 - \eta$ for all three values of the parametric gain.

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