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Added noise in homodyne measurement of field observables

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Abstract

Homodyne tomography provides a way for measuring generic field operators. Here we analyze the determination of the most relevant quantities: intensity, field, amplitude and phase. We show that tomographic measurements are affected by additional noise in comparison with the direct detection of each observable by itself. The case of coherent states has been analyzed in detail and earlier estimations of tomographic precision are critically discussed. © 1997 Elsevier Science B.V.

1. Introduction

One of the most exciting developments in the recent history of quantum optics is represented by the so-called *homodyne tomography*, namely the homodyne detection of a nearly single-mode radiation field while scanning the phase of the local oscillator [1–4]. From a tomographic data record, in fact, the density matrix elements can be recovered, thus leading to a complete characterization of the quantum state of the field. This is true also when not fully efficient photodetectors are involved in the measurement, provided that quantum efficiency is larger than the threshold value $\eta = 1/2$.

In homodyne tomography a general density matrix element is obtained as an expectation value over homodyne outcomes at different phases. In formula

$$\langle\psi|\hat{\varrho}|\varphi\rangle = \int_0^\pi \frac{d\phi}{\pi} \int_{-\infty}^{\infty} dx\, p_\eta(x;\phi)\, \langle\psi|K_\eta(x-\hat{x}_\phi)|\varphi\rangle\,, \tag{1}$$

where $p_\eta(x;\phi)$ is the probability density of the homodyne outcome x at phase ϕ for quantum efficiency η and the integral kernel is given by

$$K_\eta(x) = \tfrac{1}{2}\mathrm{Re} \int_0^\infty dk\, k \exp\left(\frac{1-\eta}{8\eta}k^2 + ikx\right)\,. \tag{2}$$

While the kernel in Eq. (2) is not even a tempered distribution, its matrix elements can be bounded functions depending on the value of η. This is the case of the number representation of the density matrix, for which the "pattern function"

$$f^{(\eta)}_{n,n+d}(x,\phi) \equiv \langle n|K_\eta(x-\hat{x}_\phi)|n+d\rangle \tag{3}$$

can be expressed as a finite linear combination of parabolic cylinder functions [3].

As it comes from the experimental average

$$\overline{\varrho_{n,m}} = \int_0^\pi \frac{d\phi}{\pi} \int_{-\infty}^{\infty} dx\, p_\eta(x;\phi)\, f^{(\eta)}_{n,n+d}(x,\phi)\,, \tag{4}$$

the tomographic determination $\overline{\varrho_{n,m}}$ for the matrix element $\varrho_{n,m} \equiv \langle n|\hat{\varrho}|m\rangle$ is meaningful only when its

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