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High efficiency optical tomography using a squeezed vacuum

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Abstract

We show that it is possible to enhance the detection efficiency of homodyne tomography by injecting a broad band squeezed vacuum into the cavity that contains the field whose state one wants to recover before it is contaminated by the injected signal.

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After the seminal work of Vogel and Risken [1] it became clear that one could reconstruct the quantum state of the radiation field from experimental data. Smithey, Beck, Raymer and Faridani [2] were able to give the first image of a squeezed vacuum, plotting its Wigner function obtained from backprojected homodyne data. That image reconstruction was since named "optical homodyne tomography". A homodyne tomography of a single electromagnetic mode a consists of an ensemble of repeated measurements of field quadratures $\hat{a}_\phi = \frac{1}{2}(a^\dagger e^{i\phi} + a e^{-i\phi})$ for various phases ϕ relative to the local oscillator of the homodyne detector. However, the method first employed in Ref. [2] and other methods suggested later [3] need a coarse-graining, because in order to reconstruct the Wigner function from the Vogel and Risken result [1] one has to perform analytical integral transforms of the homodyne probability distributions. In this way a smoothing procedure is required, and this was performed by methods which are standard in tomographic imaging [4]. This procedure sets the resolution with which the Wigner function is determined, and suffers systematic errors related to the filtering cutoff of the backprojection. Moreover, the tomographically reconstructed distributions are all affected by the nonunity efficiency η at detectors, and what is really reconstructed is, at most, a smoothed Wigner function. Hence, the limited quantum efficiency of detectors makes the reconstruction of the Wigner function problematic.

On the other hand, an exact method to reconstruct the density matrix avoiding the evaluation of the Wigner function as an intermediate step and any smoothing procedure on data has been proposed in Ref. [5], and in Ref. [6] the density matrix is reconstructed by averaging a set of sampling functions with respect to the measured quadrature values. Recently, D'Ariano, Leonhardt and Paul [7,8] provided a simple analytic form for the method of Ref. [5], giving an explicit relation between the density operator $\hat{\rho}$ of the field and the tomographic homodyne probabilities, and this relation also holds for generic quantum efficiency η of detectors. They showed, indeed, that

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the density matrix can be experimentally sampled for quantum efficiency $\eta < 1$, but there is still a lower bound for η which depends on the vector basis chosen to represent the density matrix: for number and coherent state representations such bound is $\eta = 0.5$. On the other hand, the overall quantum efficiency of homodyne detection can be enhanced using anti-squeezed preamplification at the photodetector [9], recent results have indicated indeed that noiseless amplification is possible [10].

In this work we will show that part of the overall quantum efficiency can be enhanced by squeezed vacuum techniques, similar to those introduced in Ref. [11] as a way to slow down the destruction of quantum coherence, and in this respect this paper can be considered as a generalization of Ref. [11] suitable for quantum state reconstruction.

In the notion of "quantum efficiency" one should include not only the probability of conversion of photons into electric pulses during the homodyne measurement, but also the vacuum fluctuations entering the cavity used to generate the field state that one wishes to reconstruct. As we will show below, this means that the vacuum fluctuations entering the cavity inhibit the complete reconstruction of the radiation state generated inside. Actually, in the measurement process involving a cavity – which is the most frequently used in quantum optics – one wants that the measured state remains inside the cavity, and, at the same time, a field connected to the intracavity one is detected outside. In the following we will show how this requirement actually limits the overall quantum efficiency of the measurement, whereas it is possible to overcome such efficiency loss by squeezing some vacuum modes outside the cavity. It is clear that the squeezed vacuum entering the cavity generally affects the radiation state inside the cavity after the measurement, so that only a single measurement can be done without contaminating the measured state.

Let us consider a ring cavity with a partially lossy mirror. This latter can be modeled by a beam splitter if the time at which we perform the measurement is shorter than the typical round trip time of the cavity. The lossy mirror is depicted in Fig. 1. The mode a is the intracavity mode, whose density matrix $\hat{\rho}_a$ we want to measure. The free-space mode b is initially vacuum, then it interacts with a at the mirror and transforms into the free-space mode $d = \hat{U}^\dagger b \hat{U}$, which is

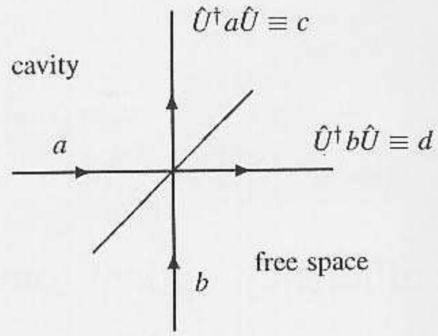

Fig. 1. Scheme of the Heisenberg evolved fields at the lossy mirror of a ring cavity (see text).

now a linear combination of a and b. Analogously, the intracavity mode a interacts with the vacuum mode b at the mirror, and transforms into the new intracavity mode $c = \hat{U}^\dagger a \hat{U}$. Our aim is to measure $\hat{\rho}_a$ through a tomography at the output mode d. This scheme can also be considered as a prototype case of an indirect measurement, and the following arguments also apply to any system coupled to another one on which eventually the measurement is performed [12]. Let us select the path lengths of all involved modes in such a way that the unitary in-out Heisenberg evolution is

$$\begin{pmatrix} c \\ d \end{pmatrix} = \hat{U}^\dagger \begin{pmatrix} a \\ b \end{pmatrix} \hat{U} = \mathbf{M} \begin{pmatrix} a \\ b \end{pmatrix} , \tag{1}$$

with unitary scattering matrix $\mathbf{M}$ given by

$$\mathbf{M} = \begin{pmatrix} \cos\kappa & -\sin\kappa \\ \sin\kappa & \cos\kappa \end{pmatrix} . \tag{2}$$

In Eq. (2) $\cos^2\kappa \equiv 1-\eta$ is the reflection coefficient, and $\sin^2\kappa \equiv \eta$ is the transmission one. The transformation (1) with matrix (2) is achieved by the unitary operator

$$\hat{U} = \exp[\kappa(ab^\dagger - a^\dagger b)] , \tag{3}$$

which can be ordered normally with respect to the mode b by using the Backer–Campbell–Hausdorff formula for the angular momentum group [13]

$$\hat{U} = e^{\tan\kappa ab^\dagger} |\cos\kappa|^{a^\dagger a - b^\dagger b} e^{-\tan\kappa a^\dagger b} . \tag{4}$$

In the Schrödinger picture the joint density matrix of the input fields evolves as follows

$$\hat{R}_{cd} = \hat{U} \hat{\rho}_a \otimes |0\rangle_{bb}\langle 0| \hat{U}^\dagger , \tag{5}$$

and any desired expectation value of operators corresponding only to the evolved mode $d = \hat{U}^\dagger a \hat{U}$ can be obtained with the reduced density matrix

$$\hat{\rho}_d = \mathrm{Tr}_b[\hat{R}_{cd}] \; . \tag{6}$$

Using Eq. (4), it is easy to obtain

$$\begin{aligned}\hat{\rho}_d &= \mathrm{Tr}_b[\mathrm{e}^{\tan\kappa a b^\dagger} |\cos\kappa|^{a^\dagger a} \\ &\qquad \times \hat{\rho}_a \otimes |0\rangle_{bb}\langle 0| |\cos\kappa|^{a^\dagger a} \mathrm{e}^{-\tan\kappa a^\dagger b}] \\ &= \mathrm{Tr}_b[\mathrm{e}^{\tan\kappa a b^\dagger} |\cos\kappa|^{a^\dagger a} \\ &\qquad \times \hat{\rho}_a |\cos\kappa|^{a^\dagger a} |0\rangle_{bb}\langle 0| \mathrm{e}^{-\tan\kappa a^\dagger b}] \\ &= \sum_{k=0}^{\infty} \frac{(-)^k (\tan\kappa)^{2k}}{k!} a^k |\cos\kappa|^{a^\dagger a} \hat{\rho}_a |\cos\kappa|^{a^\dagger a} a^{\dagger k} .\end{aligned} \tag{7}$$

Hence, the two matrices in the number representation are connected as follows

$$\begin{aligned}\langle n|\hat{\rho}_d|m\rangle &= (\cos\kappa)^{n+m} \sum_{k=0}^{\infty} (-)^k (\sin\kappa)^{2k} \\ &\quad \times \left[\binom{m+k}{m} \binom{n+k}{n} \right]^{1/2} \langle n+k|\hat{\rho}_a|m+k\rangle \; ,\end{aligned} \tag{8}$$

or, inversely,

$$\begin{aligned}\langle n|\hat{\rho}_a|m\rangle &= (\cos\kappa)^{-n-m} \sum_{k=0}^{\infty} (\tan\kappa)^{2k} \\ &\quad \times \left[\binom{m+k}{m} \binom{n+k}{n} \right]^{1/2} \langle n+k|\hat{\rho}_d|m+k\rangle \; .\end{aligned} \tag{9}$$

Eqs. (8) and (9) coincide with an analogous result obtained in Ref. [14] in a different context: here, Eqs. (8) and (9) give the way to connect the desired density matrix $\langle n|\hat{\rho}_a|m\rangle$ in the number representation to the measured one $\langle n|\hat{\rho}_d|m\rangle$. Eq. (8) represents a convolution of the number distribution with the binomial probability, which produces the undesired smearing effect due to vacuum fluctuations.

We can evaluate the relation between measured probabilities for the d mode and density matrix of the a mode in a more general way. The moment generating function $\chi_d(r,\phi)$ of the quadrature $\hat{d}_\phi = \frac{1}{2}(d e^{i\phi} + d^\dagger \mathrm{e}^{-i\phi})$ at the output (the phase ϕ is controlled by the experimentalist) can be written in the two equivalent – Schrödinger or Heisenberg – pictures as follows

$$\chi_d(r,\phi) = \mathrm{Tr}(\mathrm{e}^{ir\hat{b}_\phi} \hat{R}_{cd}) = \mathrm{Tr}(\mathrm{e}^{ir\hat{d}_\phi} \hat{\rho}_a \otimes |0\rangle_{bb}\langle 0|) \; . \tag{10}$$

The nth derivative of $\chi_d(r,\phi)$ with respect to the parameter r is connected with the nth moment of the output quadrature $\hat{d}_\phi$. We can easily factor out $\chi_d(r,\phi)$ into two components, each pertaining a single input mode. One has

$$\begin{aligned}\chi_d(r,\phi) &= {}_b\langle 0|\mathrm{Tr}_a[\mathrm{e}^{ir(\sin\kappa\hat{a}_\phi + \cos\kappa\hat{b}_\phi)} \hat{\rho}_a]|0\rangle_b \\ &= \chi_a(r\sqrt{\eta},\phi)\chi_b(r\sqrt{1-\eta},\phi) \; ,\end{aligned} \tag{11}$$

where $\chi_a(\lambda,\phi) = \mathrm{Tr}_a[\mathrm{e}^{i\lambda\hat{a}_\phi}\hat{\rho}_a]$, and $\chi_b(\lambda,\phi) = \langle 0|\mathrm{e}^{i\lambda\hat{b}_\phi}|0\rangle$ and we set $\eta = \sin^2\kappa$ which represents the mirror transmissivity. The Fourier transform of $\chi_d(r,\phi)$ gives the marginal probability distribution $p_d(x,\phi)$ of the quadrature $\hat{d}_\phi$ of the detected output field

$$p_d(x,\phi) = \int_{-\infty}^{+\infty} \frac{\mathrm{d}r}{2\pi} \mathrm{e}^{-irx} \chi_d(r,\phi) \; , \tag{12}$$

and, similarly, $\chi_a(\lambda,\phi)$ and $\chi_b(\lambda,\phi)$ are related to the quadrature probability distributions of their respective modes. Our aim is now to connect the density matrix elements of $\hat{\rho}_a$ with the detected probability distribution $p_d(x,\phi)$: this will give us the desired tomographic formula. We start from the operator identity [7]

$$\hat{\rho}_a = \int_0^{\pi} \frac{\mathrm{d}\phi}{\pi} \int_{-\infty}^{+\infty} \frac{\mathrm{d}r|r|}{4} \mathrm{e}^{-ir\hat{a}_\phi} \chi_a(r,\phi) \; , \tag{13}$$

and we substitute Eq. (11) into Eq. (13), thus obtaining

$$\begin{aligned}\hat{\rho}_a &= \int_0^{\pi} \frac{\mathrm{d}\phi}{\pi} \int_{-\infty}^{+\infty} \frac{\mathrm{d}r|r|}{4} \mathrm{e}^{-ir\hat{a}_\phi} \\ &\quad \times \chi_d\left(\frac{r}{\sqrt{\eta}},\phi\right) \chi_b^{-1}\left(r\sqrt{\frac{1-\eta}{\eta}},\phi\right) \; .\end{aligned} \tag{14}$$

Using Eq. (12) one has

$$\hat{\rho}_a = \int_0^{\pi} \frac{d\phi}{\pi} \int_{-\infty}^{+\infty} \frac{dr|r|}{4} e^{-ir\hat{a}_\phi} \int_{-\infty}^{+\infty} dx\, e^{irx/\sqrt{\eta}} \times p_d(x,\phi)\, \chi_b^{-1}\left(r\sqrt{\frac{1-\eta}{\eta}}, \phi\right) , \quad (15)$$

which provides the desired relation

$$\hat{\rho}_a = \int_0^{\pi} \frac{d\phi}{\pi} \int_{-\infty}^{+\infty} dx\, p_d(x,\phi)\, K\left(\frac{x}{\sqrt{\eta}} - \hat{a}_\phi\right) , \quad (16)$$

where

$$K(x) = \frac{1}{2}\,\mathrm{Re} \int_0^{\infty} dr\, r\, e^{irx} \chi_b^{-1}\left(r\sqrt{\frac{1-\eta}{\eta}}, \phi\right) . \quad (17)$$

For b in the vacuum state, one has

$$\chi_b\left(r\sqrt{\frac{(1-\eta)}{\eta}}, \phi\right) = \exp\left(-\frac{1-\eta}{8\eta} r^2\right) , \quad (18)$$

which gives the same kernel obtained in Ref. [7] for homodyne tomography with quantum efficiency η, i.e.

$$K(x) = \frac{1}{2}\,\mathrm{Re} \int_0^{\infty} dr\, r \exp\left(\frac{1-\eta}{8\eta} r^2 + irx\right) . \quad (19)$$

Hence, our present indirect measurement scheme is equivalent to a direct measurement in the presence of nonunity efficiency. Therefore, we can use the theorem of Ref. [7], namely: for the number representation the matrix elements of $K(x-\hat{a}_\phi)$ are bounded for $\eta > 0.5$, and in this case the density matrix can be statistically sampled from experimental data. In our case, however, η represents the cavity mirror transmissivity, and it is usually very small (i.e. good cavities). We will show now how this problem can be overcome using a low transmissivity mirror, and compensating the effect of loss by means of a squeezed vacuum at the input port b.

For b in a squeezed vacuum $|0,\zeta\rangle = \hat{S}(\zeta)|0\rangle$, with $\hat{S}(\zeta) = \exp[\frac{1}{2}(\zeta(a^\dagger)^2 - \bar{\zeta}a^2)]$ and $\zeta = |\zeta|e^{i\psi}$ one has [15]

$$\chi_b(\lambda,\phi) = \langle 0,\zeta|e^{i\lambda\hat{b}_\phi}|0,\zeta\rangle = \langle 0|\hat{S}^\dagger(\zeta)e^{i\lambda\hat{b}_\phi}\hat{S}(\zeta)|0\rangle = \langle 0|e^{i\lambda\Delta\hat{b}_\phi}|0\rangle = \exp(-\Delta\lambda^2/8) , \quad (20)$$

where

$$\Delta = \cosh|\zeta| + e^{-i(\psi-2\phi)} \sinh|\zeta| . \quad (21)$$

We lock the phase ψ of the squeezing to the homodyne phase ϕ in order to have $\psi - 2\phi = \pi$, namely $\Delta = \exp(-|\zeta|)$. The kernel in Eq. (17) becomes

$$K(x) = \frac{1}{2}\,\mathrm{Re} \int_0^{\infty} dr\, r \exp\left(irx + \Delta^2 \frac{1-\eta}{8\eta} r^2\right) , \quad (22)$$

which is of the same form of Eq. (19), but now with a renormalized quantum efficiency given by

$$\tilde{\eta} = \frac{\Delta^{-2}}{\Delta^{-2} + (1-\eta)/\eta} . \quad (23)$$

The effective efficiency (23) can be made in principle arbitrarily close to unity for sufficiently large squeezing parameter ζ, and, in principle, this can be achieved also for very low cavity transmissivity η. However, this does not mean that the measurement perturbation on the intracavity radiation can be made vanishingly small. In fact, as shown in Ref. [16], such an indirect extracavity measurement produces a von Neumann reduction with strength that depends only on the overall effective quantum efficiency, because the squeezing needed to keep $\tilde{\eta}$ as constant also amplifies the perturbation back to a finite extent.

In conclusion, we have shown that squeezing the vacuum at the unused port of a cavity output mirror, with the phase of the squeezing locked to the local oscillator, allows us to detect the state of radiation inside it, even for low mirror transmissivity, i.e. for good cavities. The measurement however, has to be performed before the input squeezing alters this state, which, on the other hand, should be reprepared after each measurement. This can be considered a technique useful for enhancing the overall quantum efficiency of tomographic detections of a field prepared inside the cavity in a transient regime. It could be useful when one wants to reconstruct the Wigner function of Schrödinger cat states, because the measurements have to be performed in a time much shorter than the decoherence time, which typically goes as the inverse of the cavity damping constant times the average photon number. Of course, within a high finesse cavity, one should inject a broad band squeezed vacuum, with frequency centered around that of the signal mode, otherwise one should have considered a non-white spectrum.

Finally, one could see from Eq. (23), that for example, by using cavity with transmissivity within 10^{-2}–10^{-6}, a squeezed vacuum of the order of 80% allows an improvement of the overall quantum efficiency by a factor 5.

From the above considerations we note how gentle the experimentalist has to master the vacuum when he/she wants to reconstruct quantum states.

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